TASMANIAN ASSESSMENT, STANDARDS & CERTIFICATION

External Assessment 2026

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MATHEMATICS SPECIALISED

MTS415118

Pages: 20
Questions: 30
Answer Booklets: 5
Information Sheet: 1

Preparation time for this exam: 15 minutes

Suggested working time: 3 hours

Instructions:

- There are **five (5)** sections to this exam paper.
- Answer **all** questions and **all** items within each question.
- You **must** show the methods you used to solve questions to receive full marks.
- Answer each section in a **separate answer booklet**. Write the section you are answering on the front cover of each answer booklet.
- Approved calculators and all their functions may be used.
- The exam is **three (3) hours** in length. The suggested working time for each section is **approximately 36 minutes**.
- The Mathematics Specialised Information Sheet can be used through this exam.
- All answers must be written in **English**.
- You **must** make sure your answers address the listed criteria.

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Guide to Exam Structure

	Questions available	Questions to answer	Suggested working time	Marks available
Section A	6	6	36 minutes	36 marks
Section B	6	6	36 minutes	36 marks
Section C	6	6	36 minutes	36 marks
Section D	6	6	36 minutes	36 marks
Section E	6	6	36 minutes	36 marks
Totals	**30**	**30**	**180 minutes (3 hours)**	**180 marks**

Criteria

You **must** make sure your answers address:

- Criterion 4 solve problems and use techniques involving finite and infinite sequences and series
- Criterion 5 solve problems and use techniques involving matrices and linear algebra
- Criterion 6 use differential calculus and apply integral calculus to areas and volumes
- Criterion 7 use techniques of integration and solve differential equations
- Criterion 8 solve problems and use techniques involving complex numbers.

Section A

- Answer **all** questions in this section in a **separate answer booklet**.
- This section is worth 36 marks. The suggested working time for this section is **approximately 36 minutes**.
- This section assesses **Criterion 4**.

Question 1

An arithmetic progression A has first term a and common difference d $(d \neq 0)$.

The 1^{st} , 5^{th} and 15^{th} terms of A are the first three terms of a geometric series.

a) Show that $3a = 8d$. (3 marks)

b) If the sum of the first 10 terms of A is 43, find the value of a. (3 marks)

Question 2

Use mathematical induction to prove that:

$$\frac{1}{1 \times 3} + \frac{1}{2 \times 4} + \frac{1}{3 \times 5} + \cdots + \frac{1}{n(n+2)} = \frac{n(3n+5)}{4(n+1)(n+2)} \qquad \text{for } n \geq 1.$$

(6 marks)

Question 3

a) Show that:

$$\sum_{r=1}^{n} (3r^2 + 8r + 3) = \frac{1}{2} n(2n+5)(n+3)$$

(3 marks)

b) Given that:

$$\sum_{r=1}^{12} [3r^2 + 8r + 3 + \lambda(2^{r-1})] = 3520$$

determine the value of λ. (3 marks)

Question 4

a) Use formal methods to prove that the sequence:

$$\left\{\frac{5n+2}{2n-5}\right\}$$

converges to $\frac{5}{2}$ as $n \to \infty$. (4 marks)

b) Prove that the sequence:

$$\left\{\frac{5n^2+2}{2n-5}\right\}$$

diverges to ∞ as $n \to \infty$. (2 marks)

Question 5

a) Use the method of differences to show that:

$$\sum_{r=2}^{n} \frac{1}{r^2-1} = \frac{3}{4} - \frac{an+b}{2n(n+1)}$$

where a and b are integers to be found. (4 marks)

b) Find the limit as $N \to \infty$ of:

$$N \sum_{r=N+1}^{3N} \frac{1}{r^2-1}$$

(3 marks)

Question 6

a) **Given** (i.e. do not attempt to prove) that the first five terms in the Maclaurin expansion of $\ln(1+x)$ are:

$$\ln(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \frac{1}{5}x^5 + \cdots$$

Write down the first five terms in the Maclaurin expansion of $\ln(1-x)$. (1 mark)

b) Hence, deduce the Maclaurin series for:

$$\ln\left(\frac{1+x}{1-x}\right)$$

(2 marks)

c) Hence, evaluate the sum of:

$$1 + \frac{1}{3 \times 2^2} + \frac{1}{5 \times 2^4} + \frac{1}{7 \times 2^6} + \cdots$$

(2 marks)

Section B

- Answer **all** questions in this section in a **separate answer booklet**.
- This section is worth 36 marks. The suggested working time for this section is **approximately 36 minutes**.
- This section assesses **Criterion 5**.

Question 7

Let **A** and **B** be matrices given by:

$$\mathbf{A} = \begin{pmatrix} 3 & -1 \\ 0 & 1 \end{pmatrix} \quad \text{and} \quad \mathbf{B} = \begin{pmatrix} 4 & 1 \\ 2 & 1 \end{pmatrix}$$

Calculate the matrix:

$$(\mathbf{A}+\mathbf{B})\,(2\mathbf{A}-\mathbf{B})$$

(3 marks)

Question 8

The matrix **C** is given by:

$$\mathbf{C} = \begin{pmatrix} 2 & c \\ -1 & -1 \end{pmatrix} \qquad \text{where } c \neq 2$$

a) Find the inverse matrix $\mathbf{C}^{-1}$. (2 marks)

b) If $\mathbf{C}+\mathbf{C}^{-1} = \mathbf{I}$, where **I** is the identity matrix, determine the value of the constant *c*.

(2 marks)

Question 9

A matrix **P** is given by**:**

$$\mathbf{P} = \frac{1}{\sqrt{2}}\begin{pmatrix} -1 & -1 \\ 1 & -1 \end{pmatrix}$$

a) Describe the single geometrical transformation *U* represented by **P.** (2 marks)

The transformation *V*, represented by the matrix **Q**, is a reflection in the line $y = x$.

b) Write down the matrix **Q**. (1 mark)

The transformation *U* followed by the transformation *V* is the transformation *W*.

The transformation *W* is represented by the matrix **R**.

c) Find the matrix **R**. (2 marks)

d) Describe the single geometric transformation *W*. (2 marks)

e) Hence, or otherwise, write down the inverse of matrix **R**. (2 marks)

Question 10

The points P and Q have coordinates $(1, 2, 3)$ and $(4, 6, -2)$ respectively.

a) Find the equation of the line L_1 that passes through the points P and Q. (2 marks)

A plane Π has the equation:

$$\Pi: \quad x + 2y + z = 24$$

b) Where does L_1 meet Π ? (2 marks)

c) Find the equation of the plane that is parallel to Π and which includes the point Q. (2 marks)

Question 11

The unit circle $x^2 + y^2 = 1$ is transformed by the matrix:

$$\mathbf{G} = \begin{pmatrix} 3 & 2 \\ 5 & -2 \end{pmatrix}$$

a) Find the equation of its image. (4 marks)

b) What is the area of the image? (2 marks)

Question 12

a) Is there a solution for the following system of equations?

$$x - 3y + 2z = 5$$

$$3x + 2y + z = 11$$

$$8x + 9y + z = 20$$

(3 marks)

b) What is the solution if the third equation is replaced by the following equation?

$$9x - 5y + 8z = 37$$

(5 marks)

Section C

- Answer **all** questions in this section in a **separate answer booklet**.
- This section is worth 36 marks. The suggested working time for this section is **approximately 36 minutes**.
- This section assesses **Criterion 6**.

Question 13

Differentiate the following with respect to x:

a) $(\log_2 x)(\arcsin x)$

(2 marks)

b) $$\frac{\cos(2x^3)}{3x}$$

(2 marks)

Question 14

a) Given that:

$$y = 2x(x^2 - 1)^5$$

find the values of x for which the curve is an increasing function. (3 marks)

b) Using the second derivative, find and classify the nature of the stationary points. The y – coordinates are not required.

(4 marks)

Question 15

The curve C is defined by the equation:

$$x = 8y\tan(2y)$$

a) Show that the point P $\left(\pi, \frac{\pi}{8}\right)$ lies on C. (1 mark)

b) Find the equation of the normal to the curve at the point P. (4 marks)

Question 16

Given the function:

$$f(x) = \frac{x}{(x-2)^2} \quad \text{where} \quad x \neq 2$$

a) Determine the location and nature of the stationary point(s) and inflection point(s) of the function.

(4 marks)

b) Sketch the graph of the function, showing all important points and features including asymptotes.

(2 marks)

Question 17

The point Q (2,9) lies on the curve C defined by:

$$y = 3^x$$

The line L is the tangent to the curve at Q.

Find the area bounded by the curve C, the line L and the two coordinate axes. (6 marks)

Question 18

a) Sketch the curve for:

$$y = 4 + \sqrt{x} \qquad \text{where} \qquad 0 \leq x \leq 4$$

(2 marks)

Consider the region R bounded by this curve, the line $y = 6$ and the $y-$ axis.

b) Find the volume generated if R is rotated completely about the $y-$ axis. (3 marks)

c) Find the volume generated if R is rotated completely about the $x-$ axis. (3 marks)

Section D

- Answer **all** questions in this section in a **separate answer booklet**.
- This section is worth 36 marks. The suggested working time for this section is **approximately 36 minutes**.
- This section assesses **Criterion 7**.

Question 19

a) Determine:

$$\int \cos(2\theta)\sin(3\theta)\ d\theta$$

(2 marks)

b) Determine:

$$\int \frac{d\theta}{\cot^2(2\theta)}$$

(2 marks)

Question 20

a) Evaluate:

$$\int_0^2 \frac{dx}{\sqrt{16-x^2}}$$

(3 marks)

b) Evaluate:

$$\int_0^2 \frac{dx}{16-x^2}$$

(3 marks)

Question 21

a) If:

$$I_n = \int_0^1 x\,(1-x^3)^n\,dx$$

then prove that

$$(3n+2)I_n = 3nI_{n-1}$$

(4 marks)

b) Evaluate I_3. (2 marks)

Question 22

Solve the differential equation:

$$2xy^2\frac{dy}{dx} - 1 - y^2 = 0 \qquad \text{given } y = 1 \text{ when } x = 1$$

(5 marks)

Question 23

Determine the solution of the equation for:

$$\frac{dy}{dx} = \frac{3x-4y}{4x+3y} \qquad \text{given } y = 7 \text{ when } x = 1$$

Write your answer in the form $f(x,y) =$ constant. (7 marks)

Question 24

Three different colours of paint are being mixed evenly together in a tank.

Initially there are 20 litres of red paint, 50 litres of green paint and 14 litres of yellow paint in the tank.

The mixture is altered by:

- adding red paint to the tank at a rate of 6 litres a minute
- adding green paint to the tank at a rate of 3 litres a minute
- adding yellow paint to the tank at a rate of 5 litres a minute
- withdrawing fully mixed paint from the tank at a rate of 14 litres a minute.

Let y litres be the volume of yellow paint in the tank, t minutes after the process starts.

a) Explain why the volume of yellow paint satisfies the differential equation:

$$\frac{dy}{dt} = 5 - \frac{y}{6}$$

(2 marks)

b) When is the yellow paint one-third of the total volume of paint in the tank?

Give your answer to the nearest second. (4 marks)

c) What is the composition of the mixture as $t \to \infty$? (2 marks)

Section E

- Answer **all** questions in this section in a **separate answer booklet**.
- This section is worth 36 marks. The suggested working time for this section is **approximately 36 minutes**.
- This section assesses **Criterion 8**.

Question 25

Determine the complex number w that satisfies:

$$w + 3i\,\overline{w} = -1 + 8i$$

(3 marks)

Question 26

In the Argand diagram shown in Figure 1, the points A, B and C are the vertices of an equilateral triangle with its centre at the origin.

The point A represents the complex number $-4 + 2i$.

Find the complex numbers represented by the points B and C, giving your answers in the form $x + iy$.

(3 marks)

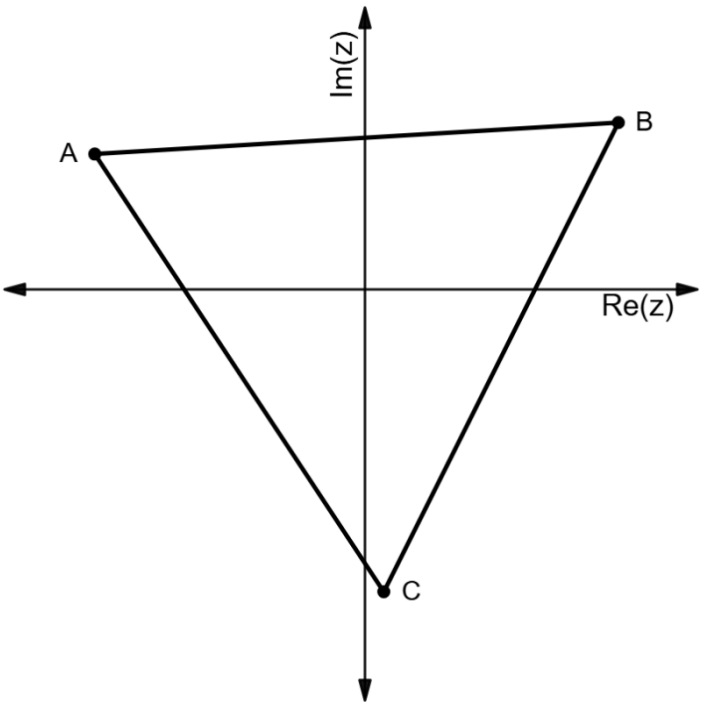


Figure 1: Argand Diagram - Triangle ABC.

Question 27

a) The complex number:

$$z = \frac{a + 3i}{2 + ai} \qquad \text{where} \quad a \in \mathbb{R}$$

If $\arg(z) = \pi/4$ find the value of a. (3 marks)

b) If the complex number w satisfies:

$$|w + z| = |w - z|$$

show that the difference in the arguments of w and z is $\pi/2$. (4 marks)

Question 28

Consider the cubic polynomial:

$$P(z) = z^3 + \alpha z^2 + 6z + \beta$$

a) Given that $z = 5 - i$ is one root of $P(z) = 0$, determine the values of the real numbers α and β.

(3 marks)

b) Write $P(z)$ as the product of a quadratic factor and a linear factor with real coefficients.

(3 marks)

Question 29

a) Sketch a curve on an Argand diagram on which:

$$2|z + i| = |z - 2i|$$

(4 marks)

b) Show on an Argand diagram the set of points given by:

$$\{z: [\mathrm{Im}(z - i)]^2 \leq 1\} \cap \left\{z: |\arg(z) - \pi| \leq \frac{\pi}{4}\right\}$$

(4 marks)

Question 30

a) Write down the five roots of the equation $z^5 = 1$ giving the roots as $z = e^{i\theta}$ with $-\pi < \theta \leq \pi$.

(2 marks)

b) Hence express $z^4 + z^3 + z^2 + z + 1$ as the product of four linear factors.

(2 marks)

c) Given that:

$$(z - e^{i\theta})(z - e^{-i\theta}) = z^2 - 2z\cos\theta + 1$$

deduce that:

$$z^2 + z + 1 + z^{-1} + z^{-2} = \left(z - 2\cos\frac{2\pi}{5} + z^{-1}\right)\left(z - 2\cos\frac{4\pi}{5} + z^{-1}\right)$$

(2 marks)

d) Use the substitution $w = z + z^{-1}$ to derive an exact expression for $\cos\frac{2\pi}{5}$

(3 marks)

End of Exam

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