2023 ASSESSMENT REPORT

MTM315117 MATHEMATICS METHODS – FOUNDATION

Section A – Non-calculator section

General comments

- Many students who understood the content covered in this section lost marks due to errors made when performing numerical calculations. Students need to develop better skills with basic multiplication, fractions and indices work, and for algebraic manipulation questions when solving for x.
- Some students were very inaccurate with the sketching of graphs resulting in graphs which were not mathematically accurate. Whilst some leeway was granted, a number of students were penalised for substantial inaccuracies caused by lack of attention to detail when sketching. Students are encouraged to use pencil for their graphs (so they can erase errors), use a ruler if they need to draw axes, ensure they draw one smooth line/curve which accurately passes through the relevant points and ensure the resulting graph passes the vertical line test.
- Any question worth 2 marks or more **must** include some appropriate working or full marks will not be gained.

Part 1 (Criterion 4)

Question	Part	Marks	Comments
Q1		2	• Rearranging this equation was generally poorly done. • Common errors included: – leaving the answer in the form of a fraction over a fraction and not simplifying – incorrect cancellation of the t variable. • Candidates who subtracted ut first then often encountered sign errors later.
Q2	(a)	2	• Candidates who took out the highest common factor first were more likely to recognise the difference of two squares required to fully factorise this expression. • Many different errors made by candidates when completing this question. • A common error was to incorrectly identify this as a perfect square.
	(b)	1	• This question was generally completed well by most candidates.
Q3	(a)	2	• This question was generally completed well. Occasionally, candidates incorrectly cancelled the 3 on the numerator with the 3 on the denominator. Or, forgot to multiply either the 2 or the 5 by three.

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Question	Part	Marks	Comments
	(b)	2	• For a 2 mark question candidates need to include Null Factor Law (NFL) to explain the method used.
Q4	(a)	2	• Candidates who converted to fractional indices and applied the correct addition principles had the most success. • Some candidates incorrectly recognised the coefficients as the denominators of the fractional indices. • Some candidates incorrectly added 3 and ½ when determining the index of the second term.
	(b)	2	• This question was generally completed well by most candidates. • Subtracting a negative power proved problematic for some candidates. As did multiplying 6 by 8. • Many candidates did not convert the final answer to positive indices.
Q5	(a)	1	• This question was generally completed well by most candidates.
	(b)	2	• This question was generally completed well by most candidates.

Part 2 (Criterion 5)

Question	Part	Marks	Comments
Q6	(a)	1	• This question was generally completed well by most candidates.
	(b)	2	• This question was generally completed well by most candidates. • Occasionally candidates incorrectly found the perpendicular function. • Arithmetic errors were the most common reason for deducted marks in this question.
Q7		3	• Care needs to be taken to ensure a continuous line is drawn and the shape is consistent with a cubic function. • Marks were deducted if the drawn function did not pass the vertical line test. • Some candidates failed to recognise that when $x = 2$ the function had a turning point and consequently they drew an incorrect quadratic function.
Q8	(a)	2	• Candidates who were successful were able to recognise the turning point form of the function and were then able to solve for a using the $y -$ intercept coordinates. • Some candidates made an arithmetic error resulting in $a = 0$. Candidates are encouraged to go back and check working when they have an illogical answer.

Question	Part	Marks	Comments
			• Candidates were not expected to expand the function as the question did not explicitly ask for this.
	(b)	2	• Candidates who were successful were able to recognise the intercept form of the function and were then able to solve for a using the $y -$ intercept coordinates. • The most common error occurred when candidates solved $-10 = -20a$ to give an answer of $\mathrm{a} = 2$ or $\mathrm{a} = -1/2$. • Candidates were not expected to expand the function as the question did not explicitly ask for this.
Q9	(a)	2	• Many candidates failed to recognise that coordinates can be a function. • When applying the vertical line test to these points a sketch was required for full marks. • Candidates who were successful recognised there was only one $y -$ value for each $\mathrm{x} -$ value and provided a clear explanation.
	(b)	1	• Many candidates incorrectly considered this question to be two separate functions. • Candidates need to remember the correct wording to use for "vertical line test". Or, clearly state that some $\mathrm{x} -$ values have more than one $\mathrm{y} -$ value. • Some candidates incorrectly implied that a relation is the opposite of a function and therefore did not fully answer the question. • Candidates who used x –axis or x –intercept instead of the correct x –value or x –coordinate did not receive full marks for their explanation.
Q10		2	• Candidates generally were able to identify a, c and Δ but had difficulty determining b. • The markers suspect that some candidates struggled with the greater than and less than symbols.
Q11		2	• Candidates did very well on this question. • Candidates who used the general function notation $y = f(x)$ rather than $y = x^3$ as their starting point were generally unsuccessful in obtaining the final function.

Part 3 (Criterion 6)

Question	Part	Marks	Comments
Q12	(a)	2	• Generally done well. Candidates who were successful with this question knew to change 16 to 2^4 and then compare powers. Whilst most candidates recognised the need to change $16$16 to a base of 2, a common error was using the wrong power.
	(b)	2	• Candidates who did well in this question used the subtraction log law correctly and divided the 2 terms. They also knew to convert from log to exponential form, a common error was leaving the lefthand side as 1.
	(c)	1	• Most candidates correctly converted into exponential form. A common error was candidates not calculation 3^2 correctly.
Q13		3	• Most candidates correctly drew an exponential shape tending towards an asymptote. Common errors included incorrect calculations for the y −intercept. Some candidates forgot to find a second point.
Q14	(a)	2	• Candidates who did well correctly used either the $\sin^2 x + \cos^2 x = 1$ identity or Pythagoras to determine $\cos x$. Several candidates did not recognise that x was in the second quadrant therefore missed that the solution should be negative. • Pythagoras' Theorem was the most successfully used method. Candidates who tried to use the Trigonometric Identity often made the error of not recognising that $sinx = \frac{4}{5}$ and instead tried to replace $\frac{4}{5}$ for x. Successful candidates recognised that $\sin^2 x = \frac{16}{25}$ and could rearrange.
	(b)	2	• Most candidates successfully used the identity $\tan\theta = \frac{\sin\theta}{\cos\theta}$ but again, many forgot the solution would be negative in the second quadrant.
Q15	(a)	1	• Most candidates recognised that the value of n was $\frac{1}{2}$ however many were unable to correctly divide by $\frac{1}{2}$. A common error was using the period formula for a sine or cosine graph rather than a tan graph.
	(b)	3	• Candidates successfully drew the shape of a tan graph that passed the vertical line test and adhered to the given domain. • Common errors include: – not having the tan function pass through the origin – asymptotes in incorrect and inconsistent locations – period were inconsistent at times.

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Part 4 (Criterion 7)

Question	Part	Marks	Comments
Q16	(a)	1	• Generally done well.
	(b)	2	• Most candidates expanded first; this is essential for the correct answer.
	(c)	2	• Mostly done well. Common errors: – not making the 3 a negative power – not subtracting 1 from the power when differentiating.
	(d)	2	• Main error was not converting a radical to a fractional index correctly.
Q17		3	• Completing this question caused the most difficulty. • Common errors included: – $\frac{dy}{dx} = 0$ or $\frac{dy}{dx} = 2$ – finding $\frac{dy}{dx}$ at $x = 2$ and then using it as the y co-ordinate.
Q18		3	• Mostly done well. • Common errors included: – not expanding $(x + h)^2$ correctly – poor notation.
Q19		3	• Candidates who answered this question (approximately half of all candidates) did it well. However, many left it blank.

Part 5 (Criterion 8)

Question	Part	Marks	Comments
Q20	(a)	2	• Very well done.
	(b)	1	• Very well done.
Q21	(a)	1	• Many candidates over complicated this question and did not use the formula given on the Information Sheet for independent events.
	(b)	2	• Completed quite well. A common error was lack of understanding of how to multiply fractions efficiently.
Q22	(a)	2	• Very well done.
	(b)(i)	1	• Very few errors (especially including error carried forward).
	(b)(ii)	1	• Very few errors (especially including error carried forward).
Q23	(a)	1	• Completed quite well. Some candidates complicated this by using nCr Common error was adding 12 and 10 instead of multiplying.
	(b)	1	• Mostly done well. However, some candidates complicated this question using nCr notation.
Q24	(a)	2	• Many candidates correctly identified $10\mathrm{C}4$ but were unable to expand and simplify it correctly.
	(b)	2	• Answers was often over complicated. • Common errors: – adding $5\mathrm{C}2$ and $5\mathrm{C}2$ rather than multiplying – multiplying $5\mathrm{C}2$ by 2, giving a numerator of 20.

Section B – Calculator section

General comments

- Students need to remember to utilise their calculators to ensure answers are correct.
- Any question worth 2 marks or more MUST include some appropriate working else full marks will not be gained.
- Inaccurate graphs with scribbled out sections, unclear lines, lack of scale on axes etc. will not achieve full marks.
- Students are encouraged to use rulers, pencils and eraser when constructing graphs.

Part 1 (Criterion 4)

Question	Part	Marks	Comments
Q25	(a)	2	• Many students attempted to complete the square without factorising first. Of those who did factorise, many did not take a factor of 2 out of each term. The second source of error was not including the last term in the final expansion.
	(b)	2	• This part was done well by students who could follow and use the quadratic formula. Most students could not cancel the resultant fraction down to the simplest form.
Q26		2	• Most students could show and use Pascal's Triangle or the Binomial Theorem to expand. A number of errors were made with positive and negative signs. Many students wrote the expansion starting with $16x^4$.
Q27	(a)	1	• Very well done; most students supplied the correct linear equations. Some students did not know how to solve simultaneous equations.
	(b)	2	• This question was completed successfully. Some students didn't complete the question by finding both the number of minutes riding and the price paid or did not state dollars.
Q28	(a)	2	• Most students identified the correct terms and their placement in the equation for Pythagoras' Theorem. Some students made errors in their expansions but generally this question was well done.
	(b)	2	• Most students could find the two solutions for the values for x. Having done so, many did not disregard the negative solution or give units.
Q29	(a)	2	• This question was not well completed. Many students did not transpose the "m" variable to the LHS. Many students ignored the "m" variable all together.
	(b)	1	• The majority of students could successfully find the solution for the discriminant.

Question	Part	Marks	Comments
Q30	(a)	1	• This question was accurately completed by most students.
	(b)	3	• Most students could compose the equation and make it equal to 500 and then solve. Various answers were given, some were even rounded to 7 hours. Few students could convert 6.78 hours to 6 hours and 48 minutes. Some students did not give an accurate final time.

Part 2 (Criterion 5)

Question	Part	Marks	Comments
Q31		2	• Question well answered overall. • The main source of error was writing the equation with a positive gradient.
Q32		2	• Question was very poorly answered overall. • Typical errors included students not realising the x and y scale were different, x and y intercepts not calculated or shown accurately, TP not calculated or shown accurately. • Some candidates had the turning point in completely different quadrants.
Q33	(a)	2	• This question posed issues to many candidates. • Main errors included miscalculating the dilation factor of $-3/8$ths. • Significant number of candidates did not attempt this question. • For those that were successful, using the TP or the x-ints was helpful.
	(b)	2	• Many candidates found this part of the question difficult to visualise and also did not attempt this part of the question. • Common sources of error included candidates substituting x for 1.5 instead of setting the equation equal to 1.5 to then solving for x.
Q34	(a)	4	• A straightforward question that saw many candidates not include endpoints. • Overall, well answered but some candidates could benefit by improving their shapes of cubics when sketching them and knowing which circle to use to indicate respective endpoints.
	(b)	1	• Well answered by those candidates who included endpoints in part (a). • Typically not attempted by those candidates who did not include endpoints in part (a) • Main source of error included incorrect brackets.
Q35		3	• Typically well answered by most candidates. • Common errors included incorrectly re-writing original equation into $y-$ form and therefore calculating an incorrect gradient, or using the parallel

Question	Part	Marks	Comments
			gradient instead of the perpendicular gradient, or algebraic errors when using gradient and a point to find the final equation.
Q36	(a)	1	• Significant number of candidates either did not attempt this part or were unable to use the clues in the question to generate an initial equation to then reduce to $h = 1.5 - 2x$.
	(b)	1	• Successfully answered by many candidates with straightforward algebra. • Errors included candidates substituting 1.5 for x and then expanding the given equation to find a final value for volume.
	(c)	2	• Many part marks given here. • Candidates were confident that x must be positive but many did not give $x < 0.75$. • Not attempted by a significant number of candidates.

Part 3 (Criterion 6)

Question	Part	Marks	Comments
Q37		1	• Most students answered correctly as 1.309 or $0.417\ \pi$ radians. The major source of error was candidates not fully interpreting the question and leaving the answer as an exact value, rather than expressing as a decimal to 3 places.
Q38	(a)	1	• Most students answered correctly. The exponential in the question appeared quite "large" and could have easily been interpreted as not an exponential; however, most candidates did not appear to misinterpret.
	(b)	2	• Most students answered correctly. Very few had algebraic working out. A number of students did not interpret the question fully and referred to their answer as "days" not "weeks".
Q39	(a)	1	• Most students answered correctly. Very few mistakes.
	(b)	2	• Most students answered correctly. Most common mistake was rounding and some students calculated the answer with their calculator in Radian mode not Degrees. Guidance to the preciseness of the answer may have been advantageous.
Q40	(a)	2	• Most students answered correctly. A common mistake was omitting the direction of the dilation factor, and some students did not write the dilation first, which is what the markers were looking for.
	(b)	3	• Sources of error for candidates included incorrect placement of the asymptote (e.g. $x = 2, x = 0, y = -2, y = 2$ etc.) and non-inclusion of the asymptote. Graph placement and determination of 2 points was generally sound. Many candidates left axes unlabelled.
	(c)	1	• This question was well done if asymptote had been determined. A significant number of "errors carried forward".
	(d)	1	• Most students answered correctly.
Q41	(a)	1	• Most students answered correctly but the most common mistake was to write 6π 6π as the period instead of 6. It is likely that many candidates had not experienced π being part of the coefficient of the function.
	(b)	3	• Not well answered by many students. Common mistakes were: incorrect periods (even when 41a was answered correctly), very poorly drawn cos waves, and x intercepts not labelled.
	(c)	2	• Most students received part marks from either having the correct answer or the working out. Very few students received full marks.

Part 4 (Criterion 7)

Question	Part	Marks	Comments
Q42	(a)	1	• This question was generally very well done by most students, though units were periodically omitted.
	(b)	2	• It was good to see that most students recognised that the average rate of change was the gradient of the line joining the 2 points. • Common errors included incorrect sign by reversing the order of the numerator terms, mixing the numerator and denominator, and omitting units from the answer.
Q43	(a)	2	• Students who knew how to approach the question generally did it reasonably well. With few marks allocated, limited working for finding the derivative was necessary and a number of students used the calculator successfully in this instance.
	(b)	1	• Completed successfully based on a candidate's response to part (a).
Q44		3	• Most students knew to find the derivative and did it correctly, yielding 1 mark out of 3. From there, if students proceeded, the most common error was to calculate the y-intercept of the normal incorrectly or to find the equation of the tangent.
Q45		2	• Generally well done. Any errors seemed to indicate that students did not fundamentally understand that the value of the gradient is the value of the derivative.
Q46	(a)	3	• Most students saw this as a straightforward use of the derivative and solving. Most errors or loss of marks arose from either not finding the associated y-values or calculating them incorrectly.
	(b)	2	• The justification part of the nature of the stationary points in part (b) was not well done by many, with a number of gradient tables set out poorly. Students should be encouraged to make a final statement about the nature of each stationary point.
Q47	(a)	2	• Most students who attempted this question completed it successfully.
	(b)	1	• Most students who attempted this question completed it successfully.
	(c)	1	• Part (c) was poorly done by many students, indicating that they had not spent much time on calculus questions related to motion.

Part 5 (Criterion 8)

Question	Part	Marks	Comments
Q48	(a)	2	• Almost all students got full marks
	(b)	1	• Again, almost all students got full marks. Correct un-simplified answers were accepted.
	(c)	2	• This part was not done as well as the previous two. The most successful approach was to compute the conditional probability, inspecting the sample space and restricting it to outcomes with a sum of 10.
Q49	(a)	2	• Well done with the correct formula being used successfully.
	(b)	2	• By and large, students understood that probability independence means that $\Pr(A) \times \Pr(B) = \Pr(A \cap B)$ and were clear in articulating this.
Q50	(a)	2	• Most filled the tree diagram correctly with only occasional misplacements of the probabilities.
	(b)	2	• Most students used the tree diagram successfully to generate the total probability. • $\Pr(top) = \Pr(rain) \times \Pr(top\|rain) + \Pr(rain') \times \Pr(top\|rain')$.
Q51	(a)	1	• Was well done by students.
	(b)	2	• Not quite as well done. Errors included multiplying the combinations for netball and cricket only rather than adding and neglecting the denominator to generate a probability.
	(c)	2	• The most successful approach was to use complementary events students who tried the alternative (involving 3 outcomes) were less successful due to the greater amount of calculation. Again there were denominator issues.
	(d)	2	• A less successful question due to the complexities of conditional probabilities.

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External Assessment 2023

Attach your candidate label here

SETTER MARKING

MATHEMATICS METHODS – FOUNDATION

SCHEME

MTM315117

Section A

✓ indicates a mark.

Pages: 24

Questions: 24

Information Sheet: 1

Preparation time for this exam: 15 minutes

Suggested working time: 80 minutes

Instructions:

Calculators are not allowed to be used in this section.

Section A will be collected after 80 minutes.

- There are **five (5) parts** to this section.
- Answer **all** questions and **all** items within each question.
- Write your answers in the spaces provided in this exam paper.
  - Spare diagrams have been provided at the end of each part. Indicate in the box provided if you have used the spare diagram.
- The exam is **three (3) hours** in length. It is suggested that you spend **approximately 80 minutes** in total answering the questions in this exam booklet.
- During the first 80 minutes you may move onto Section B, but you **cannot** use your calculator until told by your supervisor(s).
- The Mathematics Methods-Foundation Information Sheet can be used throughout the exam.
- All answers must be written in **English**.

Marker use	
C4	/ 16
C5	/ 16
C6	/ 16
C7	/ 16
C8	/ 16

Additional Exam Instructions

- You **must** make sure your answers address the listed criteria.
- For questions worth **one (1)** mark, you are not required to show workings. Markers will look at the presentation of the answer(s) and at the argument(s) leading to the final answer(s).
- For questions worth **two (2)** or more marks **you are required** to show relevant workings.
- Marks will be allocated:
  - according to the degree to which workings convey a logical line of reasoning, and
  - for suitable justifications and explanations of methods and processes when requested.

Criteria

You **must** make sure your answers address:

- Criterion 4 manipulate algebraic expressions and solve equations
- Criterion 5 understand linear, quadratic and cubic functions
- Criterion 6 understand logarithmic, exponential and trigonometric functions
- Criterion 7 use differential calculus in the study of functions
- Criterion 8 understand experimental and theoretical probabilities and of statistics.

Guide to Exam Structure

	Parts	Questions available	Questions to answer	Suggested working time	Marks available
Section A	Part 1	5	5	16 minutes	16 marks
	Part 2	6	6	16 minutes	16 marks
	Part 3	4	4	16 minutes	16 marks
	Part 4	4	4	16 minutes	16 marks
	Part 5	5	5	16 minutes	16 marks
Totals		**24**	**24**	**80 minutes**	**80 marks**
Section B	Part 1	6	6	20 minutes	20 marks
	Part 2	6	6	20 minutes	20 marks
	Part 3	5	5	20 minutes	20 marks
	Part 4	6	6	20 minutes	20 marks
	Part 5	4	4	20 minutes	20 marks
Totals		**27**	**27**	**100 minutes**	**100 marks**
Totals		**51**	**51**	**180 minutes (3 hours)**	**180 marks**

Part 1

- Answer **all** questions in this part.
- This part assesses **Criterion 4**.

Marker use

Question 1

Rearrange the following formula to make u the subject: $s = ut + \frac{1}{2}at^2$

$ut = s - \frac{1}{2}at^2$ ✓

$u = \frac{s - \frac{1}{2}at^2}{t}$ ✓ $= \frac{2s - at^2}{2t}$

/2

Question 2

Factorise:

a) $32 - 8b^2$

$8(4 - b)^2$ ✓

$8(2 - b)(2 + b)$ ✓

/2

b) $x^2 - 7x + 12$

$(x - 3)(x - 4)$ ✓

/1

Question 3

Solve each of the following for x:

a) $\frac{4x-3}{3} - 2 = 5$

$\frac{4x-3}{3} = 7$ $\frac{1}{2}$

$4x - 3 = 21$ ✓

$4x = 24$ $\frac{1}{2}$

$x = 6$ ✓

/2

Question 3 continues

Question 3 continued

Marker use

b) $(x-1)(x+2)(3-2x)=0$

$x=1$ ½ $x=-2$ ½ $2x=3$ ½

$x=1.5$ ½

/2

Question 4

Simplify the following, giving your answers in positive index form:

a) $3\sqrt{x} \times 2\sqrt{x^3}$

$3x^{\frac{1}{2}} \times 2x^{\frac{3}{2}}$

$6x^2$

/2

b) $\frac{6a^2b^{-3} \times (2ab^{-2})^3}{4a^3b^{-1}}$

$\frac{6a^2b^{-3} \times \cancel{8}^{\,2}a^3b^{-6}}{\cancel{4}a^3b^{-1}}$

$\frac{12a^2}{b^8}$

/2

Question 5

a) Without dividing, show that $(x-1)$ is a factor of $P(x)=x^3+5x^2+2x-8$

$P(1)=(1)^3+5(1)^2+2(1)-8$

$=0$ $\therefore (x-1)$ is a factor

/1

b) Fully factorise x^3+5x^2+2x-8

	x^2	$+6x$	$+8$
x	x^3	$+6x^2$	$+8x$
-1	$-x^2$	$-6x$	-8

or some other way of factorising out $(x-1)$

$(x-1)(x^2+6x+8)$

$(x-1)(x+4)(x+2)$

/2

Total C4

/16

Part 2

Marker use

- Answer **all** questions in this part.
- This part assesses **Criterion 5.**

Question 6

a) Find the equation of the line with a gradient 3 and y intercept -5 /1

$y = 3x - 5$ ✓ ½ ½

b) Find the equation of the line parallel to the line in a) which passes through the point $(1, -4)$ /2

$m = 3$ ½ $y - {}^{-}4 = 3(x - 1)$ ½

$y + 4 = 3x - 3$

$y = 3x - 7$ ✓

Question 7

Sketch the curve $y = 3(x + 1)(x - 2)^2$ below, clearly indicating all intercepts. /3

x int $y = 0$ y int $x = 0$

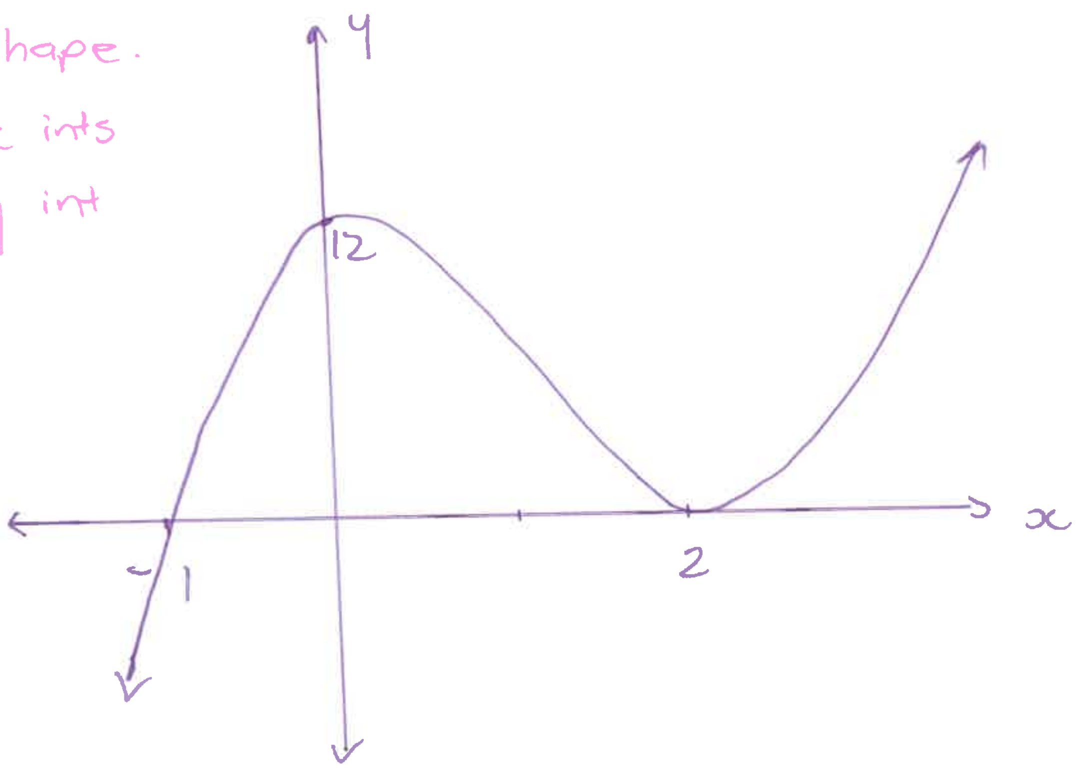
$x = -1$ $x = 2$. $y = 3(1)(-2)^2$ ✓

$= 12$

✓ shape.

✓ x ints

✓ y int



Question 8

Determine the equations of the following functions:

a)

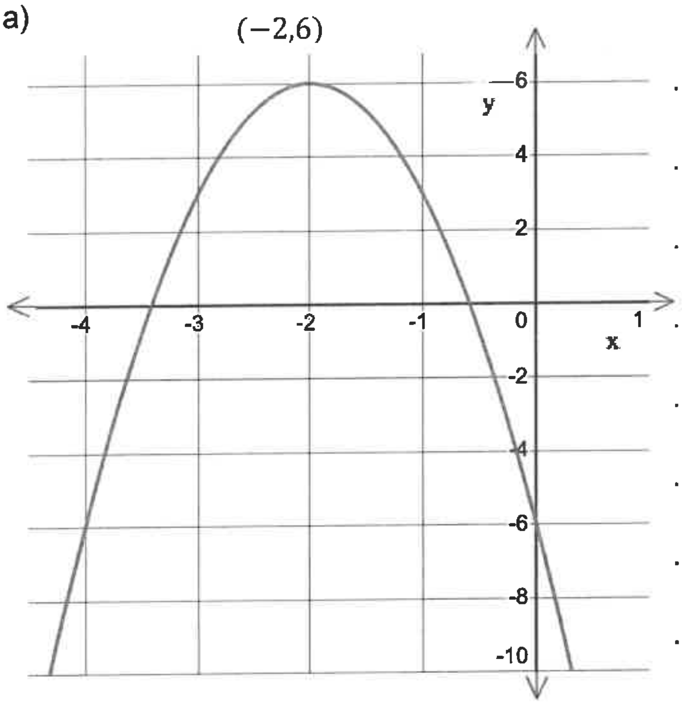


Figure 1: Graph of a function.

$y = a(x+2)^2 + 6$ ½ ½

(sub $(0,-6)$)

$-6 = a(2)^2 + 6$

$-12 = a \times 4$

$a = -3$ ½

$y = -3(x+2)^2 + 6$ ½

/2

b)

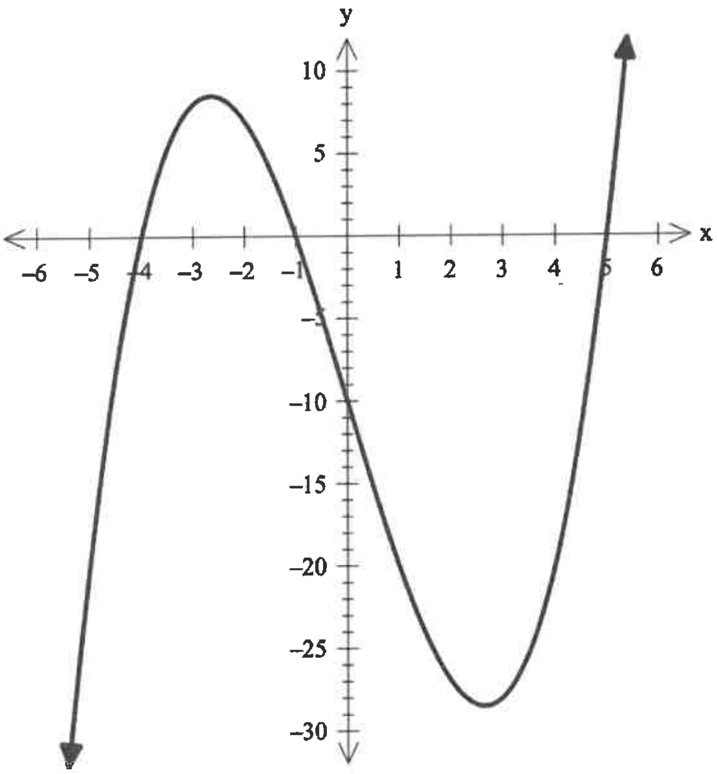


Figure 2: Graph of a function.

$y = a(x+4)(x+1)(x-5)$ ✓

sub $(0,-10)$

$-10 = a(4)(1)(-5)$

$-10 = a \times -20$

$a = \frac{1}{2}$ ½

$y = \frac{1}{2}(x+4)(x+1)(x-5)$ ½

/2

Marker use

Question 9

Marker use

State whether the following are functions. Give a **reason** for your answer.

a) $\{(-1,2),(0,3),(2,-3),(5,0)\}$

yes $\frac{1}{2}$, for each x value there is only one y value $\frac{1}{2}$.

/1

b)

/1

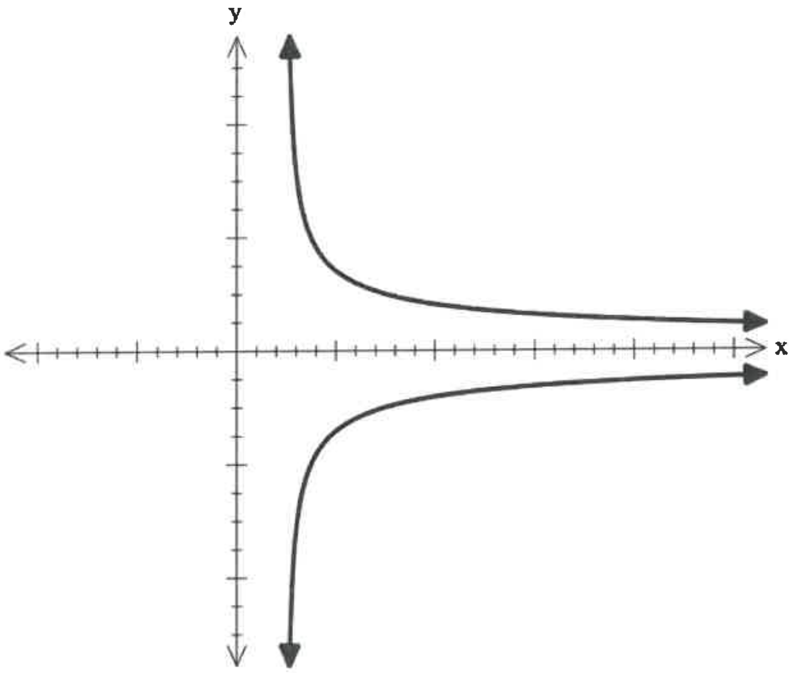


Figure 3: Graph 3.

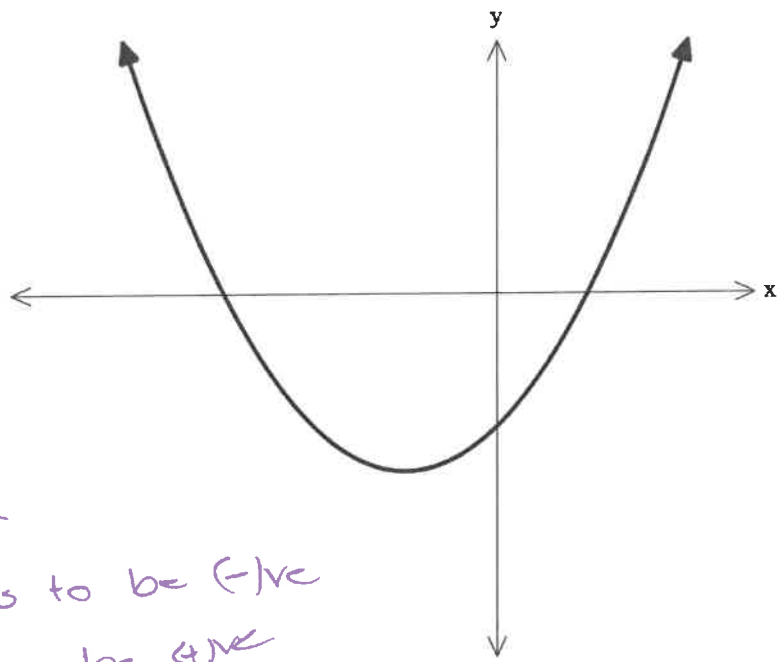
no $\frac{1}{2}$, doesn't pass the $\frac{1}{2}$ vertical line test

Question 10

Marker use

/2

For the function below, which has the form $y = ax^2 + bx + c$, circle the appropriate statements for each of the values $a,\ b,\ c$ and Δ:



axis of sym
$\frac{-b}{2a}$ has to be (-)ve
$\therefore$ b has to be (+)ve

Figure 4: Graph of a function.

$a > 0$	$b > 0$	$c > 0$	$\Delta > 0$
$a < 0$	$b < 0$	$c < 0$	$\Delta < 0$

½ for each.

Question 11

/2

The function $f(x) = x^3$ undergoes the following transformations:

- dilated by a factor of 3 in the direction of the y axis $3x^3$
- reflected in the x axis $-3x^3$
- translated left 2 units $-3(x+2)^3$
- translated up 1 unit. $-3(x+2)^3 + 1$

½ for each.

State the equation of the new function.

$g(x) = -3(x+2)^3 + 1$

Total C5

/16

Part 3

Marker use

- Answer **all** questions in this part.
- This part assesses **Criterion 6**.

Question 12

Solve algebraically for x:

a) $2^{2x} = 16$

$2^{2x} = 2^4$ ✓

$2x = 4$

$x = 2$ ✓

/2

b) $\log_3(3x + 1) - \log_3 2 = 1$

$\log_3\left[2(3x+1)\right] = 1$ $\frac{1}{2}$

$2(3x+1) = 3^1$ $\frac{1}{2}$

$6x + 2 = 3$ $\frac{1}{2}$

$6x = 1$ $\therefore x = \frac{1}{6}$ $\frac{1}{2}$

/2

c) $\log_3 x = 2$

$x = 3^2$

$x = 9$ ✓

/1

Question 13

Sketch the graph of $y = 3^{x-1} + 2$. Clearly label the asymptote, any axis intercepts and **one (1)** other point.

when $x = 0$ when $x = 2$

$y = 3^{-1} + 2 = 2\frac{1}{3}$ $y = 3 + 2 = 5$

✓ shape

✓ for asymptote and y int

✓ for point

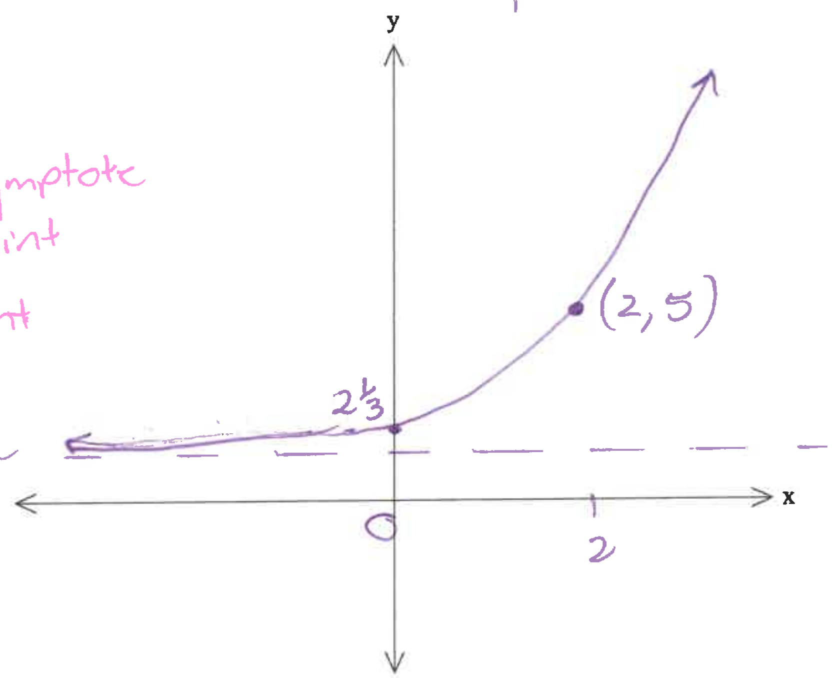


Figure 5: Axes for sketching answer to Question 13.

Spare diagram used (X) ☐

Marker use

/3

Question 14

If $\sin x = \frac{4}{5}$ and $\frac{\pi}{2} \le x \le \pi$, find the value of:

or some other working

a) $\cos x$

$\cos x = -3/5$ ✓

✓

$b^2 + 16 = 25$

$b^2 = 9$

$b = 3$

/2

b) $\tan x$, leaving your answer in simplified form.

$\tan x = \frac{\sin x}{\cos x} = \frac{4/5}{-3/5} = \frac{4}{5} \times \frac{5}{-3} = -\frac{4}{3}$ ✓

✓

/2

Question 15

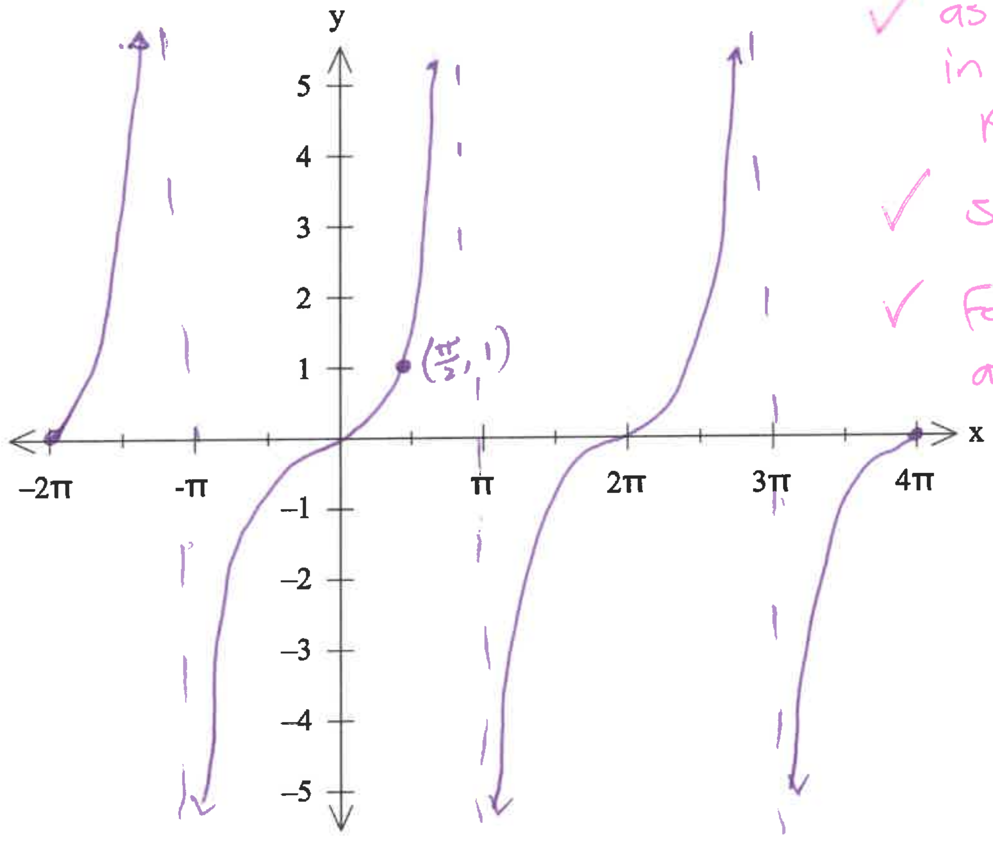
Marker use
/1
/3
Total C6 /16

a) State the period for the function $y = \tan\left(\frac{x}{2}\right)$

$y = \tan\left(\frac{1}{2}x\right)$ period $= \frac{\pi}{1/2}$

$= 2\pi$ ✓

b) Sketch a graph of this function over the domain $x \in [-2\pi, 4\pi]$



✓ asymptotes in correct place

✓ shape

✓ For endpts and labelling a pt to show dilation

Figure 6: Axes for sketching answer to Question 15 b).

Spare diagram used (X) ☐

pt asymptote $= \frac{\frac{\pi}{2}}{\frac{1}{2}}$

$= \frac{\pi}{2} \times 2$

$= \pi$

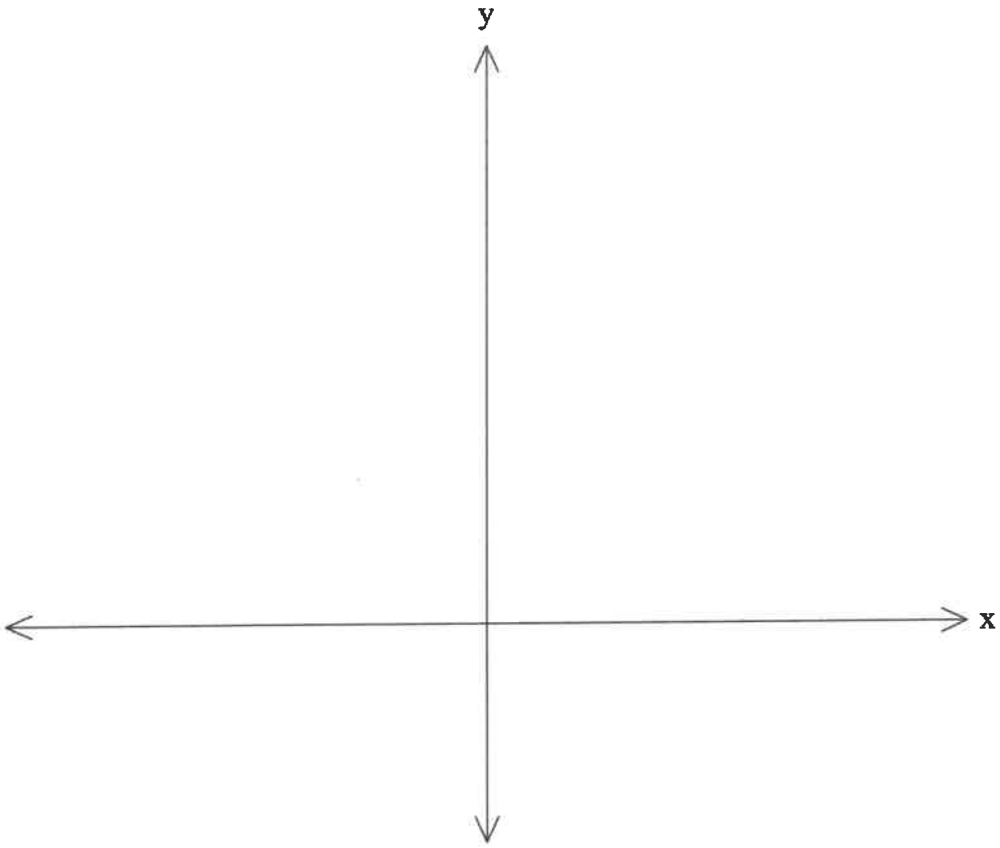
when $x = \frac{\pi}{2}$

$y = \tan\frac{\pi}{4}$

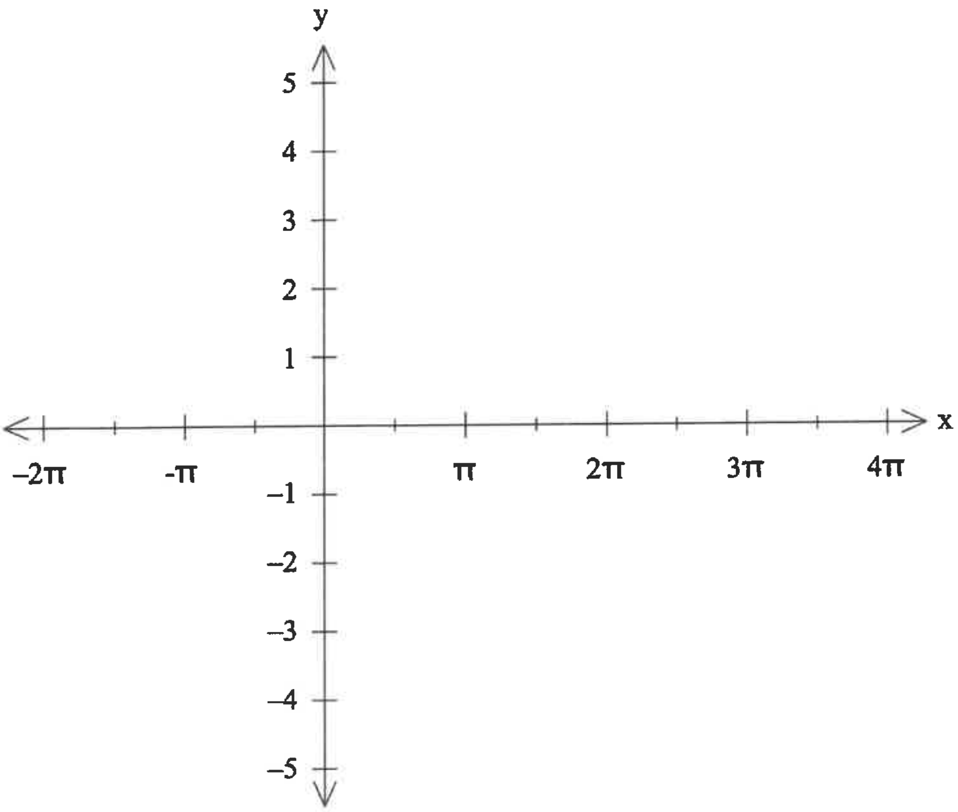
$= 1$

Spare Diagrams

Question 13



Question 15 b)



Part 4

Marker use

- Answer **all** questions in this part.
- This part assesses **Criterion 7**.

Question 16

Differentiate the following with respect to x:

a) $f(x) = 2x^4 + 3x^2 - 4x$

$f'(x) = 8x^3 + 6x - 4$ ✓

/1

b) $y = 2x^2(3 - 4x)$

$y = 6x^2 - 8x^3$ ✓

$\frac{dy}{dx} = 12x - 24x^2$ ✓

/2

c) $y = 3x^{-2} + \frac{2}{x^3}$

$y = 3x^{-2} + 2x^{-3}$ ✓

$\frac{dy}{dx} = -6x^{-3} - 6x^{-4}$ ✓

not expected $\left[\frac{-6}{x^3} - \frac{6}{x^4}\right]$ but would be nice.

/2

d) $y = \sqrt[3]{x^5} + 6\sqrt{x}$

$y = x^{5/3} + 6x^{1/2}$ ✓

$\frac{dy}{dx} = \frac{5}{3}x^{2/3} + 3x^{-1/2}$ ✓

$= \frac{5}{3}\sqrt[3]{x^2} + \frac{3}{\sqrt{x}}$

/2

Question 17

For the function $y = x^2 + 3x - 5$, find the equation of the tangent when $x = 2$

Marker use

/3

when $x = 2$

$y = 2^2 + 3(2) - 5$

$= 4 + 6 - 5$

$= 5$ ½

$\frac{dy}{dx} = 2x + 3$ ✓

$m_T = 2(2) + 3$

½ $= 7$

✓

$y - 5 = 7(x - 2)$

$y - 5 = 7x - 14$

$y = 7x - 9$ ✓

Question 18

Differentiate $f(x) = x^2 + 2x - 1$ using first principles.

/3

$f(x+h) = (x+h)^2 + 2(x+h) - 1$

$= x^2 + 2xh + h^2 + 2x + 2h - 1$ ✓

$f'(x) = \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 + \cancel{2x} + 2h - \cancel{1} - (\cancel{x^2} + \cancel{2x} - \cancel{1})}{h}$

$= \lim_{h \to 0} \frac{2xh + h^2 + 2h}{h}$ ✓

$= \lim_{h \to 0} \frac{\cancel{h}(2x + h + 2)}{\cancel{h}}$

$= 2x + 0 + 2$

$f'(x) = 2x + 2$ ✓

• must have $\lim_{h \to 0}$ throughout or lose ½ mark.

Question 19

Marker use

/3

Sketch a possible graph of the function $y = f(x)$ which has the following properties:

$f(-2) = 0,$ $\quad f'(-2) = 0$ $\quad f'(x) < 0$ for $x < -2$ and $x > 3$

$f(3) = 10$ $\quad f'(3) = 0$ $\quad f'(x) > 0$ for $-2 < x < 3$

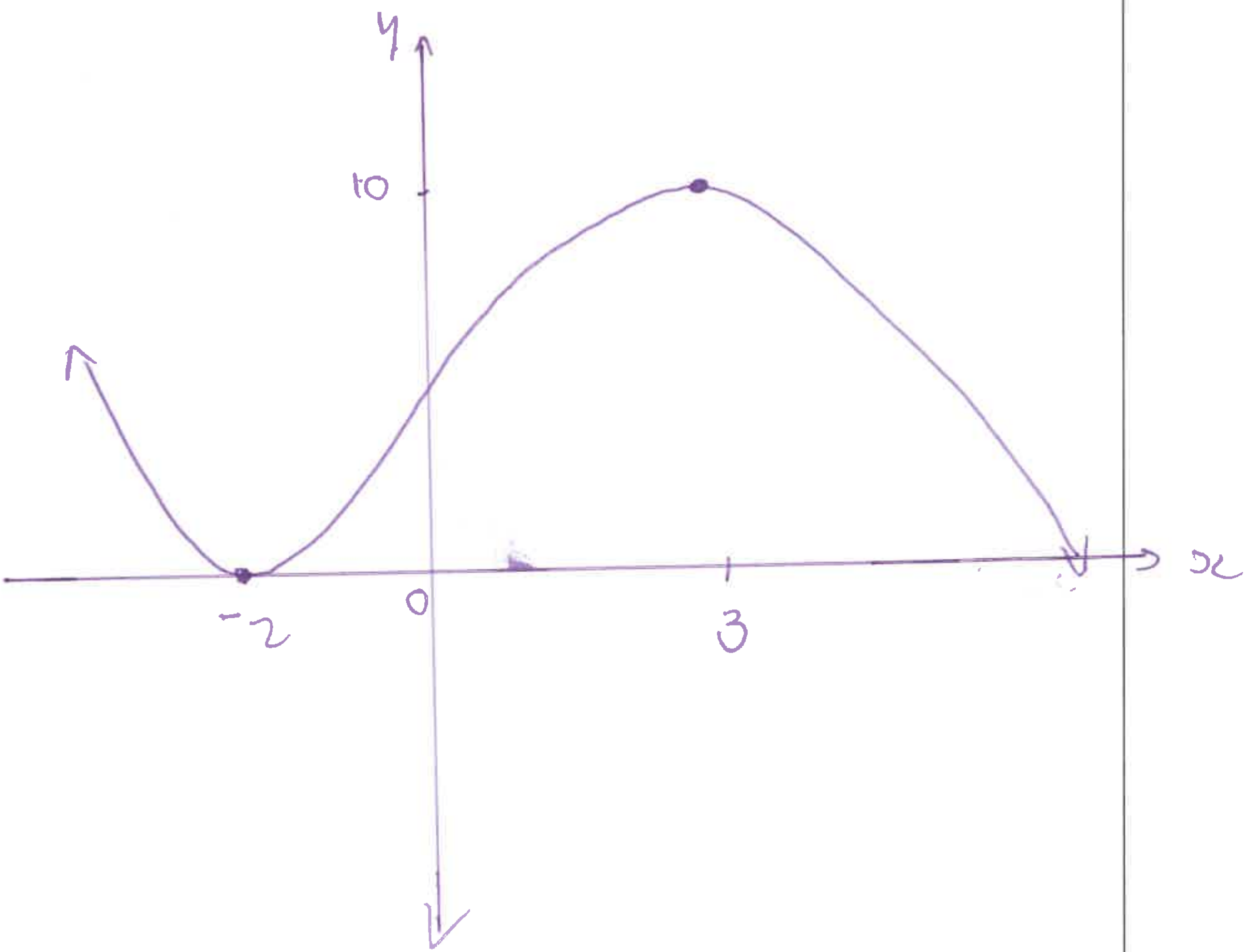
x	$x<-2$	-2	$x>-2$
$f'(x)$	$-$	0	$+$

min

for working this out ✓

x	$x<3$	3	$x>3$
$f'(x)$	$+$	0	$-$

max



✓ shape.

✓ for given points being clear

Total C7

/16

Part 5

Marker use

- Answer **all** questions in this part.
- This part assesses **Criterion 8**.

Question 20

From a survey of a group of 50 students it was found that over the past year, 35 had made a donation to a local charity, 20 had donated to an international charity and 15 students had donated to both local and international charities.

a) Show this information clearly on the Venn Diagram in Figure 7 below:

/2

$-\frac{1}{2}$ for each error

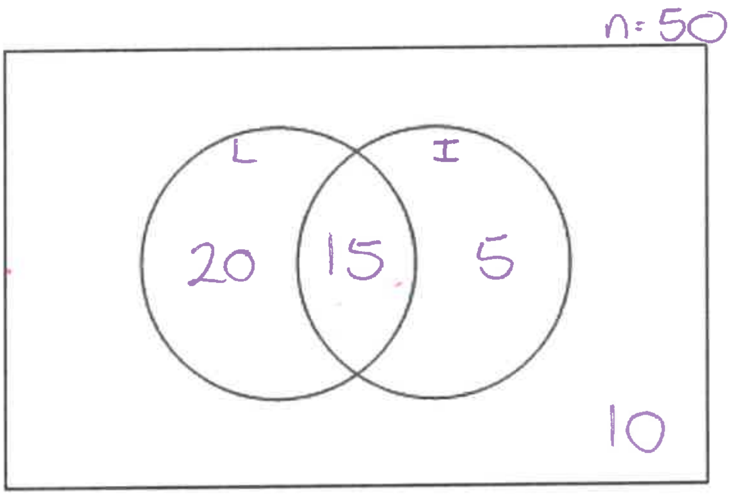


Figure 7: Venn diagram for annotation to answer Question 20 a).

Spare diagram used (X) ☐

b) If **one (1)** person is chosen at random, calculate the probability that this person donates to an international charity but not a local one.

/1

$P(I \text{ only}) = \frac{5}{50}$

$= \frac{1}{10}$ ✓

Question 21

Marker use

Events A and B are independent. If $\Pr(B) = \frac{2}{3}$ and $\Pr(A|B) = \frac{4}{5}$, find:

a) $\Pr(A)$

$Pr(A|B) = Pr(A)$ For independent

$= 4/5$

/1

b) $\Pr(A \cap B)$

$Pr(A \cap B) = Pr(A) \times Pr(B)$

$= 4/5 \times 2/3$

$= 8/15$

/2

Question 22

The probability that Harper uses a car (C) during their commute to university is 0.4, the probability they use public transport (P) is 0.7 and the probability they use both is 0.2.

a) Complete this probability table (Table 1) using this information:

/2

	C	C'	
P	0.2	0.5	0.7
P'	0.2	0.1	0.3
	0.4	0.6	1

Table 1

1/2 for placing given values correctly

1.5 for working out others.

Spare diagram used (X) ☐

b) Find the probability that on a given day, Harper:

i. uses public transport but not a car.

$Pr(P \cap C') = 0.5$

/1

ii. does not use a car at all.

$Pr(C') = 0.6$

/1

Question 23

Baby Fleur has 12 different bibs and 10 different bodysuits. She wears one (1) bodysuit and one (1) bib at a time.

a) How many different combinations of bib and bodysuit does Fleur have?

$12 \times 10 = 120$ ✓

b) If **five (5)** of her bodysuits are red and **three (3)** of her bibs are striped, what is the probability that Fleur is dressed in a red bodysuit and a striped bib?

$\frac{5}{10} \times \frac{3}{12} = \frac{1}{2} \times \frac{1}{4}$

$= \frac{1}{8}$ ✓

Question 24

A committee of four (4) is chosen from five (5) staff members and five (5) students.

a) How many different committees can be formed?

${}^{10}C_4 = \frac{10!}{4! \times 6!}$ ✓ $= \frac{10 \times \cancel{9}^{3} \times \cancel{8} \times 7 \times \cancel{6!}}{\cancel{4} \times \cancel{3} \times \cancel{2} \times 1 \times \cancel{6!}}$

$= 210$ ✓

b) What is the probability of having a committee with **two (2)** staff members and **two (2)** students?

${}^{5}C_2 = \frac{5!}{3! \times 2!} = \frac{5 \times \cancel{4}^{2}}{\cancel{2} \times 1} = 10$ ✓

$P(2 \text{ of each}) = \frac{{}^{5}C_2 \times {}^{5}C_2}{{}^{10}C_4}$

$= \frac{100}{210}$

$= \frac{10}{21}$ ✓

note: this should be simplified

Marker use

/1

/1

/2

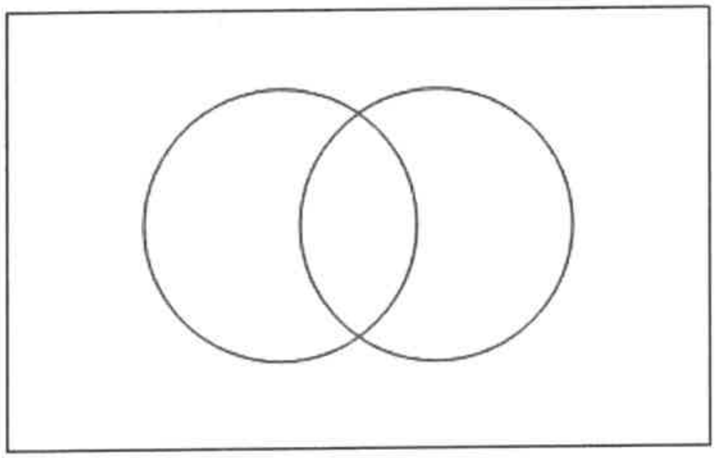
/2

Total C8

/16

Spare Diagrams

Question 20



Question 22

	C	C'	
P			
P'			
			1

End of Section A

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TASMANIAN ASSESSMENT, STANDARDS & CERTIFICATION

External Assessment 2023

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MATHEMATICS METHODS – FOUNDATION

MTM315117

Section B

Pages: 28

Questions: 27

Information Sheet: 1

Suggested working time: 100 minutes

Instructions:

Calculators are allowed to be used in this section.

- There are **five (5) parts** to this section.
- Answer **all** questions and **all** items within each question.
- Write your answers in the spaces provided in this exam paper.
  - Spare diagrams have been provided at the end of each part. Indicate in the box provided if you have used the spare diagram.
- The exam is **three (3) hours** in length. It is suggested that you spend **approximately 100 minutes** in total answering the questions in this exam booklet.
- During the first 80 minutes of this exam you may move onto Section B, but you **cannot** use your calculator until told by your supervisor(s).
- The Mathematics Methods-Foundation Information Sheet can be used throughout the exam.
- All answers must be written in **English**.
- You **must** make sure your answers address the listed criteria.

Marker use	
C4	/ 20
C5	/ 20
C6	/ 20
C7	/ 20
C8	/ 20

Additional Exam Instructions

- For questions worth **one (1)** mark, you are not required to show workings. Markers will look at the presentation of the answer(s) and at the argument(s) leading to the final answer(s).
- For questions worth **two (2)** or more marks **you are required** to show relevant workings.
- Marks will be allocated:
  - according to the degree to which workings convey a logical line of reasoning, and
  - for suitable justifications and explanations of methods and processes when requested.

Criteria

You **must** make sure your answers address:

- Criterion 4 manipulate algebraic expressions and solve equations
- Criterion 5 understand linear, quadratic and cubic functions
- Criterion 6 understand logarithmic, exponential and trigonometric functions
- Criterion 7 use differential calculus in the study of functions
- Criterion 8 understand experimental and theoretical probabilities and of statistics.

Guide to Exam Structure

	Parts	Questions available	Questions to answer	Suggested working time	Marks available
Section A	Part 1	5	5	16 minutes	16 marks
	Part 2	6	6	16 minutes	16 marks
	Part 3	4	4	16 minutes	16 marks
	Part 4	4	4	16 minutes	16 marks
	Part 5	5	5	16 minutes	16 marks
Totals		**24**	**24**	**80 minutes**	**80 marks**
Section B	Part 1	6	6	20 minutes	20 marks
	Part 2	6	6	20 minutes	20 marks
	Part 3	5	5	20 minutes	20 marks
	Part 4	6	6	20 minutes	20 marks
	Part 5	4	4	20 minutes	20 marks
Totals		**27**	**27**	**100 minutes**	**100 marks**
Totals		**51**	**51**	**180 minutes (3 hours)**	**180 marks**

Part 1

Marker use

- Answer **all** questions in this part.
- This part assesses **Criterion 4**.

Question 25

a) Transform $2x^2 + 4x - 1$ into the form $a(x-h)^2 + k$ by completing the square.

$2\left[x^2 + 2x - \frac{1}{2}\right]$

$2\left[(x+1)^2 - 1 - \frac{1}{2}\right]$ $-\frac{3}{2}$.

$2(x+1)^2 - 3$ ✓

✓ for working

/2

b) Using this information or the quadratic formula, find the exact solutions to:

$2x^2 + 4x - 1 = 0$

$2(x+1)^2 - 3 = 0$

$2(x+1)^2 = 3$

$(x+1)^2 = \frac{3}{2}$

$(x+1) = \pm\sqrt{\frac{3}{2}}$

$x = -1 \pm \sqrt{\frac{3}{2}}$

$= -1 \pm \frac{\sqrt{6}}{2}$

OR

$x = \dfrac{-4 \pm \sqrt{4^2 - 4 \times 2 \times -1}}{2 \times 2}$

$= \dfrac{-4 \pm \sqrt{24}}{4}$

$= \dfrac{\cancel{2}(-2 \pm \sqrt{6})}{\cancel{4}\,2}$

$= -1 \pm \sqrt{6}/2$

✓ for working
✓ for answer.
I wouldn't expect it simplified.

/2

Question 26

Expand $(1-2x)^4$ using Pascal's triangle or the Binomial Theorem to assist.

✓ $1(1)^4 + 4(1)^3(-2x) + 6(1)^2(-2x)^2 + 4(1)(-2x)^3 + 1(-2x)^4$

✓ $1 - 8x + 24x - 32x^3 + 16x^4$

/2

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

Question 27

Marker use

The cost of hiring an e-scooter through Venus e-scooters in Burnie is a \$1 unlocking fee and then \$0.45 per minute. Another company, Saturn e-scooters, is looking to open up their e-scooter business in Burnie and has a pricing model of a \$5 unlocking fee and then \$0.25 per minute.

a) Write **two (2)** equations for the price (**P**) of using an e-scooter through Venus and Saturn for a trip of **m** minutes. /1

$P = 1 + 0.45m$ ① Venus 0.5

$P = 5 + 0.25m$ ② Saturn 0.5

b) Solve these equations simultaneously, showing full algebraic working, to find the number of minutes riding and the price paid when both e-scooter companies charge the same amount. /2

② − ①

$0 = 4 - 0.2m$ sub in ①

$0.2m = 4$ $P = 1 + 0.45 \times 20$

$m = 4/0.2$ $P = 10$ ✓

$= 20$ ✓

∴ At 20 minutes both charge \$10

Question 28

Marker use

The hypotenuse of a right-angled triangle is $(2x + 3)$ cm and the other two (2) sides are $2x$ cm and $(x - 2)$ cm.

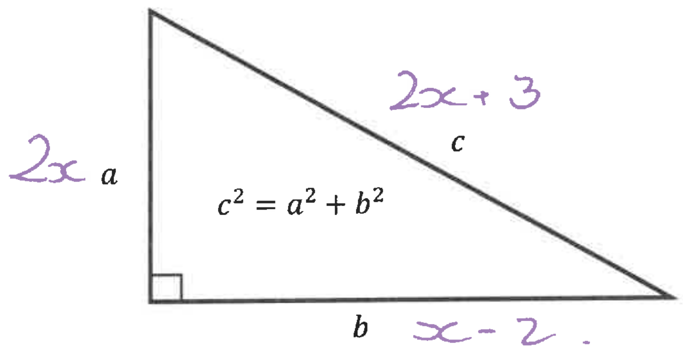


Figure 8: Diagram of a right-angled triangle.

a) Show that Pythagoras' rule as shown in Figure 8 can be simplified to give $x^2 - 16x - 5 = 0$ for this triangle. /2

$(2x)^2 + (x-2)^2 = (2x+3)^2$ ✓

$4x^2 + x^2 - 4x + 4 = 4x^2 + 12x + 9$ ½

$\therefore\ x^2 - 16x - 5 = 0$ ½

b) Hence, determine the value(s) of x. Give your answer(s) to **one (1)** decimal place. /2

$x^2 - 16x - 5 = 0$

$x = -0.3066$ OR $x = 16.3066$ ✓

$\therefore\ x = 16.3$ ✓ (can't have a negative length)

Question 29

a) Show the discriminant of the equation $2x^2 - 2x + 1 = m$ is $\Delta = 8m - 4$.

$\Delta = b^2 - 4ac$ $\quad 2x^2 - 2x + (1-m) = 0$

$\Delta = (-2)^2 - 4(2)(1-m)$

$= 4 - 8 + 8m$

$= 8m - 4$ as required

b) Hence, find the value of m for which the equation has **one (1)** solution.

for one solution $\Delta = 0$

$8m - 4 = 0$

$8m = 4 \quad m = \frac{1}{2}$

Question 30

The number of bacteria in a slow growing culture is given by $N = 50 + 20t + t^3$ where t is the number of hours after 8:00am.

a) What is the initial number of bacteria at 8:00am?

$t = 0 \quad N = 50$

b) The experiment is stopped when the number of bacteria reaches 500. At what time of day is the experiment stopped?

$500 = 50 + 20t + t^3$

$t^3 + 20t - 450 = 0$

$t = 6.797$

≈ 6 hour 48 min

∴ Stopped at 2:48 pm

Marker use
/2
/1
/1
/3
Total C4 /20

Part 2

- Answer **all** questions in this part.
- This part assesses **Criterion 5**.

Marker use

Question 31

Determine the equation of the line shown:

/2

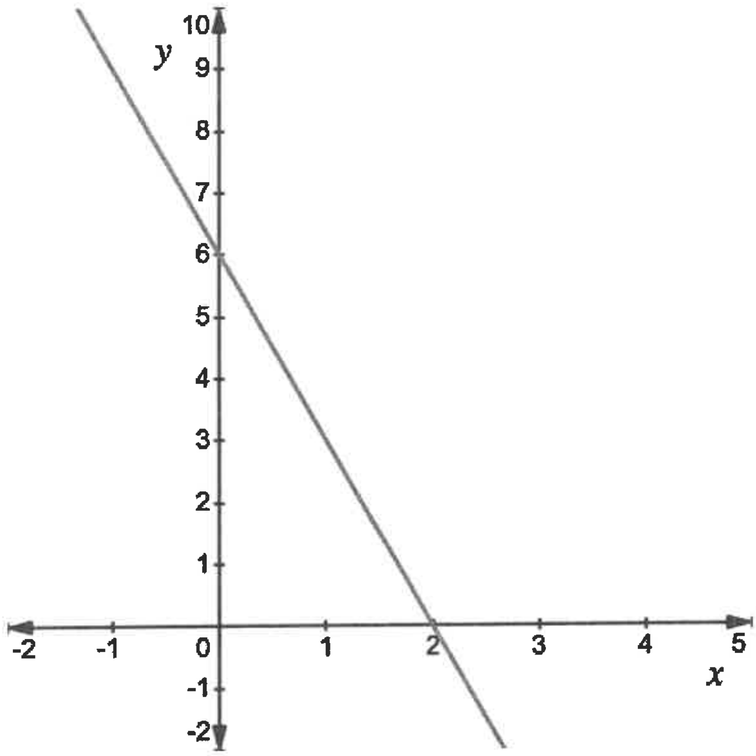


Figure 9: Graph of a straight line.

y int = 6 0.5 gradient = $\frac{\text{rise}}{\text{run}}$ 0.5 = $\frac{-6}{2}$ = -3

$y = -3x + 6$ ✓

Question 32

Marker use

/2

The function $y = f(x)$ is graphed below (Figure 10). The function $y = g(x)$ is the function $y = f(x)$ after it has been translated left two (2) units and three (3) units up. On the same set of axes, graph the function $y = g(x)$.

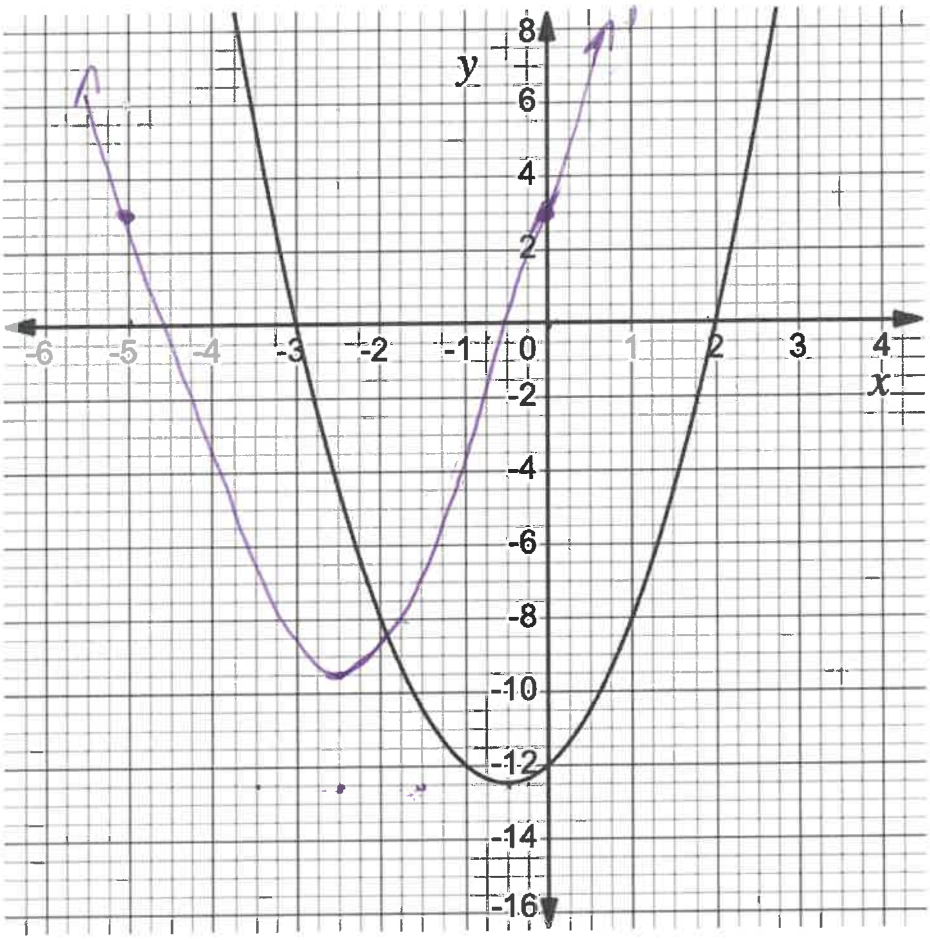


Figure 10: Graph of the function $y = f(x)$.

Spare diagram used (X) ☐

- the 3 clear points on the original must be clear on the final graph (ie. x ints and y int)
- another pt clear. (0.5 for each)

NOTE: the ~~xx~~ scale on the y axis is in 2's so this may be a common error $-\frac{1}{2}$ if the are consistent.

Question 33

Marker use

A bridge is built over a small creek. The bridge is parabolic in shape and has its feet evenly spaced from the ends of the bridge. The points A, B and C all lie on the parabola. AC is 8m.

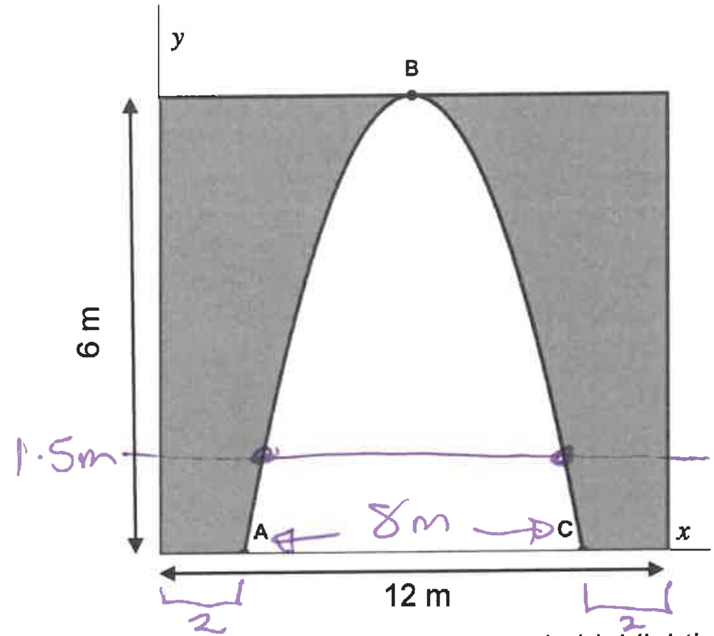


Figure 11: Diagram of a bridge built over a creek, highlighting points A – C.

a) Show the equation of the parabola which goes through A, B and C is:

$$y = \frac{-3}{8}x^2 + \frac{9}{2}x - \frac{15}{2}$$

/2

$B(6,6)$

$y = a(x-2)(x-10)$ 0.5

$6 = a(4)(-4)$

$6 = a(-16)$

$a = -3/8$

$y = -\frac{3}{8}(x-2)(x-10)$

$y = -\frac{3}{8}x^2 + \frac{9}{2}x - \frac{15}{2}$ 0.5

b) After heavy rain, the creek reaches 1.5m above AC. Find the width of water within the bridge to 1 decimal place.

/2

$-\frac{3}{8}x^2 + \frac{9}{2}x - \frac{15}{2} = \frac{3}{2}$

$x = 2.5359$ OR $x = 9.4641$

width $= 9.4641 - 2.5359$

$= 6.9282 \approx 6.9$m width

min (-20.7)
max (12.6)

Question 34

Marker use

a) Graph the function $y = x(x+3)(x-4)$ for the domain $x \in [-5, 6)$

/4

$x = -5$ $\quad$ $x = 6$ $\quad$ x int $y=0$

$y = -5(-2)(-9)$ $\quad$ $y = 6(9)(2)$ $\quad$ $x=0$ $x=-3$ $x=4$

$= -90$ $\quad$ $= 108$ $\quad$ y int $x=0$

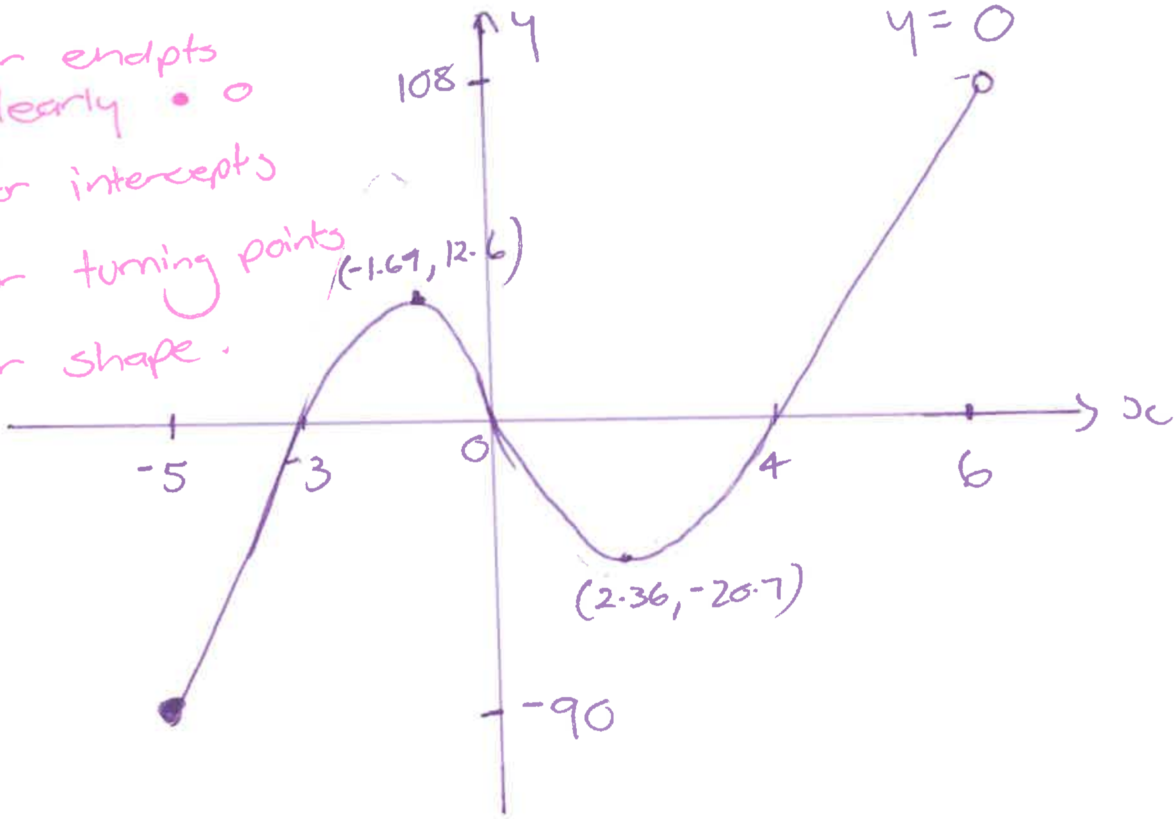
$y=0$

✓ for endpts clearly • o

✓ for intercepts

✓ for turning points

✓ for shape.



b) State the range.

/1

Range $y \in [-90, 108)$ ✓

Question 35

/3

Determine the equation of the line which is perpendicular to $4y - 2x + 3 = 5$ passing through the point $(-3, -1)$

$4y = 2x + 2$

$y = \frac{1}{2}x + \frac{1}{2}$.

$m_1 = \frac{1}{2}$ ✓

$m_2 = \frac{-1}{\frac{1}{2}} = -2$. ✓

$y + 1 = -2(x+3)$

$y + 1 = -2x - 6$

$y = -2x - 7$ ✓

Question 36

Marker use

A rectangular storage container is designed with an open top and a square base.

The base has a side length of x m and the height of the container is h m. The 12 edges are strengthened with 6m of metal.

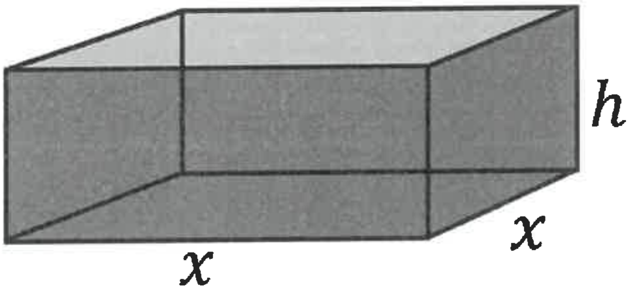


Figure 12: Diagram of a rectangular container, highlighting side length and height.

a) Show that $h = 1.5 - 2x$

$8x + 4h = 6$ ½

$4h = 6 - 8x$

$h = 1.5 - 2x$ ½

/1

b) Hence, show the expression for the volume (V) of the container is $V = 1.5x^2 - 2x^3$

$V = x \times x \times h$

$= x^2(1.5 - 2x)$ ½

$= 1.5x^2 - 2x^3$ ½

/1

c) State any restrictions on the values x can take.

$h > 0$ $x > 0$

$1.5 - 2x = 0$ ✓ ✓

$2x = 1.5$ $\therefore$ $0 < x < 0.75$

$x = 3/4$

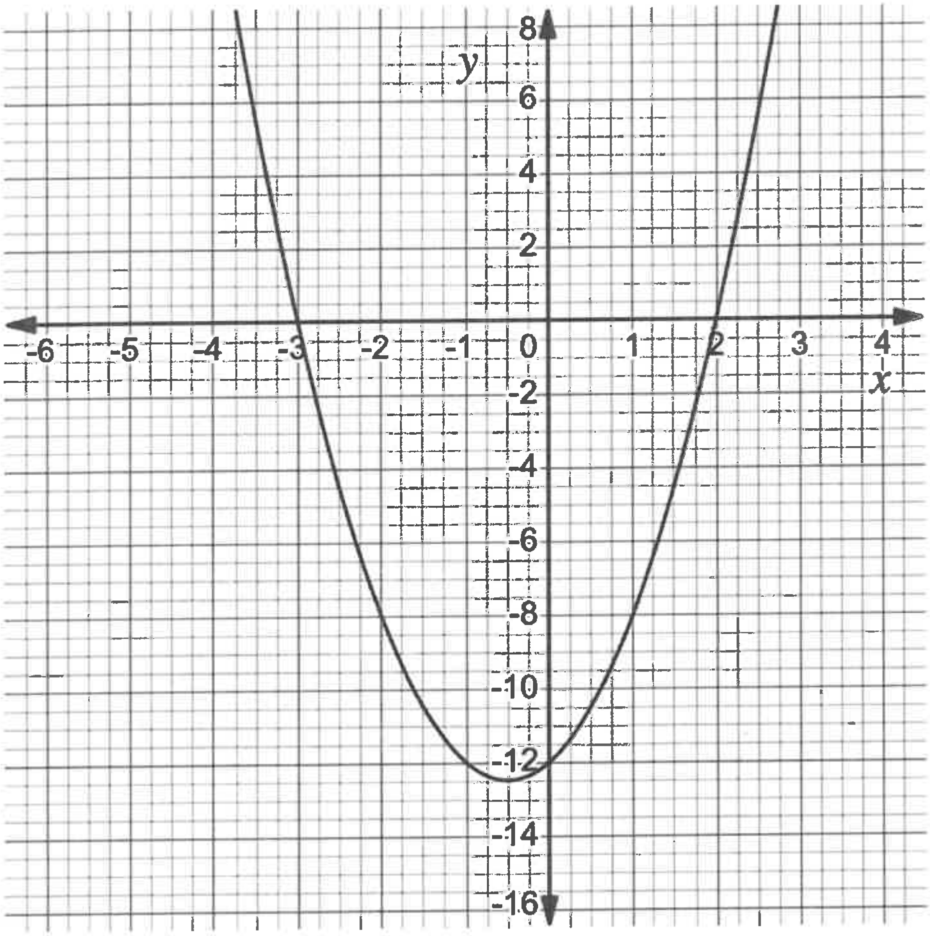
/2

Total C5

/20

Spare Diagram

Question 32



Part 3

- Answer **all** questions in this part.
- This part assesses **Criterion 6**.

Marker use

Question 37

Convert 75° into radians, giving your answer to **three (3)** decimal places.

$\frac{75 \times \pi}{180} = 1.309$ (3dp) ✓

/1

Question 38

A teacher is concerned about the amount of time he spends dealing with the number of emails he receives daily. The teacher starts keeping records and finds the average number of emails he receives each day can be modelled by:

$$E = 30 \times 2^{\frac{t}{12}}$$

Where E is the average number of emails received per day and t is the number of weeks from the time he started taking records.

a) What was the average number of daily emails the teacher was receiving when he first started taking records?

$E = 30 \times 2^{0/12}$

$= 30$ emails ✓

/1

b) How many weeks will it take before the predicted average number of emails received will double?

$60 = 30 \times 2^{\frac{t}{12}}$

$2 = 2^{\frac{t}{12}}$ ✓

$\frac{t}{12} = 1$

$t = 12$ weeks ✓

/2

Question 39

This crane has been seen in the Hobart skyline for a number of months. At the time of this photo, the largest angle inside the triangle was 100° and the smallest angle was 25°. The length of wire connecting these two (2) angles was 25m.

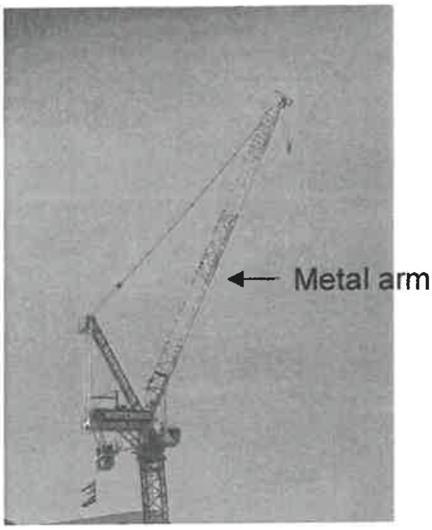


Figure 13: Photo of a mechanical crane, highlighting its metal arm.

a) Place this information on the triangle below.

✓ for notation in purple. /1

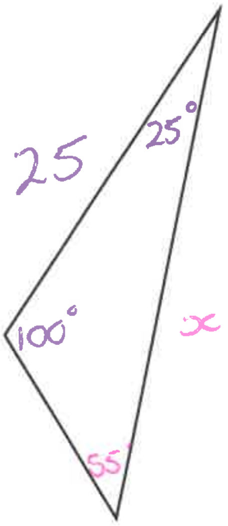


Figure 14: Diagram for annotation to answer Question 39 a).

Spare diagram used (X) ☐

b) Hence, find the length of the metal arm of the crane. /2

$$\frac{x}{\sin 100°} = \frac{25}{\sin 55°} \checkmark$$

$$x = \frac{25}{\sin 55} \times \sin 100° = 30.055$$

$\approx 30m$ ✓

Question 40

Marker use

For the function $y = 3\log_2(x+2) + 1$

a) State the transformations, in order, to transform $y = \log_2 x$ to this function.

- dilate by a factor of 3 [0.5] in the direction of the y axis
- translate left [0.5] 2 units
- translate up [0.5] 1 unit

0.5 $1/2$

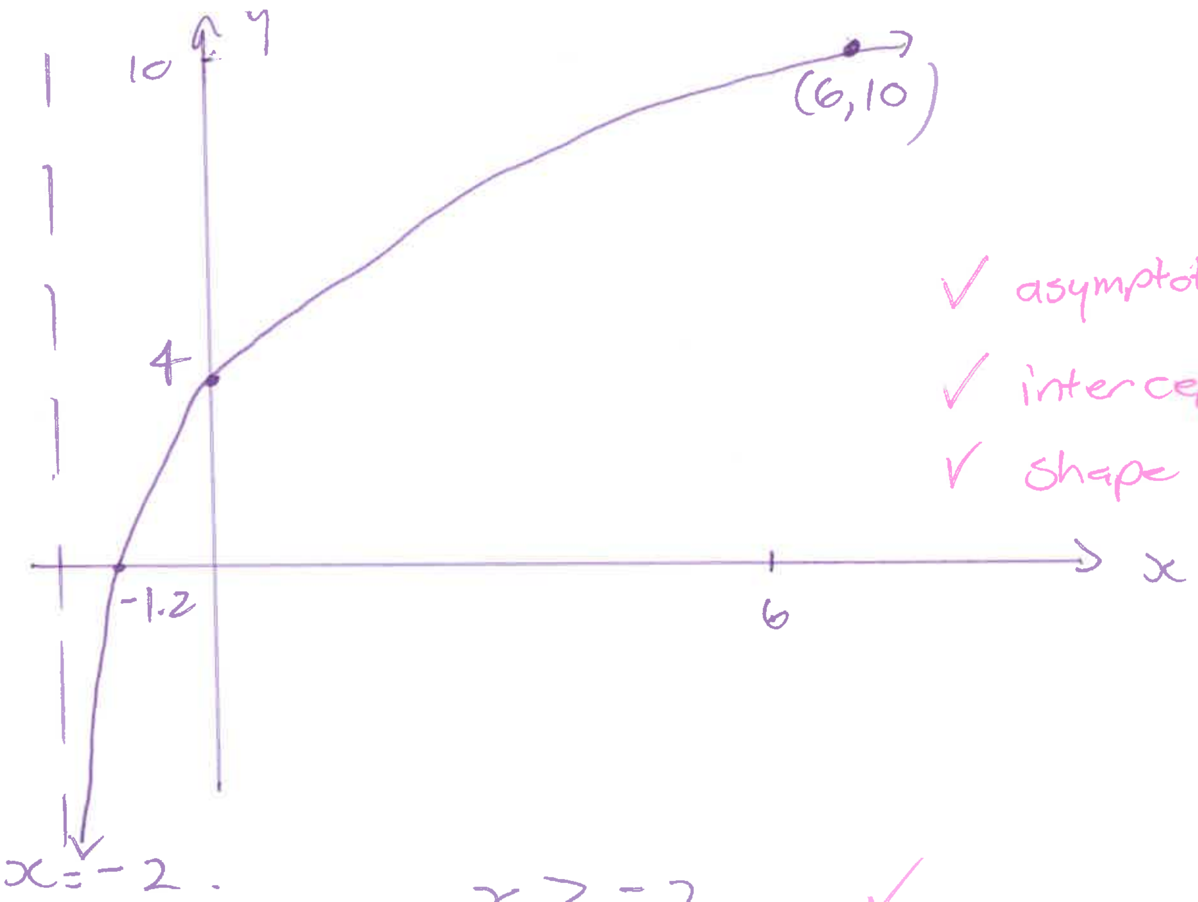
b) Sketch the graph of the function.

when $x = 0$ $y = 3\log_2 2 + 1$
$= 3 \times 1 + 1 = 4$

when $y = 0$
$x = -1.2$

when $x = 6$ $y = 3\log_2 8 + 1$
$= 10$

$/3$



✓ asymptote
✓ intercepts
✓ shape (another pt)

c) State the function's domain: $x > -2$ ✓

$/1$

d) State the function's range: $y \in \mathbb{R}$ ✓

$/1$

Question 41

Marker use

Sam plays with a yo-yo attached to a string. The yo-yo goes up and down from its rest position so that the distance above the rest position, D cm, at time t seconds is given by:

$$D = -40\cos\left(\frac{\pi}{3}t\right)$$

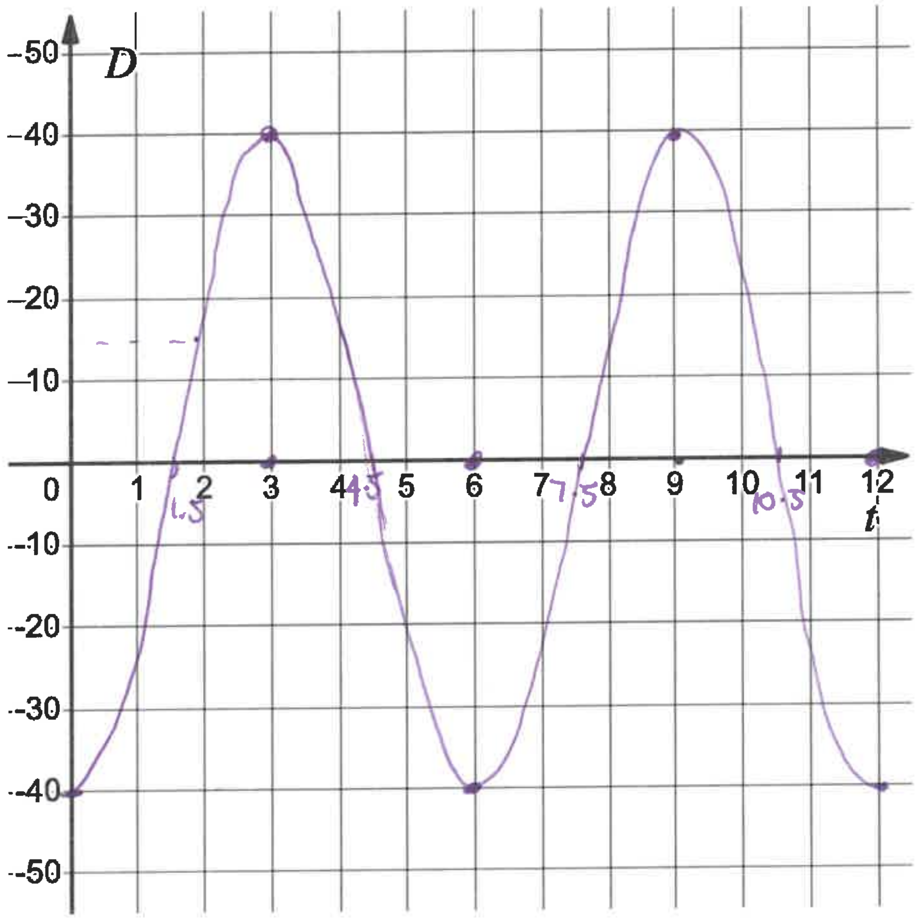
a) Determine the period of the yo-yo.

period $= \dfrac{2\pi}{\pi/3} = 2\pi \times \dfrac{3}{\pi} = 6$ ✓

/1

b) Draw a graph of the yo-yo's movement over 12 seconds. Clearly label all intercepts.

/3



✓ shape.

✓ intercepts clear

✓ period.

Figure 15: Axes for drawing answer to Question 41 b).

Spare diagram used (X) ☐

c) After how many seconds does the yo-yo first reach a height above 15cm? Give your answer to **two (2)** decimal places.

$-40\cos\left(\frac{\pi}{3}t\right) = 15$ ✓

$t = 1.87$ seconds. ✓

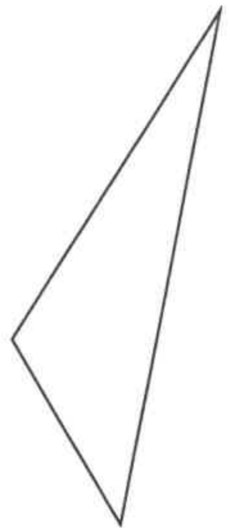
/2

Total C6

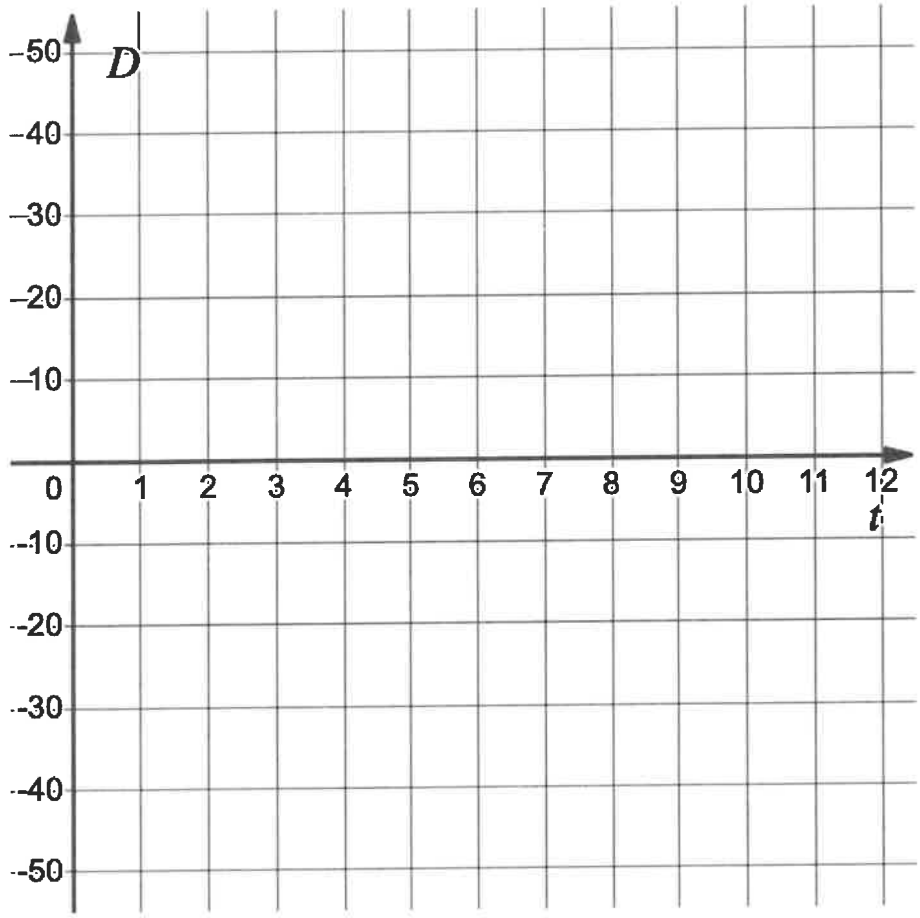
/20

Spare Diagrams

Question 39 a)



Question 41 b)



Marker use

Part 4

- Answer **all** questions in this part.
- This part assesses **Criterion 7**.

Question 42

Figure 16 shows the motion of a stone over time. The vertical height (H) is measured in metres and the time (t) is measured in seconds.

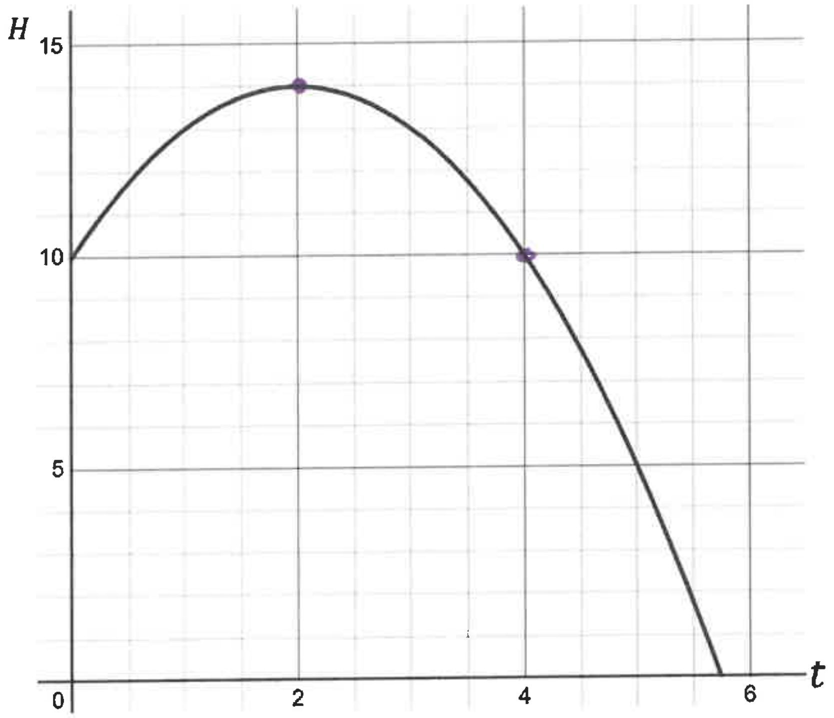


Figure 16: Graph of the motion of a stone over time.

a) Use **Figure 16** to determine the vertical height at two (2) seconds and four (4) seconds.

/1

$H_2 = 14$ m 0.5

$H_4 = 10$ m 0.5

b) Determine the average rate of change of the stone's height from two (2) seconds to four (4) seconds.

/2

$\text{Av. rate} = \dfrac{10-14}{4-2}$ ✓

$= \dfrac{-4}{2}$

$= -2$ m/s ✓ $\left(-\tfrac{1}{2}\right.$ for units incorrect

Question 43

Marker use

A bank of dirt has the cross section shown below. The curve of the bank is given by the equation:

$$H = \frac{x^2}{100}(15 - x) \quad x \in [0,15]$$

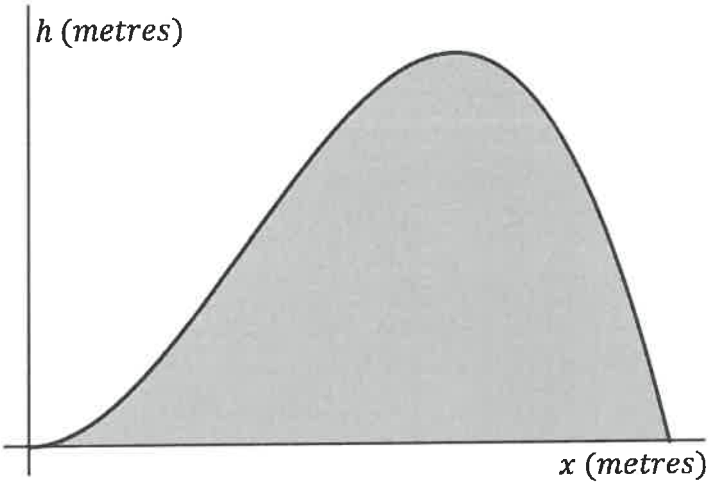


Figure 17: Diagram of a bank of dirt, highlighting h and x.

a) **Using calculus**, find the value of x for which the height is a maximum (no justification is required). /2

$H = \frac{15x^2}{100} - \frac{x^3}{100}$ 0.5

$\frac{dH}{dx} = \frac{30x}{100} - \frac{3x^2}{100}$ 0.5

$\frac{30x}{100} - \frac{3x^2}{100} = 0$ 0.5 For max

$10x - x^2 = 0$

$x(10 - x) = 0$

$\cancel{x = 0}$ or $x = 10$ 0.5 max at $x = 10$m

b) Find the maximum height of the bank. /1

$H = \frac{10^2}{100}(15 - 10)$

$= 1 \times 5$

$= 5$ m ✓

Question 44

Use calculus to determine the equation of the **normal** to the curve

$y = \frac{1}{8}x^4 + 2x^2 - 2x + 5$ at the point $(2, 11)$.

$\frac{dy}{dx} = \frac{4}{8}x^3 + 4x - 2$

when $x = 2$

$m_T = \frac{4}{8}(2)^3 + 4 \times 2 - 2$

$= 4 + 8 - 2$

$= 10$ 0.5

$m_N = \frac{-1}{10}$ 0.5

$y - 11 = \frac{-1}{10}(x - 2)$

$y - 11 = \frac{-1}{10}x + \frac{2}{10}$

$y = \frac{-1}{10}x + 11\frac{1}{5}$

Marker use

/3

Question 45

If the curve $y = 2x^2 + px - 3$ has a gradient of -2 when $x = -1$, find the value of p.

$\frac{dy}{dx} = 4x + p$

$4(-1) + p = -2$

$-4 + p = -2$

$\therefore p = 2.$

/2

Question 46

Marker use

a) Using calculus, find the stationary point(s) of $f(x) = 1 - 4x + 4x^2 - x^3$ — /3

$f'(x) = -4 + 8x - 3x^2$ 0.5 for SP $f'(x) = 0$

$-4 + 8x - 3x^2 = 0$ 0.5

$x = \frac{2}{3}$ 0.5 or $x = 2$ 0.5

$y = 1 - 4(\frac{2}{3}) + 4(\frac{2}{3})^2 - (\frac{2}{3})^3$ $y = 1 - 4(2) + 4(2)^2 - (2)^3$

$y = \frac{-5}{27}$ 0.5 $= 5$ 0.5

$\therefore$ SP $(\frac{2}{3}, \frac{-5}{27})$, $(2, 5)$

b) Justify the nature of the stationary point(s) found in a). — /2

x	0	$\frac{2}{3}$	1	2	3
$f'(x)$	−	0	+	0	−

$\therefore$ min $(\frac{2}{3}, \frac{-5}{27})$

max $(2, 5)$

min max

Question 47

The position p metres, at time t seconds ($t \geq 0$) of a particle is given by $p = t^2 - 5t + 6$

a) When does $\frac{dp}{dt}$ equal zero? — /2

$\frac{dp}{dt} = 2t - 5$ $2t - 5 = 0$

$t = 2.5$ seconds

b) What is the particle's position at this time? — /1

$p = (2.5)^2 - 5(2.5) + 6$

$= -0.25$ m.

c) What is the significance of this point in terms of the particle's motion? — /1

the particle turns at $t = 2.5$ seconds when -0.25 m behind the 0 value.

Total C7 /20

Part 5

- Answer **all** questions in this part.
- This part assesses **Criterion 8**.

Question 48

Four (4) identical balls are numbered 2, 4, 6 and 8 and put into a box. A ball is randomly drawn from the box and not returned. A second ball is then randomly drawn from the box.

a) Complete Table 2 to display the **sum** of the **two (2)** balls.

Ball 2 \ Ball 1	2	4	6	8
2		6	8	10
4	6		10	12
6	8	10		14
8	10	12	14	

Table 2

~ 0.5 each.

Spare diagram used (X) ☐

b) What is the probability the sum of the **two (2)** numbers is more than 7?

$Pr(S>7) = \frac{10}{12} = \frac{5}{6}$ ✓

c) Given that the sum of the **two (2)** balls is 10, what is the probability the number on the second ball was a 2?

$Pr(A|B) = \frac{Pr(A \cap B)}{Pr(B)}$

$= \frac{1/12}{4/12}$ ✓ $= \frac{1}{4}$. ✓

Marker use

/2

/1

/2

Question 49

Marker use

Two (2) events A and B are such that $\Pr(A) = 0.6$, $\Pr(B) = 0.55$ and $\Pr(A \cup B) = 0.86$

a) Find $\Pr(A \cap B)$ /2

$0.86 = 0.6 + 0.55 - \Pr(A \cap B)$ ✓

$0.86 = 1.15 - \Pr(A \cap B)$

$\Pr(A \cap B) = 0.29$ ✓

b) Determine, showing mathematical working, if the events A and B are independent. /2

Independent if $\Pr(A) \times \Pr(B) = \Pr(A \cap B)$

ie $0.6 \times 0.55 = 0.29$ ✓

$0.33 \neq 0.29$ $\therefore$ not independent ✓

Question 50

Incy Wincy Spider likes to climb up the water spout. His chance of falling is affected by the weather. If it is raining, the probability he will fall is 0.75. If it is not raining, the probability he will fall is 0.05. On average, it rains **three (3)** days out of 10 where Incy Wincy lives.

a) Complete the tree diagram for Incy Wincy's water spout climbing using this information. A sample space is not required. /2

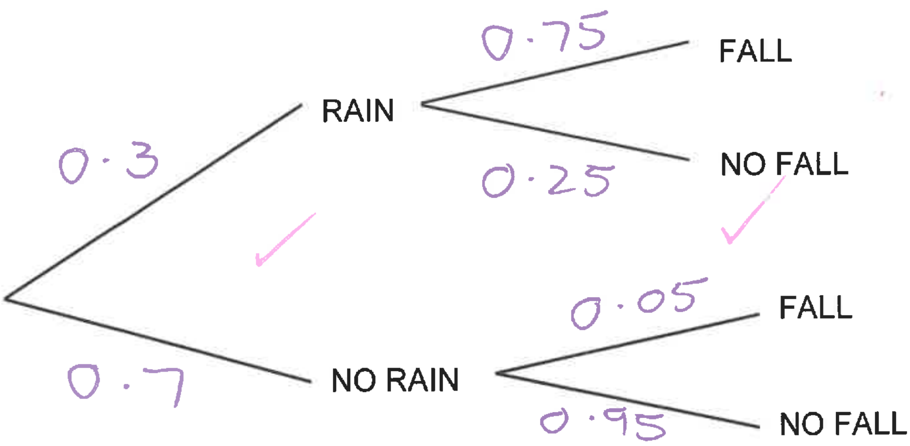


Figure 18: Tree diagram for completion to answer Question 50 a).

Spare diagram used (X) ☐

b) Find the probability that Incy Wincy will make it to the top of the spout without falling. /2

$\Pr(\text{success}) = 0.3 \times 0.25 + 0.7 \times 0.95$ ✓

$= 0.74$ ✓

Question 51

Marker use

A seven-member committee is to be made up of representatives from a number of sports:

- 12 cricket players
- 10 netball players and
- 3 badminton players.

a) How many seven-member committees are possible?

$^{25}C_7 = 480\,700$ ✓

/1

b) What is the probability the committee is made up of only cricket players **or** only netball players?

$^{10}C_7 = 120$ ✓

$^{12}C_7 = 792$

$Pr(\text{only } C \text{ or } N) = \frac{120+792}{480\,700}$

$= \frac{912}{480\,700}$

$= \frac{12}{6325}$ OR 0.001897 ✓

/2

c) What is the probability the committee contains at least **one (1)** badminton player?

$Pr(B \geq 1) = 1 - Pr(0)$

$^{3}C_0 \times {}^{22}C_7 = 170\,544$

$= 1 - \frac{170\,544}{480\,700}$ ✓

$= \frac{371}{575}$ ✓

/2

d) What is the probability the committee contains exactly **three (3)** badminton players, given that it contains at least **one (1)** badminton player?

$Pr(3 | \geq 1) = \frac{Pr(3 \cap \geq 1)}{Pr(\geq 1)}$

$= \frac{Pr(3)}{Pr(\geq 1)} = \frac{\frac{170\,544}{480\,700}}{\frac{371}{575}}$ ✓

$^{3}C_3 \times {}^{22}C_7 = 170\,544$

$= \frac{204}{371}$ ✓

/2

Total C8

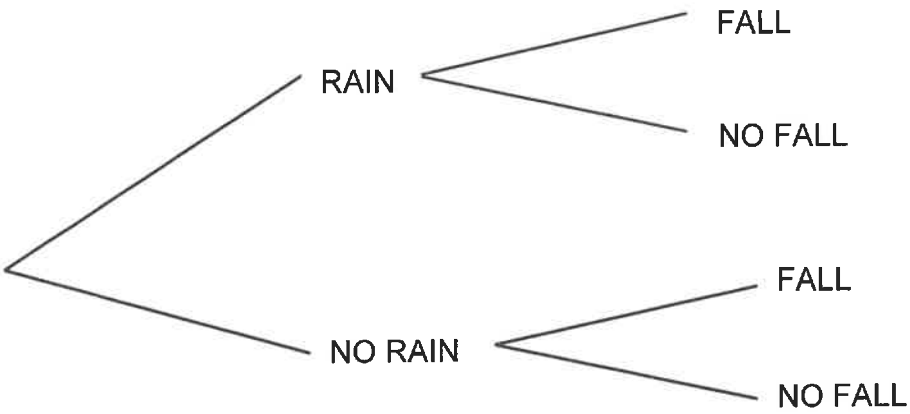
/20

Spare Diagrams

Question 48 a)

Ball 1 / Ball 2	2	4	6	8
2		6	8	10
4	6			
6	8			
8	10			

Question 50 a)



End of Section B

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ASSESSMENT, STANDARDS

& CERTIFICATION