2023 ASSESSMENT REPORT

MTG315123 GENERAL MATHEMATICS

Section A Bivariate Data Analysis

General Comments

Students were generally quite well prepared for the examination of this section and they appeared to have enough time to complete the section. Students are reminded to carefully read each question then address all the specific requirements of the question. Units should be included in all answers and rulers used when drawing graphs. Students are advised to use words in explanations, not symbols such as # or –.

Question 1

a) This question was well done by most students. Percentages were expected to be given to 1 decimal place so students who rounded to whole numbers were penalised. Students are reminded to stack their divided bar graphs in the same order as the table, so "in favour" should be above "against".

b) This question was very well done by most students. Statements needed to be backed up with data to receive full marks.

c) Similar to Question 1b), this question was very well done by most students. Statements needed to be backed up with data to receive full marks.

Question 2

a) This question was reasonably well done, most students correctly completed the table; however, several did not correctly use the regression formula to find the linear equation.

b) This question was reasonably well done, most students were able to find two points using their equation. Several students forgot to draw the line or incorrectly used data points from the table.

c) This question was very well done, most students remembered to include units.

Question 3

a) This question was quite poorly done by most students, many students calculated the correct value of the gradient; however, few used the correct units.

b) This question was poorly done by most students, many were unable to demonstrate understanding of the gradient in terms of this application.

c) This question was very poorly done by most students, there was a general lack of understanding about the x intercept and some confused it with the y intercept.

d)

i) This question was reasonably well done, some were penalised for being unable to explain that there would be a negative number of swimmers which is impossible.

ii) This question was well done by most students.

e) This question was reasonably well done, few students achieved full marks but most received some credit. Successful students made a comment that included what to expect in the long-term.

f) This question was reasonably well done. As this was a multiple-choice question, students who are unsure of the answer are encouraged to make a guess rather than leave it blank.

Question 4

a) This question was quite well done by most students. Some were penalised for not rounding correctly to 1 decimal place.

b) This question was well done by most students.

c)

i) This question was well done by most students.

ii) This question was poorly done by many students. Common errors were using the residual value from the calculator rather than calculating the residual by hand using the formula and forgetting to plot the residual value on the graph.

iii) This question was reasonably well done by some students. Several students did not state that the actual time was more than the predicted time. Students are encouraged to give their answer in terms of the question rather than the definition of a residual.

d) This question was quite well done.

e) This question was reasonably well done by some students. Several showed a lack of understanding about the meaning of handicap. Successful students were specific in their explanation and referred to the variables in the question.

f) This question was well done by those who understood part e).

Question 5

a) This question was very poorly done. Very few students received full marks by saying how the seasonal index compares to the average quarter.

b) This question was quite well done. Acceptable answers found the winter index by subtracting the other three indices from 4 or dividing an actual winter value by its deseasonalised amount.

c) This question was reasonably well done. Some students were penalised for not showing working.

d) This question was reasonably well done by most. Unfortunately, some students missed this question.

e) This question was quite well done by most students. Many correctly chose Quarter 11 but some were unable to explain its meaning.

f) This question was well done by those who chose the correct formula.

Section B: Growth and Decay in Sequences

General Comments

This section was generally done poorly. Students had difficulty recognising percentage increase or decrease. Students should remember that numbers of people/animals/objects need to be whole and that money should be rounded to 2 decimal places. Some students struggled with using correct terminology and/or explaining their answers. Students should be aware that using tn can be confused for a + by the marker.

Question 6

Whole question generally well done.

a) There was confusion as to which was term 1, but this was not penalised. Most students wrote the 4th term as 1216.7 students.

b) Many students did not read the question carefully and were adding $20m^3$. Many put -20 after the bracket which makes it look like you are subtracting but were able to generate the terms correctly.

c) This part was poorly done. Most common errors included not converting 2.5% to a decimal, treating the $100 deposit as an annuity and adding the $10 account keeping fee.

d) Most students identified "Neither" but had difficulty explaining their answer.

Question 7

This question was very well done with many students gaining full marks. Most common errors were in part d) where students either had difficulty using the formula or added T0 using the calculator app. This was given follow through marks in Q8. Students also often dropped a 0 in one of the numbers.

Question 8

a) Many students divided the big number by the little number instead of doing T2/T1 etc. Some students only divide one pair of terms which doesn't prove a geometric relationship unless you show that it works for the third term.

b) Quite well done.

c) Quite well done.

d) This question was followed through from Q7: if students included T0, they got more than 34km in part c). Many students realised the walker would not make it to their destination, but not all students supported this with calculations.

e) Many students chose 7km or a larger number and used it to show the walker would make the distance but did not use sum to infinity.

Question 9

a) Quite poorly done. Many students referred to features of Aps or GPs, not features of the graph. Students often used the language given which did not really show understanding. Many students gave overly long answers, which sometime contradicted themselves.

b) There were many different valid ways to answer this question. Only the last term needed to be calculated to show that the ratio was correct. Given the initial term was T0, not T1 students need to be careful that their calculation shown gives that answer they are expecting.

c) This was generally well done; however, some students did not recognise that they had been given the ratio in the previous part and used a different number.

d) Generally well done, however some students did not round to a whole number of fish.

e) Some students were confused by the word "rate" and divided terms to get a percentage. Many did not interpret the negative number as a positive decline rate. Again, the use of T0 confused some students who divided the difference by 6 or 4.

f) Mistakes from part e) were carried through to this question. It was generally well done except by students who assumed it was a geometric sequence.

g) Fairly well done. Students should be reminded to show their working and also check the question for a starting year. Many did not convert their answer to a calendar year or did this incorrectly.

h) See part g)

i) This was poorly done. Most students referred to the shape of the graph and explained the relationship, but very few talked about correlation vs causation.

Question 10

Many students left this question blank or stopped part way through. A significant number of students did not fill the tables. An even larger number of students did not attempt the graph even if the other parts were perfect.

The C3 parts of this question were generally well done, with students calculating the long-term membership. However, some students had difficulty describing this as an equilibrium or steady state.

a) This question was generally missed by students, however those who attempted it often did it very well. Many students did not show the asymptotes. While it was appropriate to join the points given that n represent time, it was not necessary for full marks. However, students were penalised for joining with straight lines when they should have been curved. Students should be aware that if their points don't fit the scale given, they have probably made an error.

Section C: Finance

Question 11

a) Poorly done, most candidates ignored the minimum payment, most students used the simple interest formula.

b) Well done, most candidates were able to give some advice on how to avoid "interest fees".

c) Moderately well done, most candidates could identify the correct formula but substituted the wrong numbers, particularly n.

d) Poorly done, most candidates received part marks but missed to refer to the original question.

Question 12

a) Moderately well done, most candidates found the right column (0.005) but not the correct row (120), some candidates calculated f and were unable to reach full marks. Some candidates did not use the table but their calculator in finance mode. It was useful if candidates circled the correct factor in the interest factor table.

b) Mostly well done, although not using the interest factor table, candidates used CAS to find the monthly repayment figure.

c) Mostly well done, although not using the interest factor table, candidates used CAS to find the number of periods, some candidates used the wrong unit (years instead of months).

Question 13

a) Mostly well done, some candidates included a scrap value, some only found R and an equation but did not provide the "equation for the value of the machine". Some candidates misinterpreted the question.

b) Mostly well done, most student added up the hours before continuing with the equation in part a). Some candidates substituted the number of years instead of the number of operating hours.

c) Very poorly done.

d) Mostly well done, some candidates did not calculate 15% of the purchase price (D).

e) Mostly well done, most candidates realised that the unit cost method is not dependent on years, many candidates wrongly thought that this question is comparing straight line depreciation and reducing balance depreciation.

f) Poorly done, a few candidates misinterpreted the question, most candidates did not support their argument with calculations and missed out on full marks. Some candidates found the minimum number of operating hours to 1200 hours.

Question 14

a) Poorly done, either the wrong formula, initial term, r or d or any permutation were not correct.

b) Poorly done, most candidates used term 10 instead of term 40.

c) Poorly done, most candidates lost marks by not subtracting $2500 from the initial deposit.

d) See part b) comment.

e) Poorly done, most candidates did not use the "present value of an annuity" from the information sheet.

f) Poorly done, most candidates were not able to express their understanding of the "present value" and its application in this question.

Question 15

a) Poorly done, most candidates did not know to use the compound interest formula to find the expected price, allowing for inflation. Very few candidates did not know how to do an inflation calculation.

b) Moderately well done, some candidates used annuities in arrears incorrectly.

c) Well done. Marks were awarded for the answer in (a) or the payment in (b) times 36.

d) Moderately well done, most candidates did not "show", transposition was required for full marks or a final result with extra decimal places.

e) Well done.

f) Moderately well done, most candidates provided only 1 benefit for each option.

g) Poorly done, many candidates found the remaining loan without using the annuities in arrears formula or CAS.

Section D Networks or Trigonometry

General Comments

Students were generally quite well prepared for the examination of this section and the time allocation was appropriate. Students understood that they needed to complete just one part, Networks or Trigonometry. Most students chose to complete Part 1 Networks.

Students are reminded that to be successful they must carefully read each question then address the specific requirements of the question. Definitions of key terms such as cycle, trail, cut and dummy edge should be learnt. Take care with terminology; vertices can be even or odd degree not uneven or an odd number of vertices. The use of a highlighter pen or cross hatching method to mark routes is encouraged as those marked over given lines in black ink or pencil are very difficult for markers to see. Spare graphs should be used to correct work. Units are expected in each answer. Students are encouraged to develop a strategy for keeping account of adding weights.

In Trigonometry, students should practise drawing and labelling trig diagrams because some questions cannot be tackled without a detailed diagram.

Part 1 - Networks

Question 16

a) This question was poorly done with students confusing the terminology. Some credit was given for stating Hamiltonian or closed path.

b) This question was reasonably well done. Some students need to take more care to clearly mark their routes.

c) This question was quite well done. Some students received no marks for stating a route rather than the length and others were penalised for not giving units.

d) This question was quite well done. Most students recognised Eulerian but not all remembered trail.

e) This question was quite well done. Most students knew about odd degree vertices, however some explanations were unclear.

f) Students who did well in part e) also did well in this question.

g) This question was generally well done by those successful in parts e) and f).

h) This question was done quite poorly by many students. Unfortunately, several students did not recognise that a minimum spanning tree was needed, and some did not attempt it. Prim's algorithm was done well by students who knew what was required.

i) This question was done well by those who were successful in part h).

Question 17

a) This question was quite poorly done. Some students did not know the definition of a cut and received no credit for describing why cuts were made.

b) This question was very well done.

c) This question was quite well done. Most students correctly labelled 3 out of 4 cuts. A common error was to incorrectly add the Concourse B edge when it was running from sink to source.

d) This question was well done. Some students were penalised for incorrect or no units.

e) This question was reasonably well done. Many students correctly chose the Food Hall but few could correctly explain that the Food Hall is on the minimum cut.

f) This question was poorly done. Many students recognised that Cut B would increase to 170 people/minute but few identified the new minimum cut of 160 people/minute and even fewer stated the increase.

Question 18

a) This question was done reasonably well. Most students received some marks for mentioning the connection between A, B and C; however, few achieved full marks by also mentioning F.

b) This question was quite poorly done. More than half of students did not take the dummy into account.

c) This question was reasonably well done. Some students had difficulty identifying the start of the critical path. Unfortunately, a few students missed this question.

d) This question was quite well done. Full marks were given if the completion time was consistent with the student's completed diagram and correct units stated.

e) This question was quite poorly done. Successful students were able to calculate the available time and use the formula to calculate float.

f) This question was poorly done. Most students correctly stated 2 days, however, few could explain why.

Question 19

a) This question was reasonably well done. Most students could successfully perform the row reduction, however, there were several errors made in the column reduction and the Hungarian algorithm steps.

b) This question was quite well done. Solutions reflecting part a) answers were given full marks.

c) This question was very well done with full marks given for total times consistent with answers to part b).

d) This question was not attempted by some students and very poorly done by others. Most students did not understand what this question was asking. Part marks were given for finding the total time for each person. Some students correctly found the total saving for 4 units but few found the time saved per unit.

Part 2 - Trigonometry

Question 20

a) This question was very well done by most students.

b) This question was very well done by most students.

c) This question was well done. Some students rounded values too soon, resulting in errors later on.

d) This question was very well done.

Question 21

a) This question was very poorly done due to not understanding angle of depression.

b) This question was very poorly done as a result of the diagram in part a) and for choosing the incorrect trip ratio.

Question 22

a) This question was quite well done.

b) This question was quite well done. Some students did not use 650 for the angle at B.

c) This question was quite well done.

Question 23

a) This question was very poorly done. Many students struggled to explain travelling on great circle.

b) This question was quite well done. Errors were made by not calculating θ correctly.

c) This question was quite well done. Some students made the error of not rounding to a whole number of hours.

d) This question was quite well done. A common error was not correctly changing 12 hour time into 24 hour time.

Question 24

a) This question was quite well done. Some students did not recognise this as a great circle distance question and unsuccessfully tried using a trig formula.

b) This question was reasonably well done. Some students incorrectly added rather than subtract to find the latitude of Rome.

OFFICE OF TASMANIAN ASSESSMENT, STANDARDS & CERTIFICATION

External Assessment 2023

[redacted]

GENERAL MATHEMATICS

MTG315123

Section A Bivariate Data Analysis

Pages: 20

Questions: 5

Information Sheet: 1

Preparation time for this exam: 15 minutes

Suggested working time: 48 minutes

Instructions:

- Answer **all** questions and **all** items within each question.
- Write your answers in the spaces provided in this exam paper.
  - Spare diagrams have been provided at the end of each section. Indicate using the box provided if you have used the spare diagram.
- TASC approved calculators are allowed.
- The exam is **three (3) hours** in length. The suggested working time for this section is **approximately 48 minutes**.
- The General Mathematics Information Sheet can be used throughout this exam.
- All answers must be written in **English**.
- You **must** make sure your answers address the listed criteria.

Marker use	
C5	/ 36
C3	/ 12

Guide to Exam Structure

	Parts	Questions available	Questions to answer	Suggested working time	Marks available
Section A		5	5	48 minutes	48 marks
Section B		5	5	48 minutes	48 marks
Section C		5	5	48 minutes	48 marks
Section D	Part 1 OR	4	4	36 minutes	36 marks
	Part 2	5	5	36 minutes	36 marks
Totals		**24**	**19 or 20**	**180 minutes (3 hours)**	**180 marks**

Criteria

You **must** make sure your answers address:

- Criterion 3 apply mathematical and statistical models to investigate, represent and analyse real-world situations and solve problems
- Criterion 5 interpret concepts and apply mathematical techniques to solve problems involving bivariate data analysis and time series analysis using the statistical investigation process.

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Question 1 (approximately 6 minutes)

Marker use

A city council surveys local business owners and local residents to gauge their opinion on a multi-story hotel development that is proposed for a prime waterfront site.

	Local Residents	Local Business Owners	Total
In favour of the development	401	122	523
Against the development	589	12	601
Total	990	134	1124

Table 1

a) Convert the survey data into percentage form in Table 2, and display it on the segmented bar chart in Graph 1.

C5

/4

	Local Residents %	Local Business Owners %
In favour of the development	40.5%	91.0
Against the development	59.5%	9.0
Total	100	100

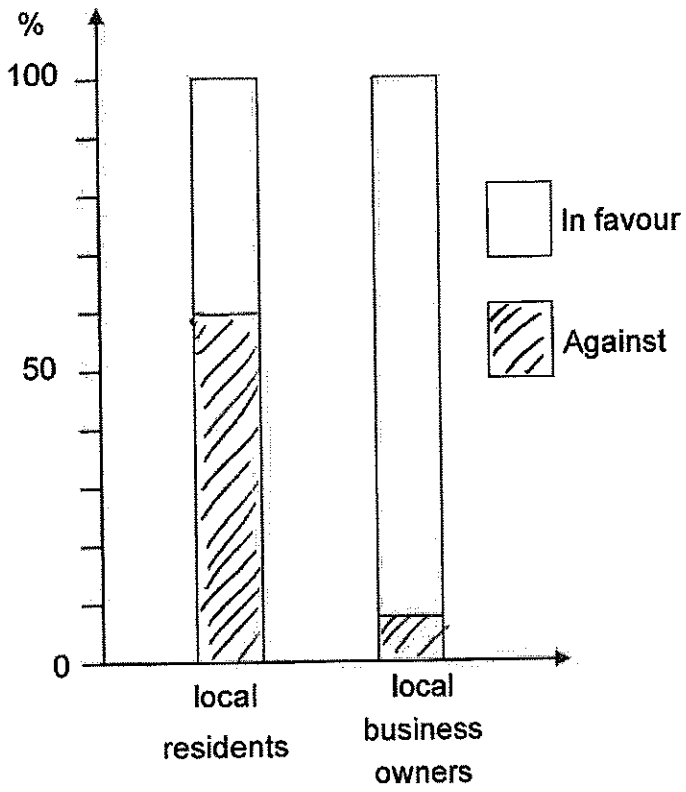
Table 2

$\frac{401}{990} \times 100\% = 40.5\%$

$\frac{589}{990} \times 100\% = 59.5\%$

Question 1 continues

Marker use



Graph 1

Spare diagram used (X) ☐

b) Use the survey data to support the case in favour of the hotel development.

Business owners are united in their support for the development (91% in favour) but residents differ in their opinions (40.5% in favour, 59.5% against)

C3 /1

c) Use the survey data to support the case against the hotel development.

Overall more people were against the development than in favour of it

($\frac{601}{1124}$ = 53.5% against the development)

C3 /1

Total C5 /4

Total C3 /2

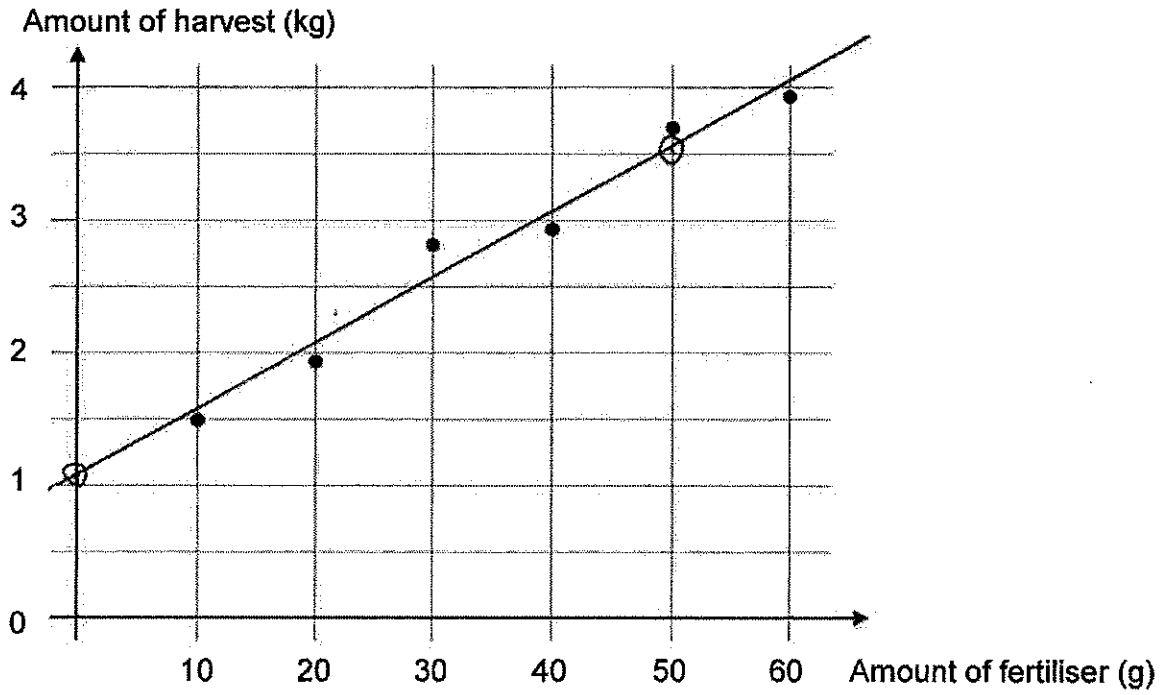
Question 2 (approximately 9 minutes)

Marker use

Six (6) zucchini plants are grown in the same greenhouse, but each is given a different amount of fertiliser. The total harvest (in kg) of zucchini from each plant is shown in the data in Table 3 and Graph 2.

	Amount of fertiliser (g)	Amount of harvest (kg)	XY	X^2
	10	1.5	15	100
	20	1.9	38	400
	30	2.8	84	900
	40	2.9	116	1600
	50	3.7	185	2500
	60	3.9	234	3600
Totals	210	16.7	672	9100

Table 3



Graph 2

Spare diagram used (X) ☐

Question 2 continues

Question 2 continued

Marker use

a) Complete the data in Table 3 and use a formula to determine the linear relationship between amount of fertiliser (f) and size of the zucchini harvest (Z). Express numbers to two (2) decimal places.

C5 /4

gradient $= \frac{n\Sigma xy - \Sigma x \Sigma y}{n\Sigma x^2 - (\Sigma x)^2} = \frac{6(672) - (210)(16.7)}{6(9100) - (210)^2} = \frac{525}{10500} = 0.05$

y intercept $= \frac{\Sigma y - m\Sigma x}{n} = \frac{16.7 - 0.05(210)}{6} = 1.03$

Equation $Z = 0.05f + 1.03$

b) Use your equation to find **two** points on the regression line and use them to locate the regression line on Graph 2.

C5 /3

when $f = 0$ $z = 1.03$ $\therefore (0, 1.03)$ is on graph

when $f = 50$ $z = 3.53$ $\therefore (50, 3.53)$ is on graph

c) Use your equation to estimate the amount of fertiliser necessary to produce a harvest of 3.2 kg of zucchinis.

C5 /2

$3.2 = 0.05f + 1.03$

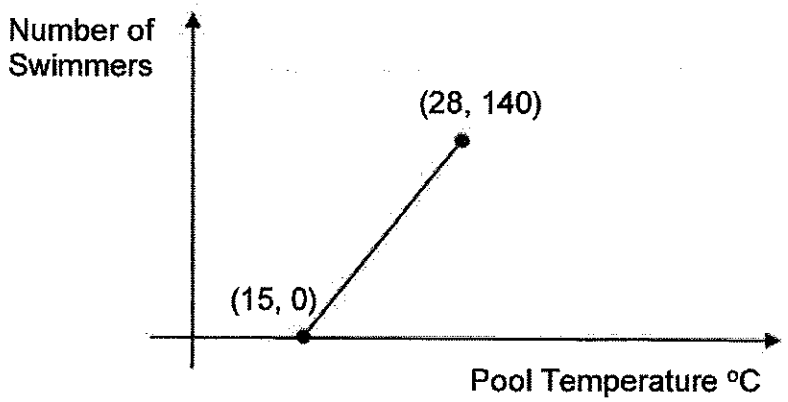
$f = 43.49$

Total C5 /9

Question 3 (approximately 9 minutes)

Marker use

The manager of a swim centre finds that there is a linear relationship between the number of swimmers using the facility each day and the water temperature of the pool as shown by Graph 3 below.



Graph 3

a) What is the gradient of the line? (Include units in your answer.)

C5 /2

gradient = $\frac{140-0}{28-15}$ = 10·77 swimmers/°C

b) What does the gradient represent?

C3 /1

The gradient gives the number of extra swimmers that will use the centre for every degree rise in the water temperature

c) What does the 'x' intercept represent?

C3 /1

The x intercept is the pool temperature at which no swimmers will use the facility

Question 3 continues

Question 3 continued

Marker use

d)

i. Why does the relationship become irrelevant for water temperatures of less than 15°C?

C3 /1

Because it gives negative results. You cannot have a negative number of swimmers

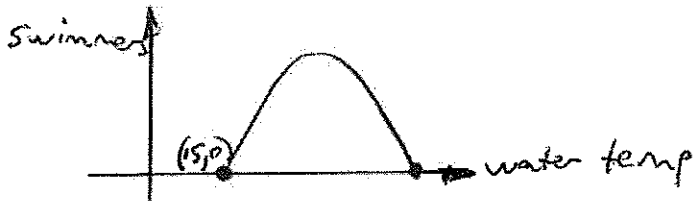
ii. How many swimmers would actually be expected when the water temperature is less than 15°C?

C3 /1

No swimmers ie 0

e) What do you suppose might happen to pool attendance for water temperatures of above 28°C? Illustrate your answer by including a sketch graph.

C3 /2



As the water temperature increases past 28°C it will become uncomfortable or even dangerous for swimming. So it is likely that there will be less swimmers as the temperature increases

f) The manager charges swimmers $4 for pool entry.

A graph is drawn showing the daily takings (in $) against the temperature of the pool (for temperatures between 15°C and 28°C). Which of the following would be true if this graph was compared with Graph 3 at the start of this question.

C3 /1

Tick **the** one (1) correct answer.

- ☐ Its gradient and y intercept would be unchanged.
- ☐ The whole graph would be would be moved up 4 units.
- ☐ Its gradient would be multiplied by 4 but it would have the same y intercept.
- ☑ Its gradient would be multiplied by 4 but it would have the same x intercept.

Total C5 /2

Total C3 /7

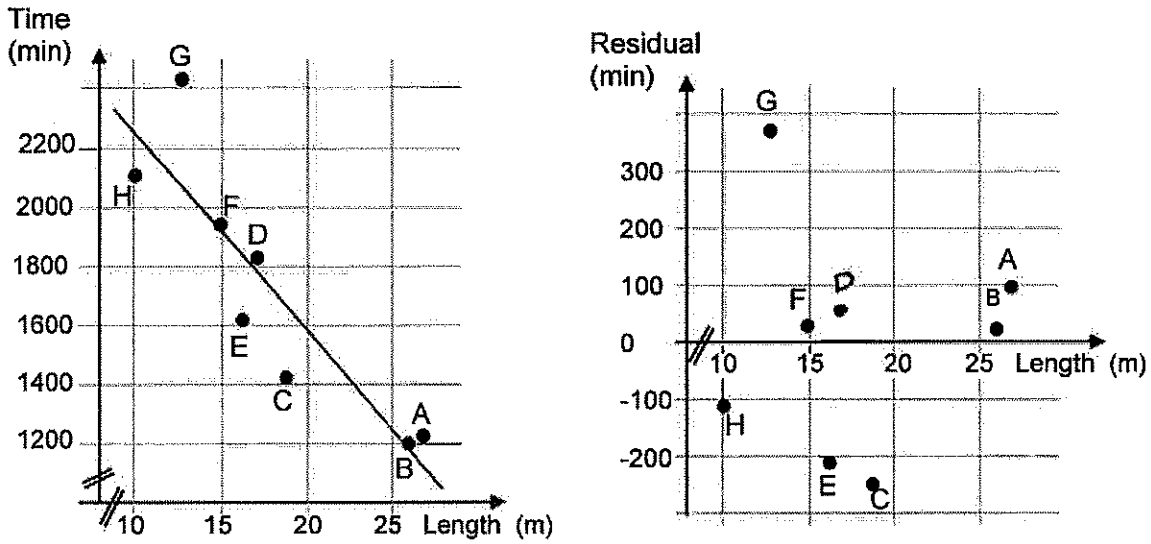
Question 4 (approximately 14 minutes)

Marker use

Table 4 and Graph 4 give information about the length of the each of the yachts in an ocean race and the time taken for them to complete the race.

	Yacht Name	Yacht Length (m)	Time taken to complete race (min)	Residual (min)
A	Gravy Train	26.5	1230	98.4
B	Constellation	25.8	1200	21.4
C	Grace	18.4	1420	-255.4
D	Helter Skelter	17.0	1830	
E	Box Seat	16.2	1613	-210.2
F	Fire Fly	14.7	1950	26.1
G	Unshackled	12.6	2435	370.1
H	Bondi Bullet	10.2	2115	-111.0

Table 4



Graph 4

Spare diagram used (X) ☐

a) Use your calculator to find the equation of the linear relationship between time taken to complete the race (T) and the length of the yacht (L). Express numbers to one (1) decimal place.

C5

/2

..... $T = -67.1L + 2910.9$ .....

Question 4 continued

Marker use

b) Find the correlation coefficient and interpret it in terms of the variables.

C5 /3

$r = -0.8926$ There is a strong negative correlation between the variables. The greater the yachts length, the less the time taken.

c) The residual for '*Helter Skelter*' is missing from the data in Table 4 and Graph 4.

C5 /1

i. Use your equation to predict the time taken for '*Helter Skelter*' to finish the race.

$T = -67.1(17) + 2910.9 = 1770.2$ mins

ii. Use this to find the residual time for '*Helter Skelter*' and plot it on the residuals graph.

C5 /3

Residual = Actual − Predicted

$= 1830 - 1770.2$

$= 59.8$ min

iii. State in words what this residual means.

C3 /1

'Helter Skelter' took 59.8 minutes longer than what the equation predicted

In ocean sailing there are often two (2) trophies presented:

- The 'line honours winner' awarded to the first yacht home, and,
- The 'handicap winner' awarded to the boat who performed best for its size.

d) Which boat would have been the line honours winner?

C5 /1

Constellation (Finishes in least time)

e) Explain how residual analysis could be useful in determining the handicap winner.

C3 /2

The residuals show how much quicker or slower a yacht is than what is expected for its size. So the yacht with the greatest negative residual is performing best for its size.

C5 /1

f) Which boat should be the handicap winner?

'Grace' should be the handicap winner.

Total C5 /11

Total C3 /3

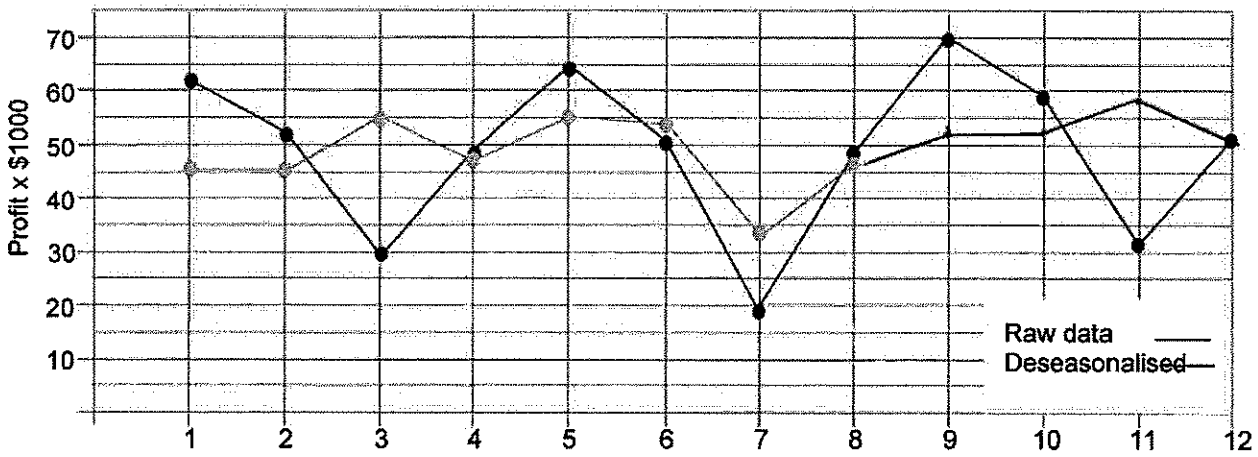
Question 5 (approximately 10 minutes)

Marker use

A shop sells specialist bushwalking equipment. The data in Table 5 and Graph 5 below give information about the profit of the shop both in raw and seasonally adjusted terms.

	Summer	Autumn	Winter	Spring	Summer	Autumn	Winter	Spring	Summer	Autumn	Winter	Spring
Quarter	1	2	3	4	5	6	7	8	9	10	11	12
Profit (x $1000)	62	52	30	48	64	51	18	48	70	58	32	52
De-seasonalised Profit (x $1000)	46.2	47.2	55.4	47.3	47.7	46.3	33.2	47.3	52.2	52.7	58.7	51.3

Table 5



Graph 5

Spare diagram used (X) ☐

Some seasonal indices are shown in Table 6 below.

	Quarter			
	Summer	Autumn	Winter	Spring
Index	1.342	1.101		1.014

Table 6

Question 5 continues

Question 5 continued

Marker use

a) What is a seasonal index?

A seasonal index is a multiplication factor used to normalise seasonal data so that data from one season can be compared with data from another

C5 /1

b) Find the index for winter.

$1.342 + 1.101 + f + 1.014 = 4$

$\therefore f = 0.543$

C5 /2

c) Use the seasonal indices to complete the de-seasonalised data in Table 5.

Summer $= \frac{A}{I} = \frac{70}{1.342} = 52.2$

Autumn $= \frac{A}{I} = \frac{58}{1.101} = 52.7$

Winter $= \frac{A}{I} = \frac{32}{0.543} = 58.9$

Spring $= \frac{A}{I} = \frac{52}{1.014} = 51.3$

C5 /2

d) Complete the graph of deseasonalised data (Graph 5).

C5 /1

e) Which of the quarters 1 – 12 had the greatest sales in de-seasonalised terms? Explain what this means.

(Q11) Although the actual sales for this quarter were low they were the greatest allowing for the fact that winter is a poor time for selling bush walking equipment.

C5 /2

f) The next quarter (Q13) recorded a deseasonalised profit of $44 000. What was the actual profit?

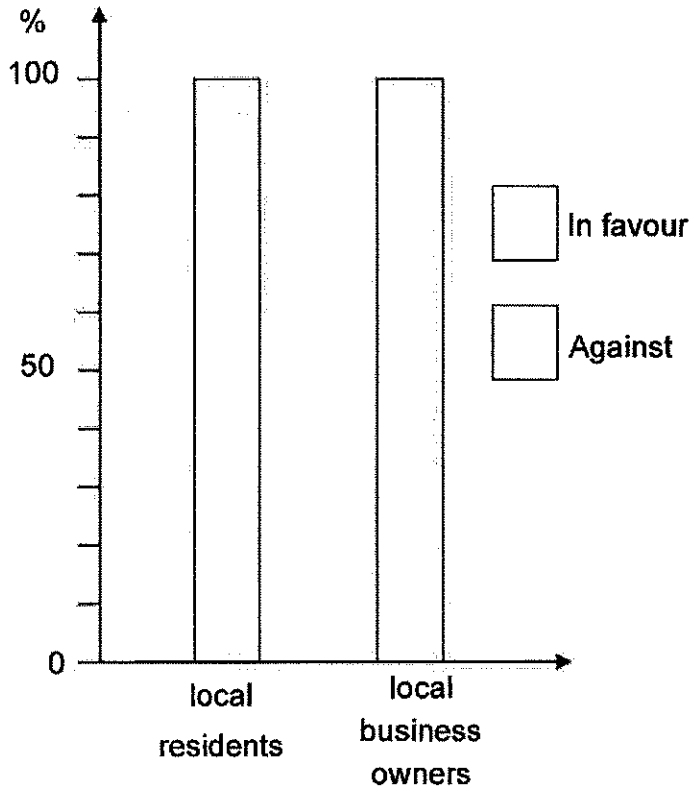
$A = D \times I = 44000 \times 1.342 = \$59048$

C5 /2

Total C5 /10

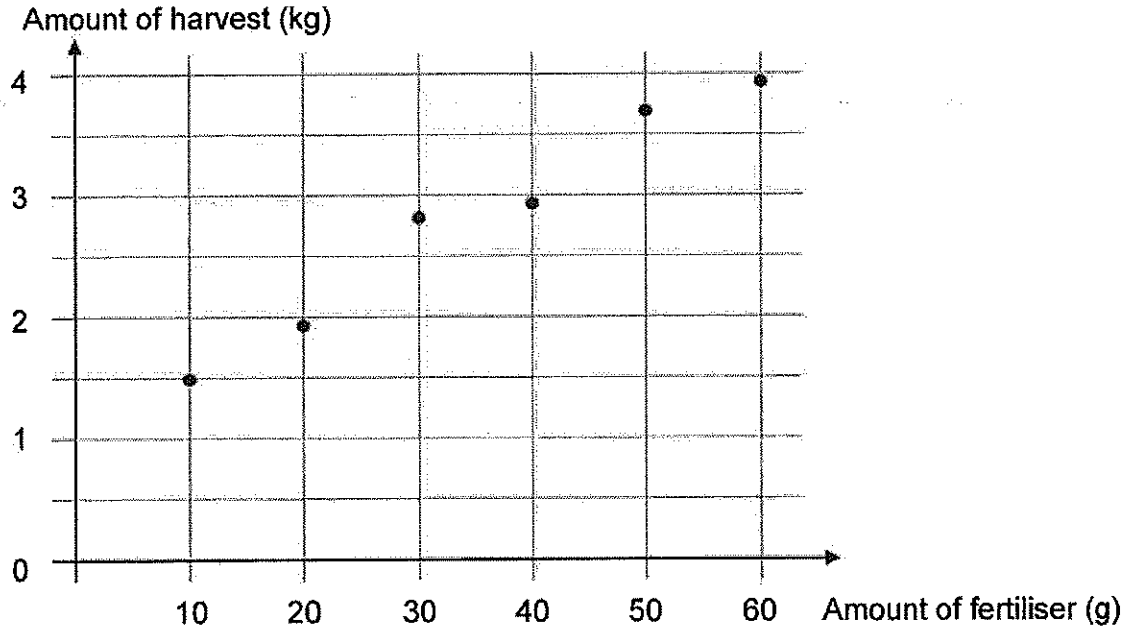
Spare Diagrams

Question 1 a)

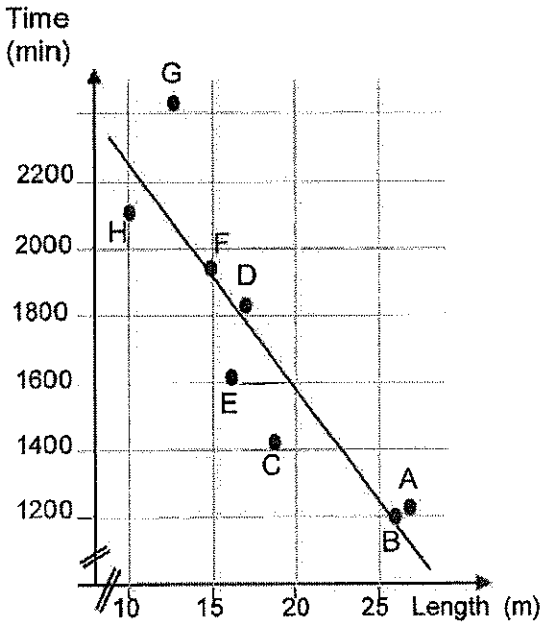


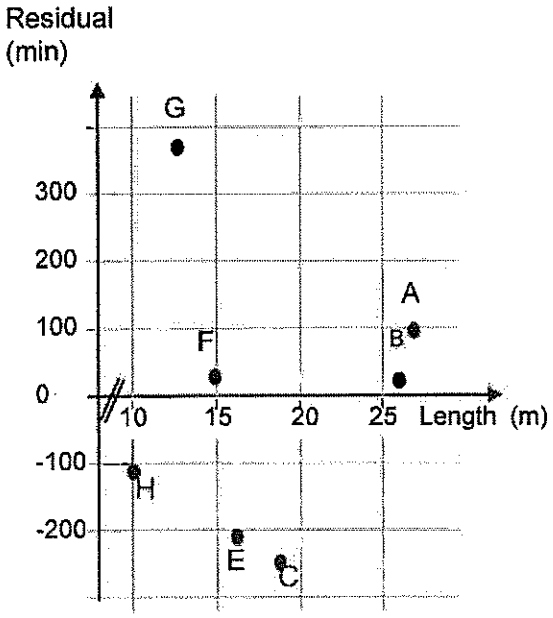
Spare Diagrams

Question 2 b)



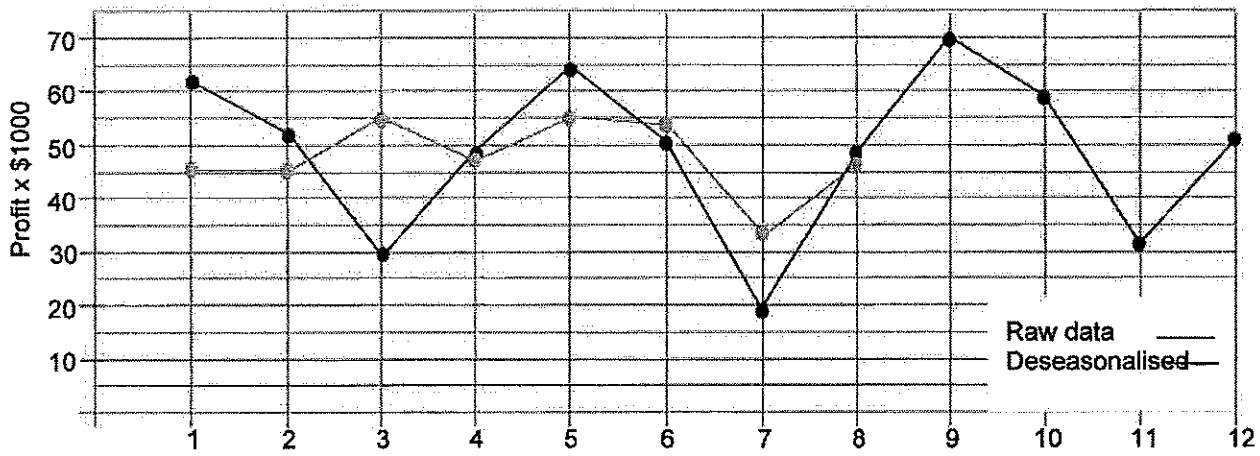
Question 4 c)





Spare Diagrams

Question 5 d)



End of Section A

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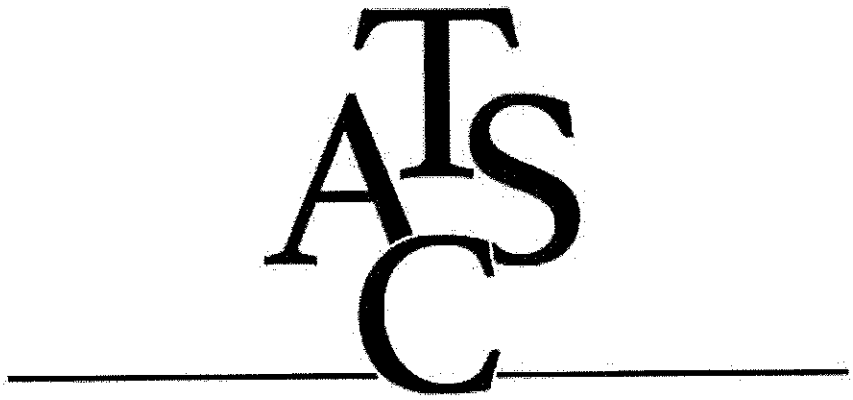

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External Assessment 2023

[redacted]

GENERAL MATHEMATICS

MTG315123

Section B Growth and Decay in Sequences

Pages: 12

Questions: 5

Information Sheet: 1

Suggested working time: 48 minutes

Instructions:

- Answer **all** questions and **all** items within each question.
- Write your answers in the spaces provided in this exam paper.
  - Spare diagrams have been provided at the end of each section. Indicate using the box provided if you have used the spare diagram.
- TASC approved calculators are allowed.
- The exam is **three (3) hours** in length. The suggested working time for this section is **approximately 48 minutes**.
- The General Mathematics Information Sheet can be used throughout this exam.
- All answers must be written in **English**.
- You **must** make sure your answers address the listed criteria.

Marker use	
C6	/ 36
C3	/ 12

Guide to Exam Structure

	Parts	Questions available	Questions to answer	Suggested working time	Marks available
Section A		5	5	48 minutes	48 marks
Section B		5	5	48 minutes	48 marks
Section C		5	5	48 minutes	48 marks
Section D	Part 1 OR	4	4	36 minutes	36 marks
	Part 2	5	5	36 minutes	36 marks
Totals		**24**	**19 or 20**	**180 minutes (3 hours)**	**180 marks**

Criteria

You **must** make sure your answers address:

- Criterion 3 apply mathematical and statistical models to investigate, represent and analyse real-world situations and solve problems
- Criterion 6 interpret concepts and apply mathematical techniques to model patterns and solve problems involving growth and decay in sequence.

Question 6 (approximately 7 minutes)

Marker use

A school has 800 students, but it is expected that the student population will grow by 15% every year. Consider the sequence formed by the annual student population.

a) Write the sequence rule and use it to generate the first four terms.

C6 /2

$T_n = 800(1{\cdot}15)^{n-1}$

$T_1 = 800$ $T_2 = 800(1{\cdot}15)^1 = 920$ $T_3 = 800(1{\cdot}15)^2 = 1058$

$T_4 = 800(1{\cdot}15)^3 = 1216{\cdot}7 \approx 1217$

b) A stockpile of iron ore 560 m^3 in size is being moved by dump-trucks which can carry 20 m^3 each load. Consider the sequence formed by the changing size of the pile.

C6 /2

Write the sequence rule and use it to generate the first four terms.

$T_n = 560 - (n-1)20$ or $T_n = 580 - 20n$

$T_1 = 560$ $T_2 = 540$ $T_3 = 520$ $T_4 = 500$

c) A bank account pays 2.5% p.a. interest which is compounded annually. A deposit of $100 is made into the account. Every year the bank charges a $10 account keeping fee. Consider the sequence formed by the annual balance of the account.

C6 /2

Write a difference equation that models the sequence and use it to generate the first four terms.

$T_{n+1} = 1{\cdot}025\,T_n - 10$, $T_0 = 100$

$T_0 = 100$ $T_1 = 92{\cdot}5$ $T_2 = 84{\cdot}81$ $T_3 = 76{\cdot}93$

d) Is the sequence generated in item c) arithmetic, geometric or neither? Explain your answer.

C3 /1

Neither arithmetic nor geometric

No common difference - so not arithmetic

No common ratio - so not geometric

Total C6 /6

Total C3 /1

Question 7 (approximately 4 minutes)

A new recruit has a salary package starting at \$58 000, but with scheduled annual increases of \$2100 for the next ten years.

Marker use

a) What mathematical term could be used to describe the sequence formed? C3

Arithmetic sequence (or linear increase) /1

b) Write, in algebraic form, a rule for the sequence. C6

$T_n = 58000 + (n-1)2100$

$= 2100n + 55900$ /1

c) Find the salary in the 10th year of work. C6

$T_{10} = 2100(10) + 55900$

$= \$76900$ /1

d) Find the total paid to the recruit over the 10-year period. C6

$S_n = \frac{n}{2}[2a + (n-1)d]$ \$674500

$S_{10} = \frac{10}{2}[2(58000) + 9(2100)] = \$$ ~~\$_____~~ /1

Total C6 /3

Total C3 /1

Question 8 (approximately 7 minutes)

Marker use

A bushwalker who is attempting to reach a mountain cabin that is 34 km away walks:

- 6 km in the first hour,
- 4.8 km in the second hour,
- 3.84 km in the third hour.

a) Show that the terms form a geometric sequence.

C6 /1

Consider $T_2/T_1 = 4.8/6 = 0.8$

Consider $T_3/T_2 = 3.84/4.8 = 0.8$

$\therefore$ GP as terms have common ratio $(r = 0.8)$

b) Find the distance walked in the 10th hour.

C6 /1

$T_{10} = 6(0.8)^{10-1} = 0.805$ km

c) Find the total distance covered after 10 hours of walking.

C6 /1

$S_n = \frac{a(1-r^n)}{1-r}$

$S_{10} = \frac{6(1-0.8^{10})}{1-0.8} = 26.779$ km

d) Will the walker ever reach his destination? (Assume that he can keep walking forever). Support your answer with calculations.

C3 /2

$S_\infty = \frac{a}{1-r} = \frac{6}{1-0.8} = 30$ km

No he will not reach the cabin that is 34 km away. Even if he walks forever he will only cover 30 km

e) Given that the walker's rate of fatigue remains unchanged, find the minimum distance that he should have covered in the first hour to make reaching the cabin a possibility.

C6 /2

Require $S_\infty = 34$

$\therefore \frac{a}{1-0.8} = 34$

$\frac{a}{0.2} = 34$

$a = 6.8$ km

$\therefore$ Must cover 6.8 km in the first hour.

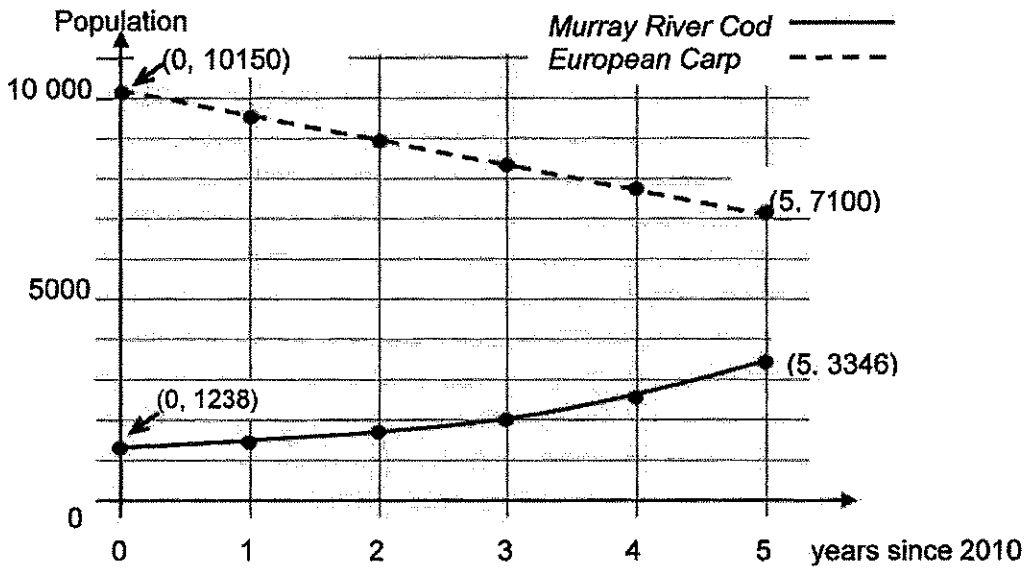
Total C6 /5

Total C3 /2

Question 9 (approximately 13 minutes)

Marker use

The *Murray River Cod* is a native Australian freshwater fish. Last century the *Murray River Cod* was listed as endangered. The depletion of numbers was thought to be due to the introduction of another fish species - the *European Carp*. A program was introduced in 2010 to cull the numbers of *European Carp*. Graph 2 below shows how the estimated populations of both fish in a river reserve have changed since the introduction of the program.



Graph 1

a) What features of the graph show that:

i. *Murray River Cod* population numbers are showing exponential increase.

Graph is curved upwards - increasing distance between terms

C3 /1

ii. *European Carp* population numbers are showing linear decline.

Graph is a straight line sloping down - constant difference between terms.

C3 /1

b) Show that the rate of exponential growth for *Murray River Cod* is 22% p.a.

C6 /2

$T_0 = 1238$

$T_1 = 1238(1.22) = 1510$

$T_2 = 1510(1.22) = 1843$ Which confirms with

$T_3 = 1510(1.22) = 2248$ graph

$T_4 = 2248(1.22) = 2743$

$T_5 = 2743(1.22) = 3347$

Question 9 continues

Question 9 continued

Marker use

c) Find a difference equation that models the population numbers of *Murray River Cod*.
Use initial term T_0 =

$T_{n+1} = 1.22T_n \quad T_0 = 1238$ C6 /1

d) Estimate the number of cod in 2020.

$T_{10} = 9043$ (Calculator sequence mode) C6 /1

e) Find the annual rate of decline in *European Carp* numbers.

$D = \frac{7100 - 10150}{5 - 0} = -610$ Fish/year C6 /2

f) Find a difference equation that models the population numbers of *European Carp*.
Use initial term T_0 =

$T_{n+1} = T_n - 610 \quad T_0 = 10150$ C6 /1

g) Estimate the year when there will no longer be a population of carp.

$T_{16} = 390 \quad T_{17} = -220$ ∴ 2027 will be the first year with no carp population C6 /1

h) Find the year in which cod numbers exceed carp numbers.

Cod $T_8 = 6076$ Carp $T_8 = 5270$
∴ Year will be 2018 C6 /1

i) Does the data show that removing European Carp has caused an increase in *Murray River Cod* numbers. Explain your answer.

No! There is a strong negative correlation between carp numbers and cod numbers - The greater the carp the lower the cod but cause and effect cannot be assumed on the basis of correlation. C3 /2

Total C6 /9

Total C3 /4

Question 10 (approximately 17 minutes)

Marker use

a) A lawn bowls club has 800 members but each year its membership declines by 20%.

i. Write a difference equation for the situation. Use initial term T_0 =

$T_{n+1} = 0.8T_n \quad T_0 = 800$ C6 /1

ii. Set up the difference equation on your calculator and use it to complete Table 1 below. C6 /1

Years	0	5	10	15
Membership	800	262	86	28

Table 1

iii. Describe what happens to membership numbers over time. C3 /1

Membership is in exponential decline
Eventually there will be no members

b) A marketing analyst estimates that if the club promotes itself with television advertising 100 new members will join the club each year (while still losing 20% of the older members).

i. Write a difference equation for this situation. Use initial term T_0 =

$T_{n+1} = 0.8T_n + 100 \quad T_0 = 800$ C6 /1

ii. Set up the difference equation on your calculator and use it to complete Table 2 below. C6 /1

Years	0	5	10	15
Membership	800	598	532	511

Table 2

iii. Describe what happens to membership numbers over time.

Membership declines towards a long term steady state. Eventually there will be 500 members. C3 /1

Question 10 continues

Marker use

c) The analyst estimates that if the club renovates the club rooms and conducts the television advertising then 240 new members will join each year (while still losing 20% of the older members).

i. Write a difference equation for this situation. Use initial term T_0 =

$T_{n+1} = 0.8T_n + 240 \quad T_0 = 800$ C6 /1

ii. Set up the difference equation on your calculator and use it to complete Table 3 below. C6 /1

Years	0	5	10	15
Membership	800	1069	1157	1186

Table 3

iii. Describe what happens to membership numbers over time. C3 /1

Membership increases towards a long-term steady state. Eventually there will be 1200 members.

d) The club's committee decide that what they would really like is to attract just enough new members each year to replace the 20% of members who are leaving the club.

i. Write a difference equation for this situation. Use initial term T_0 = C6 /2

Required annual gain x = annual loss

$x = 0.2(800) = 160$

$T_{n+1} = 0.8(T_n) + 160 \quad T_0 = 800$

ii. What mathematical name is given to this type of situation? C3 /1

Equilibrium state (or steady state)

iii. Complete Table 4 below. C6 /1

Years	0	5	10	15
Membership	800	800	800	800

Table 4

Question 10 continues

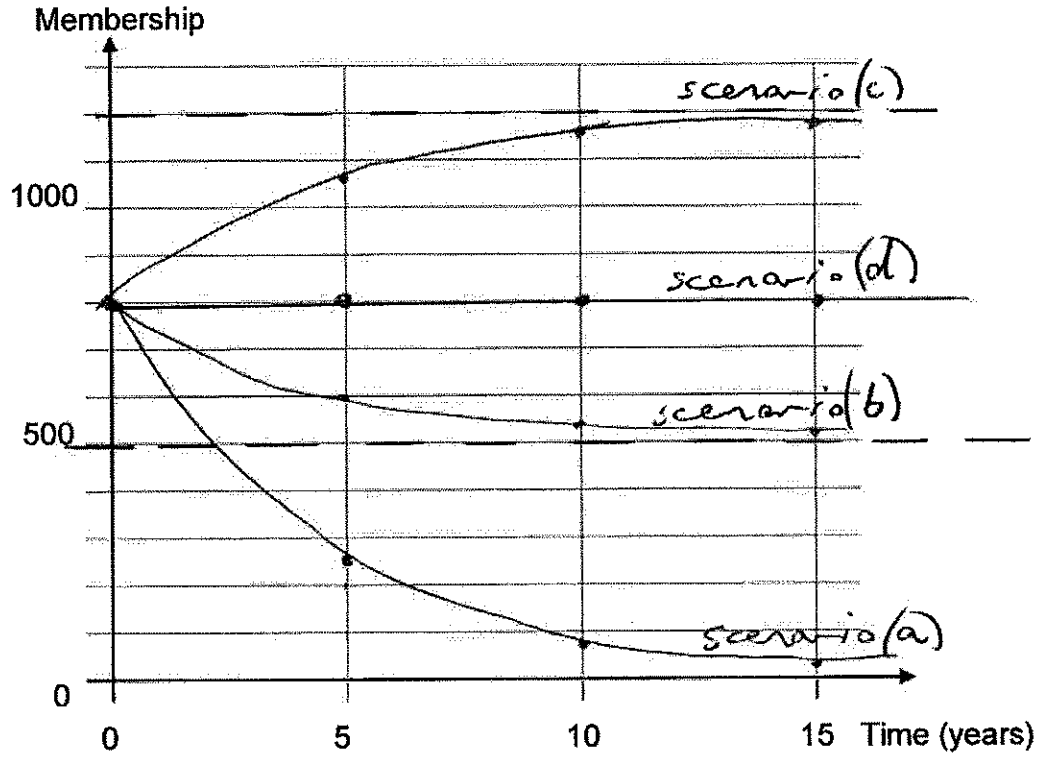
Question 10 continued

Marker use

C6

/4

e) Complete Graph 1 below showing the four (4) scenarios (a) to (d). Show clearly the shape of each graph and any asymptotes or limits.



Graph 2

Spare diagram used (X) ☐

Total C6

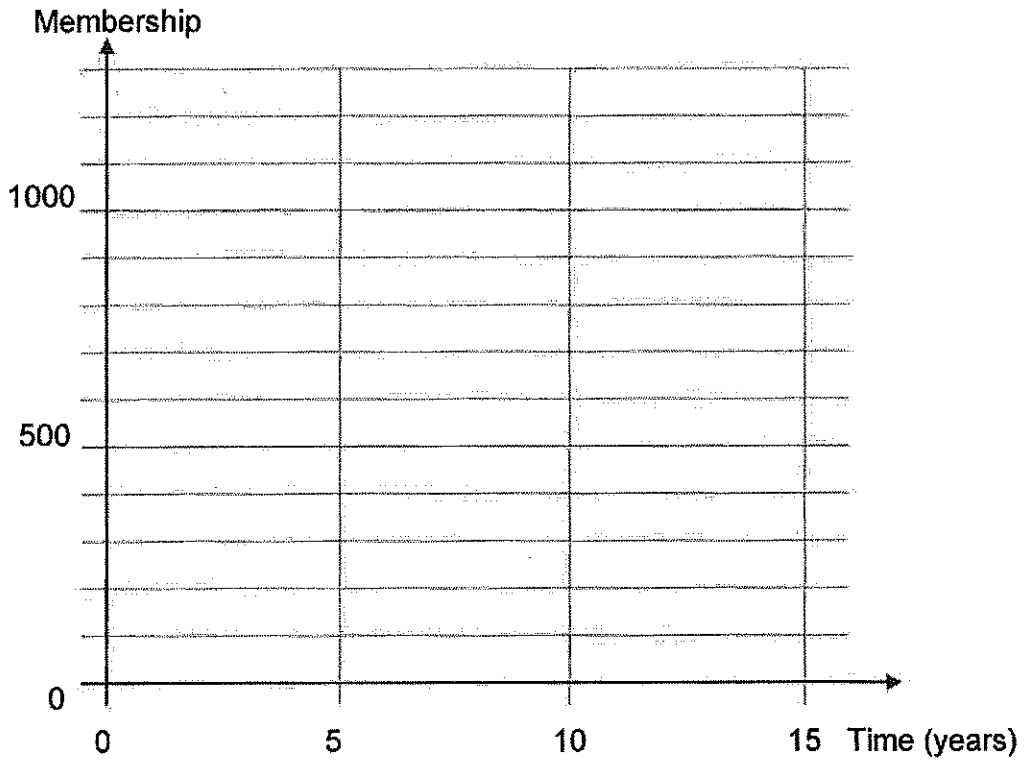
/13

Total C3

/4

Spare Diagrams

Question 10 e)



End of Section B

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External Assessment 2023

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GENERAL MATHEMATICS

MTG315123

Section C Finance

Pages: 12
Questions: 5
Information Sheet: 1

Suggested working time: 48 minutes

Instructions:

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C7	/ 36
C3	/12

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Criteria

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- Criterion 3 apply mathematical and statistical models to investigate, represent and analyse real-world situations and solve problems
- Criterion 7 interpret concepts and apply mathematical techniques to solve problems involving standard financial models.

Question 11 (approximately 7 minutes)

Marker use

Vanessa's credit card has an annual interest rate of 17.2% p.a. compounded daily. Before purchasing a laptop computer for $1350 Vanessa owed nothing on her credit card account. When Vanessa receives her account statement she notes that the minimum payment required is 3.5% of the balance – which she pays by the due date. When Vanessa gets her next statement she finds that she is charged 30 days interest on the outstanding balance.

a) How much interest was charged?

C7 /3

Payment required = 3.5% of 1350 = $47.25

∴ Balance = 1350 − 47.25 = $1302.75

$FV = 1302.75\left(1 + \frac{.172}{365}\right)^{30} = \$1321.29$

∴ Interest = $1321.29 − 1302.75 = $18.54

b) What advice can you give Vanessa for avoiding interest fees on her credit card account?

C3 /1

Avoid paying interest by paying the account in full by the due date

c) Use an algebraic formula to find the effective interest rate that Vanessa is paying on her credit card.

C7 /1

$E = \left(1 + \frac{.172}{365}\right)^{365} - 1 = 18.76\%$

d) Explain, with reference to this question, what is meant by the term 'effective interest rate'.

C3 /2

Every day $\frac{1}{365}$th of the annual interest rate is charged to the account but as this is added to the balance outstanding in subsequent periods the principal will be greater so over the course of the year more than 17.2% interest will be charged.

Total C7 /4

Total C3 /3

Question 12 (approximately 6 minutes)

Marker use

The following interest factors in Table 1 can be used in situations where monthly compound interest on a loan is calculated:

Annuities in arrears interest factor table: $f = \frac{[1 - (1+i)^{-n}]}{i}$

Present value of a $1 regular payment

Number of periods	Interest rate per period	0.00083	0.00167	0.00250	0.00333	0.00417	0.00500	0.00583	0.00667
	1	0.9992	0.9983	0.9975	0.9967	0.9959	0.9950	0.9942	0.9934
	2	1.9975	1.9950	1.9925	1.9900	1.9876	1.9851	1.9826	1.9802
	3	2.9950	2.9900	2.9851	2.9801	2.9752	2.9702	2.9653	2.9604
	4	3.9917	3.9834	3.9751	3.9669	3.9587	3.9505	3.9423	3.9342
	5	4.9875	4.9751	4.9627	4.9504	4.9381	4.9259	4.9137	4.9015
	6	5.9825	5.9652	5.9478	5.9306	5.9135	5.8964	5.8794	5.8625
	7	6.9767	6.9536	6.9305	6.9076	6.8848	6.8621	6.8395	6.8170
	8	7.9701	7.9403	7.9107	7.8813	7.8521	7.8230	7.7940	7.7652
	9	8.9626	8.9255	8.8885	8.8518	8.8153	8.7791	8.7430	8.7072
	10	9.9543	9.9089	9.8639	9.8191	9.7746	9.7304	9.6865	9.6429
	11	10.9452	10.8908	10.8368	10.7831	10.7299	10.6770	10.6245	10.5724
	12	11.9353	11.8710	11.8073	11.7440	11.6812	11.6189	11.5571	11.4958
	24	23.7518	23.5071	23.2660	23.0283	22.7939	22.5629	22.3351	22.1105
	36	35.4508	34.9131	34.3865	33.8708	33.3657	32.8710	32.3865	31.9118
	48	47.0335	46.0933	45.1787	44.2888	43.4230	42.5803	41.7602	40.9619
	60	58.5009	57.0524	55.6524	54.2991	52.9907	51.7256	50.5020	49.3184
	72	69.8543	67.7946	65.8169	63.9174	62.0928	60.3395	58.6544	57.0345
	84	81.0948	78.3243	75.6813	73.1593	70.7519	68.4530	66.2573	64.1593
	96	92.2235	88.6456	85.2546	82.0393	78.9895	76.0952	73.3476	70.7380
	108	103.2415	98.7628	94.5453	90.5718	86.8261	83.2934	79.9599	76.8125
	120	114.1499	108.6798	103.5618	98.7702	94.2814	90.0735	86.1264	82.4215
	240	217.4413	197.6740	180.3109	165.0219	151.5254	139.5808	128.9825	119.5543
	360	310.9072	270.5485	237.1894	209.4613	186.2818	166.7916	150.3076	136.2835

Table 1

Question 12 continues

Question 12 continued

Marker use

a) Use the interest factor table to find the amount borrowed over a 10 year loan period if monthly repayments of $1665.31 are made and the interest rate is 6% p.a. compounded monthly. C7 /2

$F = 1665.31 \times 90.0735 = \$150000.30$

ie $\simeq \$150000$

b) Find the size of the monthly repayments on a loan of $200 000 taken over a 20 year period at 5% p.a. compounded monthly. C7 /2

$R = \frac{200\,000}{151.5254} = \$1319.91$

c) How long would it take to repay $5000 if monthly repayments of $425.75 were made and the interest rate was 4% p.a. interest compounded monthly. C7 /2

$F = \frac{5000}{425.75} = 11.7440 \Rightarrow n = 12 \text{ months}$

Total C7 /6

Question 13 (approximately 10 minutes)

Marker use

An accountant is trying to determine the value of a wood-working machine which cost $9200 when new. It has an estimated lifetime of 8000 hours of operating time after which it will be worthless.

a) Find an equation for the value of the machine if the accountant uses the unit cost method.

C7 /2

Depreciation rate per use $D = \frac{9200}{8000} = \$1.15$ per hour

$\therefore V = -1.15n + 9200$ where n is number of hours use

b) Find the value of the machine after 3 years if the machine is used for 910 hours in the first year, 360 hours in the second year and 570 hours in the third year of ownership.

C7 /1

Total use $= 910 + 360 + 570 = 1840$ hours

$\therefore V = -1.15(1840) + 9200 = \$7084$

c) If a graph was drawn of the 'Value of the machine' against the 'Number of uses' It would be:

C3 /1

(Tick **the** one (1) correct answer)

- ☐ A straight line with 'y' intercept $ 9200 and gradient 1.15
- ☐ A graph made of several straight sections because it is used for differing amounts each year.
- ☑ A straight line with 'y' intercept $ 9200 and 'x' intercept 8000
- ☐ A curved line indicating exponential decrease in value.

Question 13 continues

Question 13 continued

Marker use

Another way that the accountant may find the value of the machine is by using the straight line depreciation method.

d) Find the value of the machine after 3 years if the accountant applies the straight line depreciation method based on 15% of its purchase price every year.

C7 /2

Annual depreciation = 15% of 9200 = $1380

$\therefore V = -1380(n) + 9200$ where n is number of years

$= -1380(3) + 9200$

$= \$5060$

e) Why do the two (2) methods give different valuations?

C3 /2

The two methods give different valuations because they are different ways of estimating the value. Unit cost depends upon amount of use while straight line depends upon age

f) Under what circumstances would you expect unit cost depreciation to give a lower valuation than straight line depreciation? Support your answer with calculations.

C3 /2

Equal valuation if $\$1.15 \times n = 1380$

$\therefore n = 1200$ hours/year

∴ Unit cost will give lower valuation if the machine is used for more than 1200 hours per year

Total C7 /5

Total C3 /5

Question 14 (approximately 11 minutes)

Marker use

A business has recently purchased a new vehicle that the management expect will last about 10 years. In preparation for replacing the vehicle the business pays $2500 every quarter into an investment fund which pays 5.8% p.a. (compounding quarterly).

a) Use a difference equation to model the amount in the fund. Use initial term T_0 = C7 /2

$i = \frac{.058}{4} = 0.0145$

$T_{n+1} = 1.0145(T_n + 2500) \quad T_0 = 0$

b) How much is in the fund after 10 years? C7 /2

Calculator sequence mode $n=40$ $T_{40} = \$136190.58$

Instead, the company make an initial deposit of $10 000 into the account followed by the quarterly instalments of $2500.

c) Remodel the difference equation to reflect the new situation. Use initial term T_0 = C7 /2

$T_{n+1} = 1.0145(T_n + 2500) \quad T_0 = 7500$

d) How much will be in the account after 10 years? C7 /1

Calculator sequence mode $n=40$ $T_{40} = \$149530.20$

e) What would the present value of the investment's total be? C7 /2

$PV = \frac{FV}{(1+i)^n} = \frac{149530.20}{1.0145^{40}} = \$84071.12$

Question 14 continued

Question 14 continues

Marker use

C3

/2

f) Why is the concept of 'present value' useful in situations like this?

The concept of present value allows you to compare the earnings of an investment that will mature in the future with the current value of ~~the~~ money. In this case $149530.20 in ten years time is like having $84071.12 today.

Total C7

/9

Total C3

/2

Question 15 (approximately 14 minutes)

Marker use

Jude would like to buy a new car which would cost $32 000. She is considering two (2) options:

OPTION A SAVE TO BUY	OPTION B BORROW AND BUY NOW
By making a suitable deposit every month into an account paying 5.9% pa compound interest (compounding monthly) over a 3-year time frame. This option would mean that she would also have to allow for 3% pa increase in the price of the vehicle due to inflation.	Purchase the $32000 vehicle by taking out a personal loan offering the following terms: • No deposit • monthly repayments at 8.2% pa (compounding monthly) for 3 years.

Image 1

If Jude chooses to save to buy the car (Option A):

a) What would the expected price of the car be allowing for the effects of inflation?

C7 /2

Savings target = $32000(1.03)^3 = \$34967.26$

b) What should her monthly savings payment be?

C7 /2

Calculator Finance mode ⇒ Begin

$n = 36$

$r = 5.9$

$PV = 0$ ⇒ $PMT = \$885.91$

$PMT = ?$

$FV = 34967.26$

$PY/CY = 12$

c) How much will she pay for the car in total?

C7 /1

Total Paid = $886.42 \times 36 = \$31892.76$

Question 15 continues

Question 15 continued

Marker use

If Jude chooses to borrow to buy the car now (Option B):

d) Use an algebraic formula to show that the size of the loan repayment is $1005.72.

C7 /3

$P = \frac{R[1-(1+i)^{-n}]}{i}$ $\quad i = \frac{.082}{12} = .00683333$

$32000 = \frac{R[1-(1.00683333)^{-36}]}{.00683333}$

$32000 = R \times 31.818044$

$R = \$1005.72$

e) How much will she pay in total for the car?

C7 /1

Total paid $= 1005.72 \times 36 = \$36205.92$

f) List **two (2)** benefits of each option.

C3 /2

Option A • Save over $4000 in total price

• End up with a later model car

Option B • Get to have it right now!

• Avoid paying fees for public transport etc

Jude decides to follow option B (Borrow to buy now). After making payments for 12 months Jude thinks that she can pay the loan off more quickly.

g) Find the monthly payment required to pay off the remaining loan over the next 6 months.

C7 /3

Calculator Finance mode ⇒ End

• Amount owed after 12 months

$n=24 \quad I=8.2 \quad PV=? \quad PMT=-1005.72 \quad FV=0 \quad PY/CY=12$

Amount owing after 12 months $= \$22192.20$

• To repay in 6 months

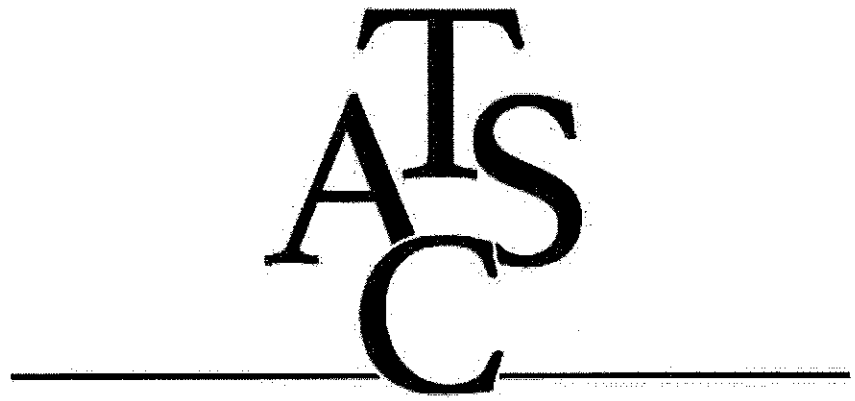
$n=6 \quad I=8.2 \quad PV=22192.20 \quad PMT=? \quad FV=0 \quad PY/CY=12$

Required repayment $= \$3787.66$

Total C7 /12

Total C3 /2

End of Section C



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GENERAL MATHEMATICS

MTG315123

Section D Networks or Trigonometry

Pages: 20

Questions: 9

Information Sheet: 1

Suggested working time: 36 minutes

Instructions:

- There are **two (2)** parts to this section. Answer **all** questions in **one (1)** part. Either:
  - **Part 1** – networks and decision mathematics

    OR
  - **Part 2** – trigonometry and Earth geometry
- Write your answers in the spaces provided in this exam paper.
  - Spare diagrams have been provided at the end of each section. Indicate using the box provided if you have used the spare diagram.
- TASC approved calculators are allowed.
- The exam is **three (3) hours** in length. The suggested working time for this section is **approximately 36 minutes**.
- The General Mathematics Information Sheet can be used throughout this exam.
- All answers must be written in **English**.
- You **must** make sure your answers address the listed criterion.

Marker use	
C8	/ 36

Guide to Exam Structure

	Parts	Questions available	Questions to answer	Suggested working time	Marks available
Section A		5	5	48 minutes	48 marks
Section B		5	5	48 minutes	48 marks
Section C		5	5	48 minutes	48 marks
Section D	Part 1 OR	4	4	36 minutes	36 marks
	Part 2	5	5	36 minutes	36 marks
Totals		**24**	**19 or 20**	**180 minutes (3 hours)**	**180 marks**

Criterion

You **must** make sure your answers address:

- Criterion 8 interpret concepts and apply mathematical techniques to represent, analyse and solve practical problems in the two-dimensional plane.

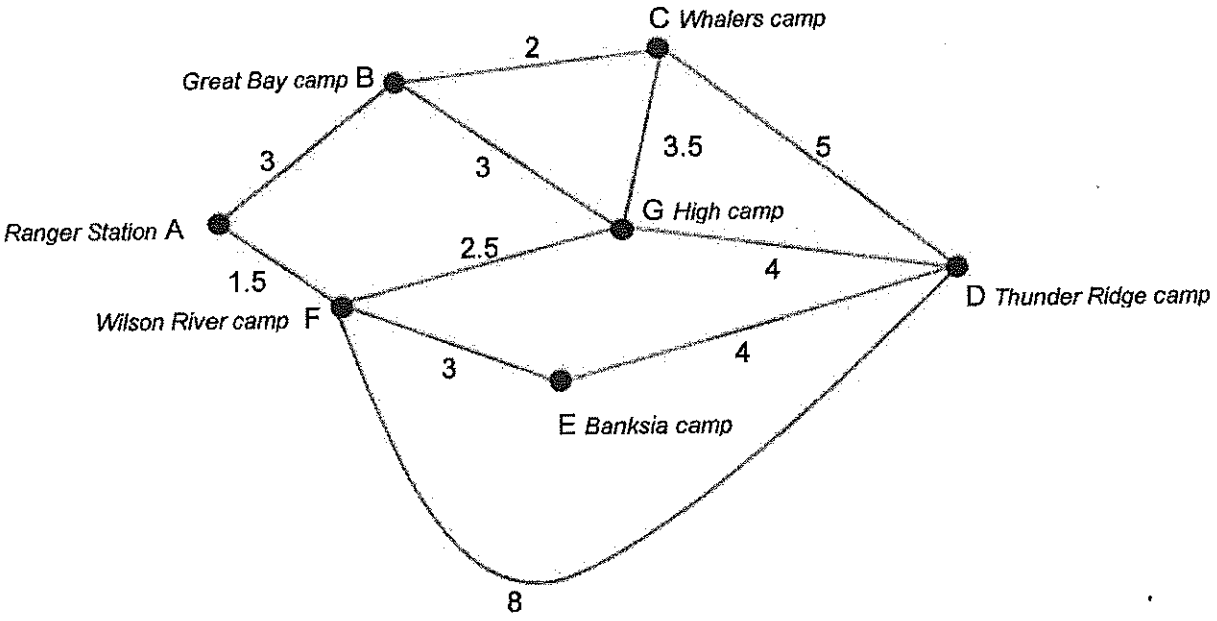
Part 1 - Networks

Marker use

- **Either** answer **all** questions in this part **OR all** questions in Part 2.

Question 16 (approximately 11 minutes)

Graph 1 below represents the network of walking tracks that join campsites in a national park. The weights on the edges of the graph represent the length of each track in km.



Graph 1

The ranger wishes to visit every campsite to check that they are clean. She wishes to do this without having to walk the same track twice, and she wants to start and finish at the Ranger Station.

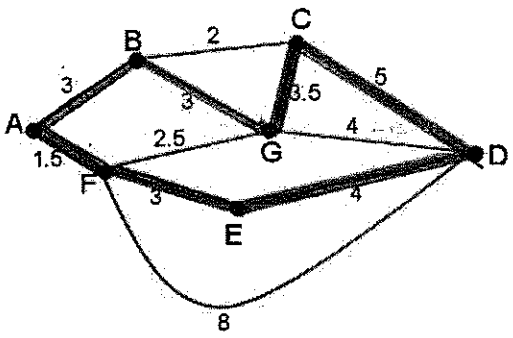
a) What mathematical name is given to this type of path?

Hamiltonian Cycle /1

Question 16 continues

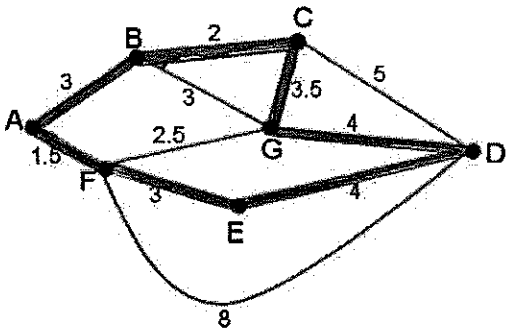
Marker use

b) Give **two (2)** examples of such a path by marking them on the graphs below (Graph 2). /2



Graph 2

A B G E D E F A = 23 Km



A B C C D E F A = 21 Km

Spare diagram used (X) ☐

c) Hence, find the length of the shortest path that the ranger should follow to accomplish her mission. /1

21 Km

On other occasions the ranger wishes to walk every track in the park to check that they are all in good condition. She wishes to do this without travelling the same track twice and wants to start and finish at the Ranger Station.

d) What mathematical name is given to this type of path? /1

Eulerian Trail

e) Is such a path possible? How can you tell? /1

No - There are vertices of odd degree.

f) What is the shortest distance that the ranger would have to walk to be able to check all the tracks (starting and finishing at the Ranger station)? /1

$3+(2\times2)+3+3.5+5+4+2.5+1.5+3+4+8$

$= 41.5$ Km

g) Which track, if any, would need to be walked twice? /1

The track between B and C

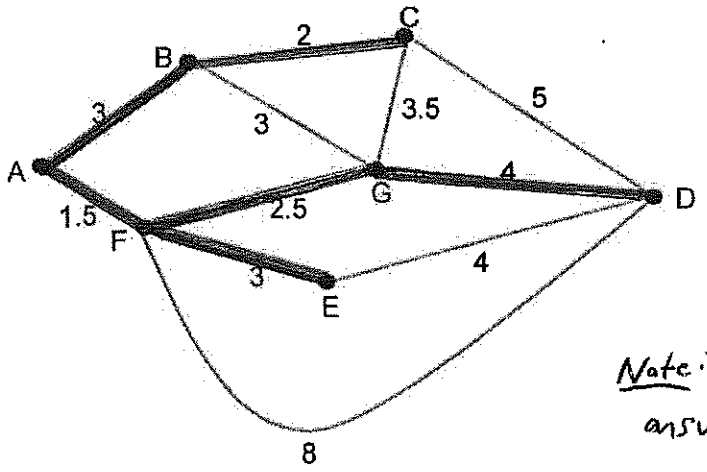
Question 16 continued

Marker use

The management of the park want to upgrade some tracks so that all campsites may be reached with bicycle transport. They wish to do this minimising the total amount of track that needs upgrading.

h) Mark the recommended sections for upgrade on Graph 3 below.

/2



Note: Alternative answer is possible

Graph 3

Spare diagram used (X) ☐

i) What is the minimum length of track that must be upgraded to bicycle standard?

/1

Total in need of upgrade = 1.5+2.5+3+3+2+4

= 16 km

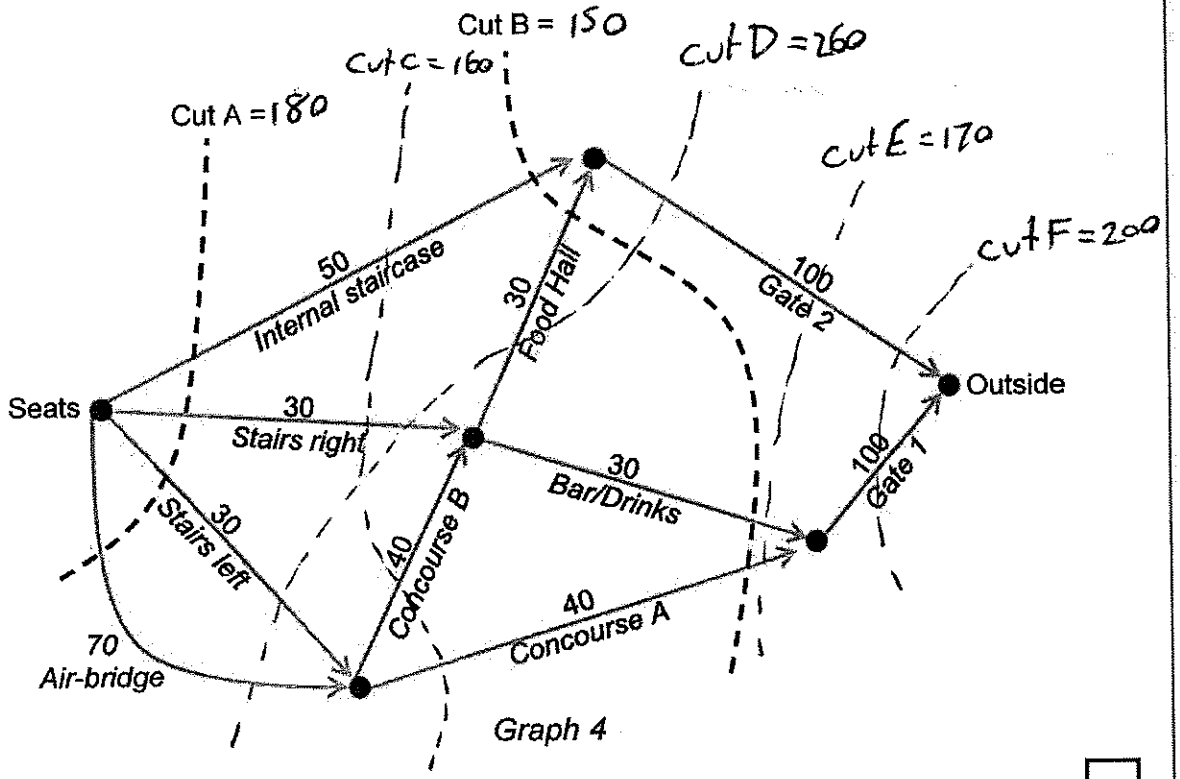
Total Q16

/11

Question 17 (approximately 9 minutes)

Marker use

At the end of a football game spectators must exit the grandstand. The weights on the graph below show the number of people per minute that can be moved through the different areas of a grandstand as it empties.



Graph 4

Spare diagram used (X) ☐

a) What is meant by the term 'cut' as it relates to a flow graph? /1

A cut in a flow graph is a line which crosses edges in a way that if those edges were removed there would be no flow from source to sink.

b) Find the capacity of cut A and cut B, and mark them on Graph 4. /1

cut A = 180 cut B = 150

c) Draw four (4) more cuts and find the capacity of each. /2

d) What is the maximum flow of people exiting the grandstand? /1

Min cut = Max flow = 150 people/min

Question 17 continues

Question 17 continued

Marker use

e) Which of the following would be likely to produce a greater flow of people exiting the grandstand? (Explain your answer):

- Widen the exit gates.
- Remove obstructions in the Food Hall.
- Put a high-speed travelator (conveyor belt) in the air-bridge.

/2

Only by removing obstructions in the food hall. To gain improvement overall, improvement must be made to these edges on the minimum cut

f) By how much would the overall flow through the network increase if the flow through the 'Drinks/Bar' area was increased to 50 people/min? (Explain your answer).

/2

Would improve the flow by 10 people/min. There would be a new minimum cut (cut C = 160 people/min)

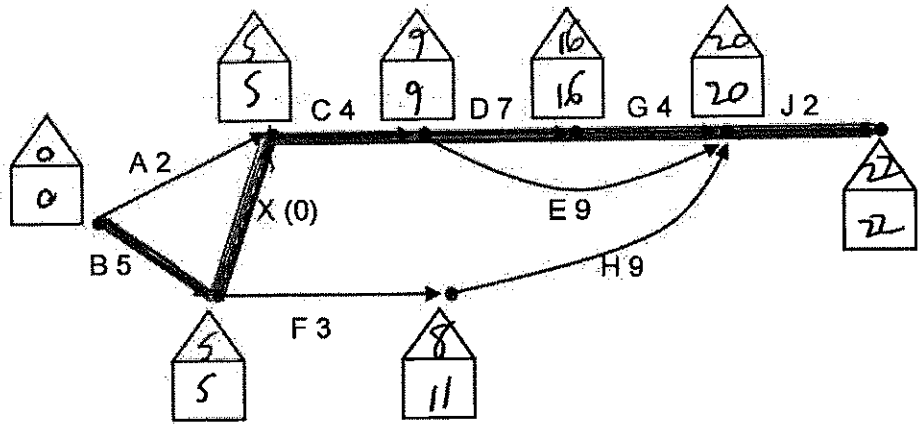
Total Q17

/9

Question 18 (approximately 9 minutes)

Marker use

A football club is building new locker rooms. The project manager is preparing a critical path analysis of the project and has made the Graph 5. The weights represent the time taken for each activity in days.



Graph 5

Spare diagram used (X) ☐

a) Explain why the dummy edge X had to be used between the end of B and the beginning of C. /2

Activity C relies upon the completion of both A and B but activity F relies only upon B being finished.

b) Find the EST and LFT for each activity and write them appropriately on Graph 5. /2

c) Mark the critical path on the graph. /1

d) What is the earliest completion time for the project? /1

22 days

e) Find the 'float' involved in activity H. /1

Float = Time available - activity duration
= (20-8) - 9
= 3 days

Question 18 continues

Question 18 continued

Marker use

f) The project manager suggests that the builders could complete the project earlier by saving time on activity D. What is the maximum worthwhile saving of time on this activity? (Explain your answer).

/2

2 days is the maximum worthwhile time saving. If more than 2 days is saved there will be a new critical path and extra time saved will not lead to an earlier completion of the project

Total Q18

/9

Question 19 (approximately 7 minutes)

Marker use

A factory manager wishes to minimize the time that it takes to manufacture items. He is considering allocating his four workers so that each specialises in one of the four tasks that are involved in producing the item.

He times how long it takes each worker to do each of the tasks. The results are shown in Table 1 below.

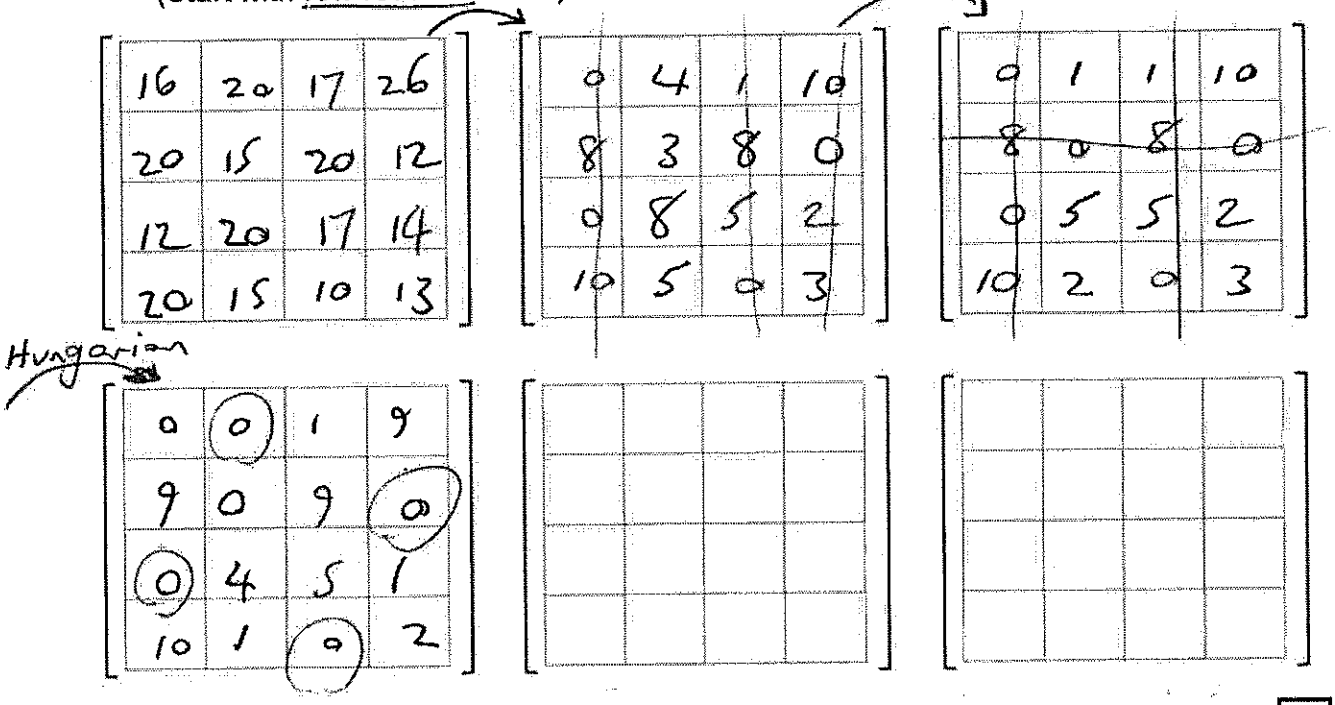
	Time taken (minutes)			
	Case assembly	Motor installation	Electronics	Control panel
Allie	16	20	17	26
Bob	20	15	20	12
Chloe	12	20	17	14
Dave	20	15	10	13

Table 1

a) Use the Hungarian algorithm to reduce the matrix below into a form where an assignment which minimises production time can be made. /3

(Start with row reduction first.)

column reduction



Spare diagram used (X) ☐

Question 19 continues

Question 19 continued

Marker use

b) Who should be assigned to each task?

Allie — Motor Installation

Bob — Control Panel

Chloe — Case Assembly

Dave — Electronics

/1

c) What is the expected production time to produce one complete unit?

Production time = 20 + 12 + 12 + 10 = 54 mins

/1

d) Find the amount of time saved per unit produced if task specialisation is introduced.

(Hint: To do this compare the time taken to produce four units with the workers specialising against the time taken if they were working individually)

/2

Working alone Allie takes 79 mins

Bob takes 67 mins

Chloe takes 63 mins

Dave takes 58 mins

267 mins for 4 units

∴ Time saving on 4 units $= 267 - (4 \times 54) = 51$ mins

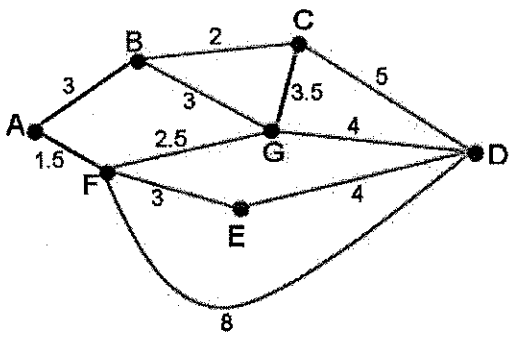
∴ Time saving per unit $= \frac{51}{4} = 12.75$ mins

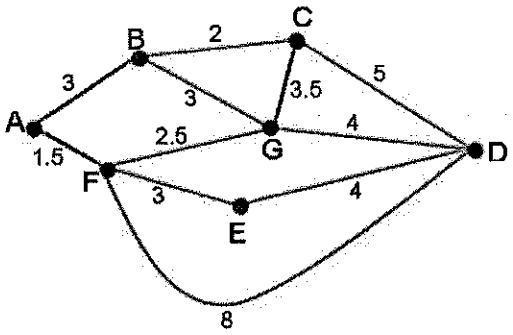
Total Q19

/7

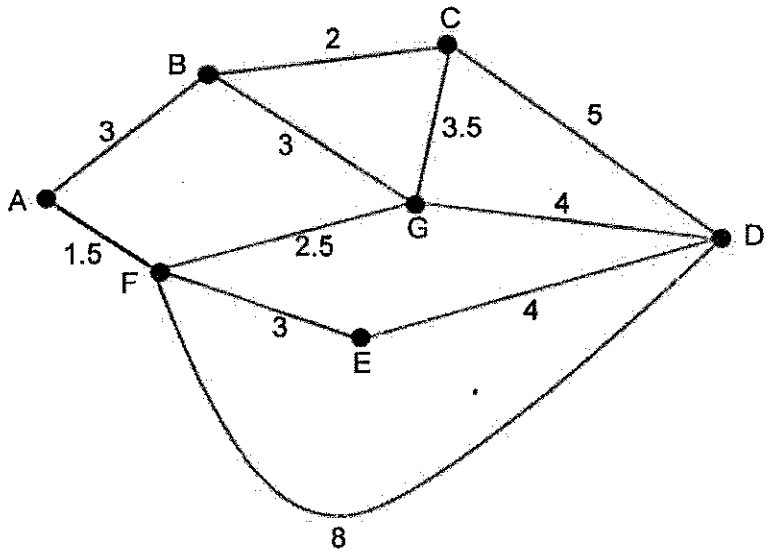
Spare Diagrams

Question 16 b)

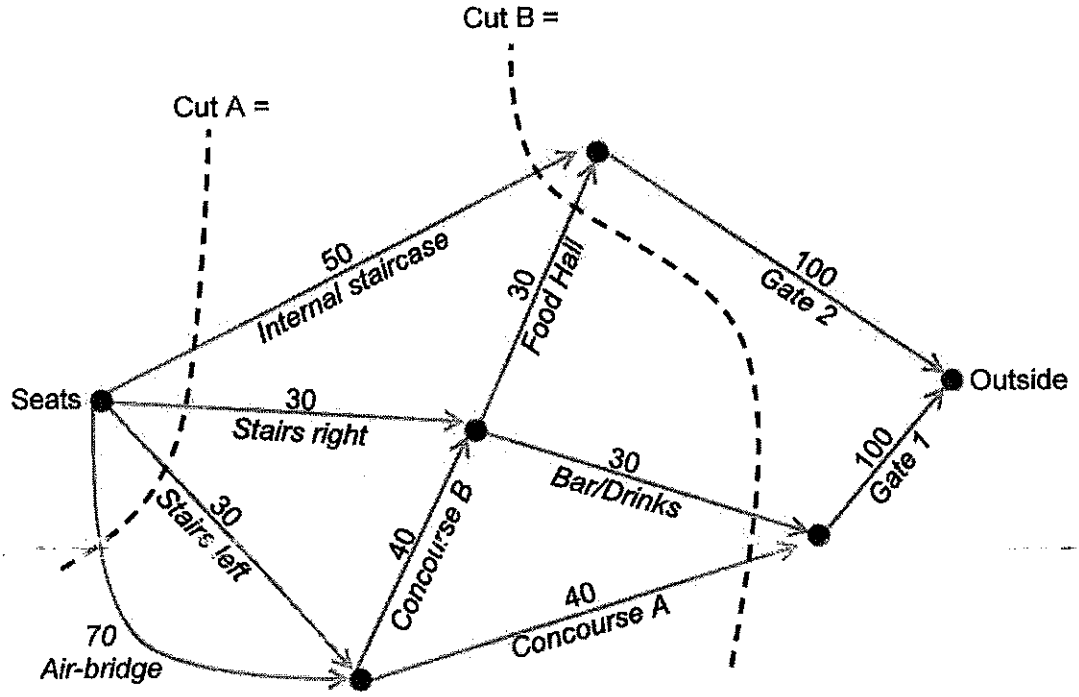




Question 16 g)

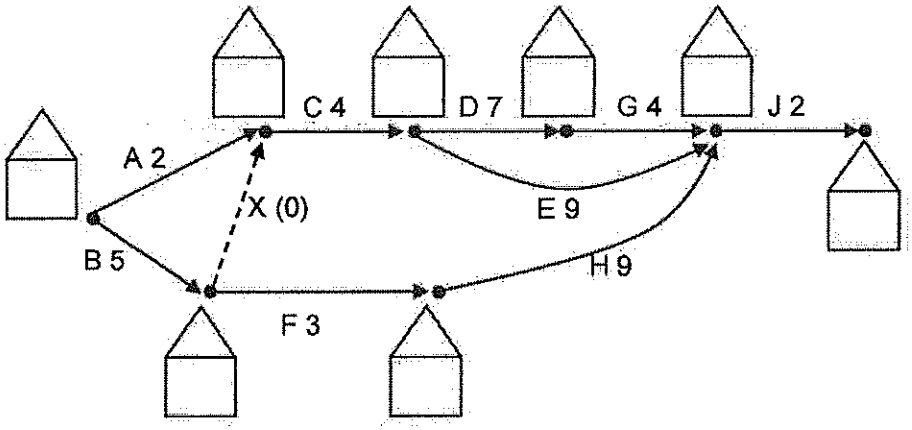


Question 17 b) and c)

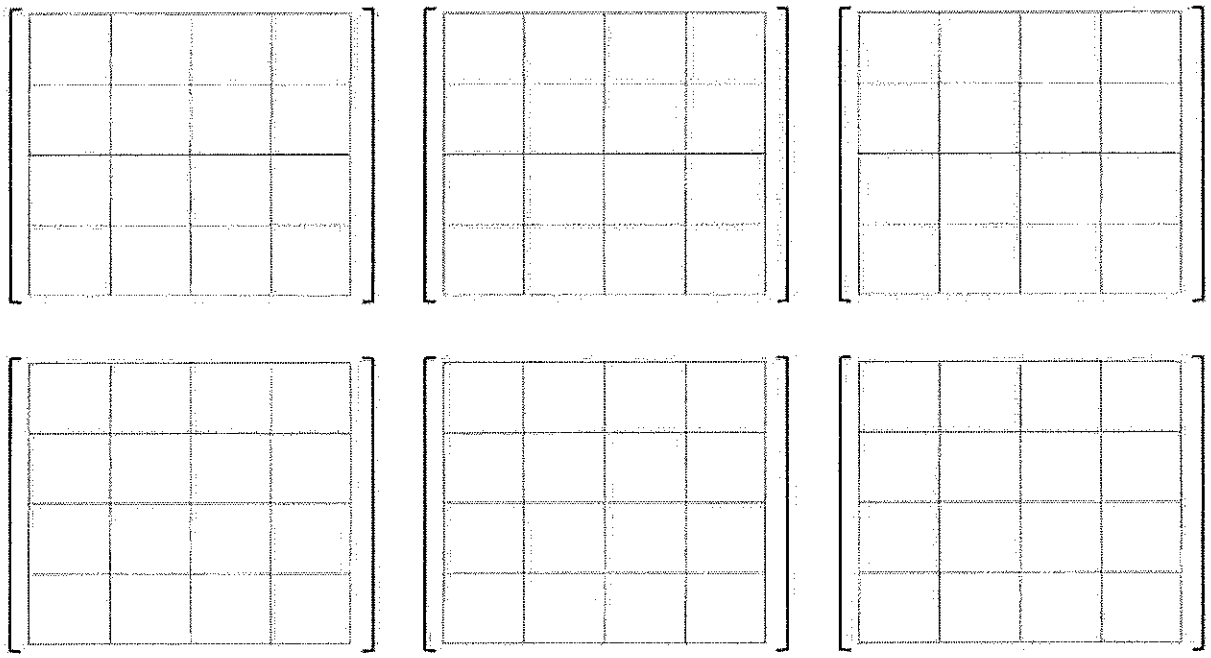


Spare Diagrams

Question 18 c)



Question 19 a)



End of Part 1

Part 2 - Trigonometry

Marker use

- **Either** answer **all** questions in this part **OR all** questions in Part 1.

Question 20 (approximately 7 minutes)

A surveyor who wishes to find the area of a block of land divides it into two (2) parts and makes measurements as shown by Diagram 1.

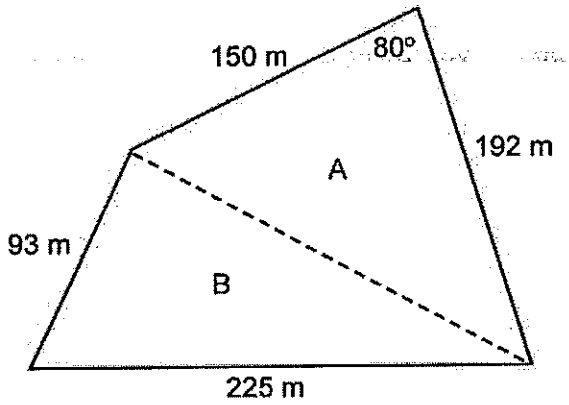


Diagram 1

a) Find the area of triangle A.

Area $A = \frac{1}{2}(150)(192)\sin 80$

$= 14181.23\ m^2$ /2

b) Find the length of the dotted line.

$x^2 = 150^2 + 192^2 - 2(150)(192)\cos 80$ /2

$x^2 = 49361.86$

$x = 222.18\ m$

c) Hence find the area of triangle B.

$s = \frac{1}{2}(93 + 225 + 222.18) = 270.09$ /2

$A = \sqrt{270.09(270.09 - 222.18)(270.09 - 93)(270.09 - 225)}$

$= 10164.80\ m^2$

d) Find the total area of the block of land, expressing your answer in **hectares**. /1

(1 ha = 10 000 m^2)

Total $= 24346\ m^2 = 2.4346\ ha$

Total Q20 /7

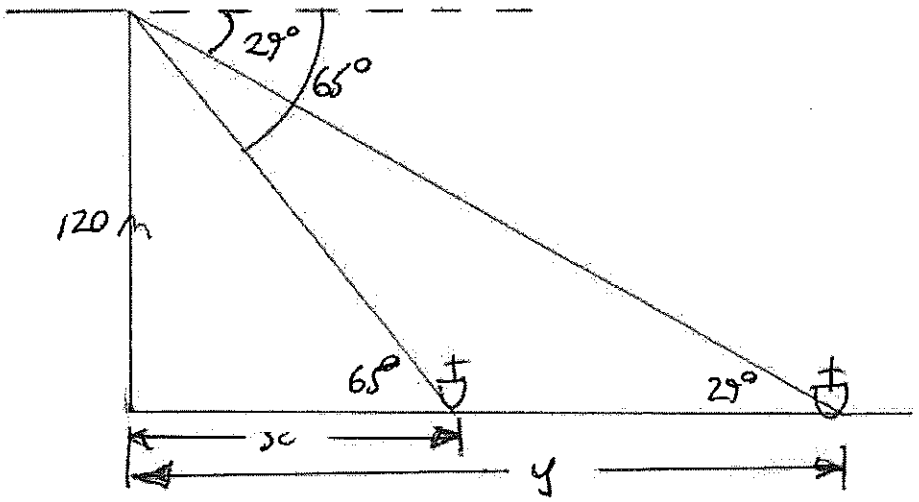
Question 21 (approximately 7 minutes)

Marker use

From the top of a 120 m high cliff the angle of depression to a small boat at sea is 29°. After some time the observer notices that the boat has moved and the angle has become 65°.

a) Draw a clear diagram that shows this information.

/3



b) Find the distance that the boat has moved between the two observations.

/4

$\tan 29 = \frac{120}{y}$ $\tan 65 = \frac{120}{x}$

$y = \frac{120}{\tan 29}$ $x = \frac{120}{\tan 65}$

$y = 216.49$ $x = 55.96$

Distance moved $= 216.49 - 55.96 = 160.53$ m

Total Q21

/7

Question 22 (approximately 8 minutes)

Marker use

The bearing of a beacon on a rocky reef is N40°W when observed from a harbour 300 m away. The captain of a ship at sea identifies the beacon on a bearing S75°W and 420 m away.

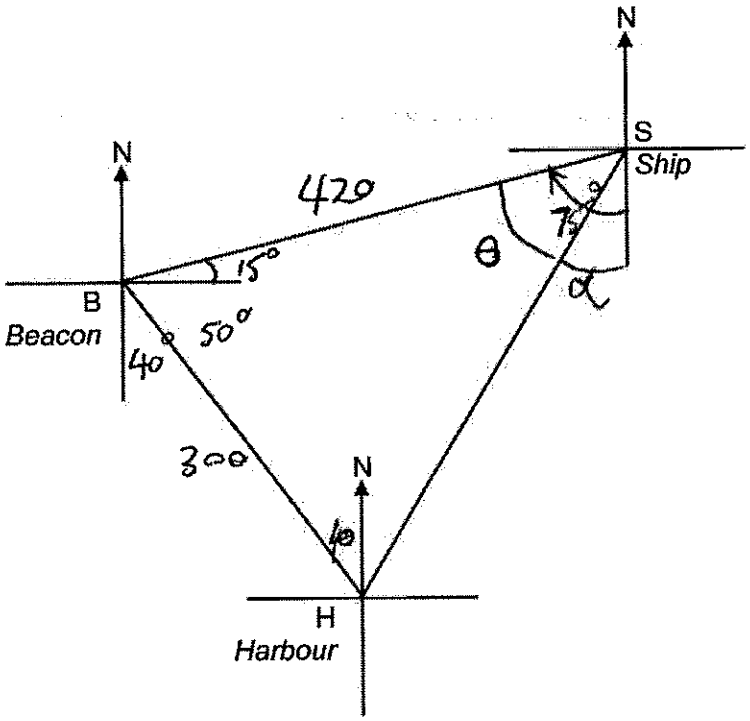


Diagram 2

Spare diagram used (X) ☐

a) Mark the bearings on diagram 2. /2

b) Find the distance of the ship from the harbour. /3

$d^2 = 300^2 + 420^2 - 2(300)(420)\cos 65$

$d^2 = 159900$

$d = 399.875$ m

c) On what bearing should the captain be heading to safely make the harbour? Answer to the nearest minute. /3

$\cos\theta = \frac{399.875^2 + 420^2 - 300^2}{2(399.875)(420)}$

$\theta = 42.8391°$

$\therefore \alpha = 90 - (49.8391 + 15) = 32.1608°$

$\therefore$ Required bearing S 32°10' W

Total Q22 /8

Question 23 (approximately 9 minutes)

Marker use

A jet leaves Perth (-31.95, 115.86) at 7:15 p.m. on Tuesday. It flies to Dubai (25.08, 55.31) by the shortest possible route and maintains a speed of 840 km/h. (Note: GPS bearings)

a) What is meant by the expression 'shortest possible route' in the context of distances on the surface of a sphere? /1

The shortest possible route on a sphere is always by travelling on a 'great circle'. ie A circle of the same radius as the sphere

b) Find the shortest distance from Perth to Dubai. /3

$\cos\theta = \sin 25.08 \sin -31.95 + \cos 25.08 \cos 31.95 \cos(115.86 - 55.31)$

$\cos\theta = 0.1535$

$\theta = 81.167$

$\therefore Dist = 2\pi 6371 \times \frac{81.167}{360} = 9025.5 \text{ km}$

c) Find the time zones (UTC) of Perth and Dubai. /2

Perth $\frac{115.86}{15} \simeq 8\text{h} +$ on UTC

Dubai $\frac{55.31}{15} \simeq 4\text{h} +$ on UTC

d) Find the estimated time of arrival (ETA) of the jet in Dubai. /3

Travel time $= \frac{Dist}{speed} = \frac{9025.5}{840} = 10:45$

$\therefore ETA = 7:15\text{ PM Tues} + \text{Travel time} \pm \text{zone time diff}$

$= 7:15\text{ PM} + 10:45 - 4$

$= 2:00\text{ AM Wednesday}$

Total Q23 /9

Question 24 (approximately 5 minutes)

Marker use

Rome and Copenhagen are both situated on the same meridian of longitude.

Copenhagen has location (56°N, 13°E). It is 1557 km due North of Rome.

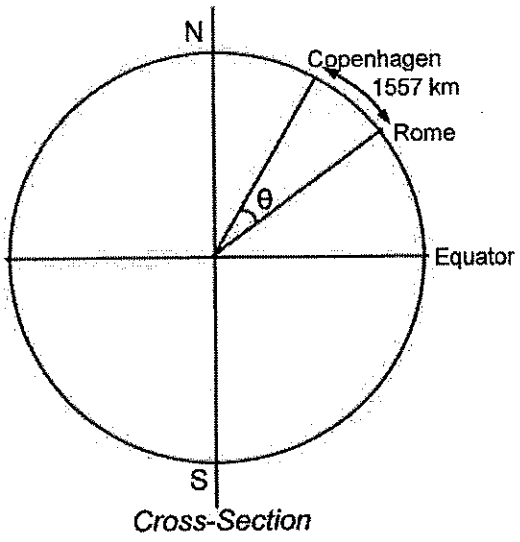


Diagram 3

a) Find the angle θ subtended by the two (2) cities.

$2\pi 6371 \times \frac{\theta}{360} = 1557$ /3

$\therefore 2\pi 6371 \times \theta = 560520$

$\therefore \theta = 14.0024^\circ$

b) Hence find the co-ordinates of Rome. (Answer to nearest whole degree).

$\therefore$ Latitude Rome $= 56^\circ - 14^\circ = 42^\circ N$ /2

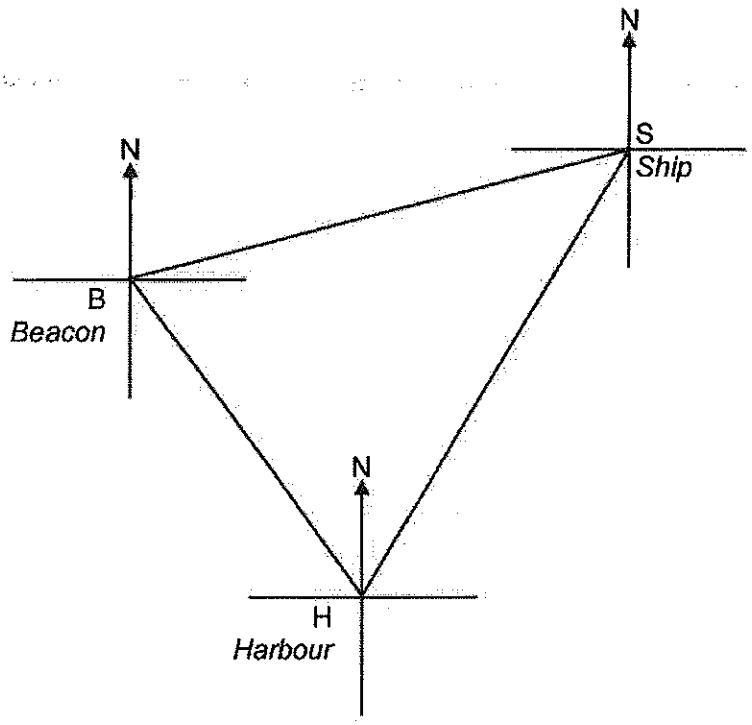
Co-ordinates of Rome are $(42^\circ N\ 13^\circ E)$

End of Part 2

Total Q24 /5

Spare Diagrams

Question 22 a)



End of Section D

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