2024 ASSESSMENT REPORT

MTM315117 MATHEMATICS METHODS FOUNDATION

Section A

General Comments

Students who understood the content of this course performed well in this section. Many errors were made from not answering questions completely. For example, students may have factorised an expression correctly, then not provided the solutions in their answer. Several errors were also made with basic mathematical operations, particularly fractions and directed numbers. The presentation and drawing of graphs had a noticeable improvement this year with most students using a ruler for straight lines and taking more care when sketching curves. The labelling of intercepts, axes of symmetry, turning points and points of inflection have improved as well. Students are reminded that questions worth 2 or more marks must include mathematical working to show how the question was started, worked through, and arrived at the answer. Of all the criteria assessed, Criterion 6 (Logarithmic, Exponential and Trigonometric Functions) is an area that students need to be more prepared for.

Part 1 – Criterion 4

Question 1

a. 1 mark

Many students made the same error with this question by giving only one possible solution. Students who factorised the expression using the difference of squares were more successful in gaining the full mark as they recognised and gave the two possible solutions.

b. 2 marks

Well completed overall. Most students factorised the expression correctly then gave the two solutions.

c. 2 marks

This question was not completed well. Most students could not correctly put the LHS expression over a common denominator, then go on to rearrange the equation and solve for x. A number of errors were made in the expansion of $2(x + 1)$.

Question 2

a. 1 mark

This question was completed well.

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b. 3 marks

Students who used the Binomial Theorem or Pascal's Triangle correctly were successful in expanding the expression accurately. Some errors were made when calculating the resultant coefficients and these errors related mostly to squaring and cubing negative numbers. Students need to be aware that when a question states to expand using the Binomial Theorem or Pascal's Triangle, they must demonstrate their understanding of this technique, using either method to gain full marks.

Question 3

a. 1 mark

The question was completed well. Some students did not give their answer as a positive index.

b. 2 marks

Students did not complete this question well. Errors varied widely with students not squaring the 2 in the first term, and then incorrectly applying the first index law of adding indices when multiplying. Some students did not express their final answer in positive index form.

Question 4

a. 1 mark

This question was well completed.

b. 3 marks

Most students could successfully divide $(x+1)$ into $x^3 - 5x^2 + 2x + 8$ using long or synthetic division. Some students did not then factorise the quotient to give the full factorisation. A number of students also did not finish the question by giving the three solutions for x.

Part 1 – Criterion 5

Question 5

a. 2 marks

Most students identified the intercept correctly. The calculation of the gradient was marred by errors, mainly from the subtraction of negative numbers. Students who were successful at finding the correct gradient then went on to find the equation of the line.

b. 1 mark

Most students could use the equation they found in item a) to solve for x.

Question 6

2 marks

Most students could draw a straight-line graph with a positive gradient, yet many answers showed a greater gradient than 2. Most responses showed a negative y intercept, yet many were not to scale. The negative y intercept needed to be the same distance from the origin as the positive y intercept in the original line graph.

Question 7

a. 2 marks

Most students successfully factorised the expression to find the correct x intercepts. The y intercept was found correctly by most students.

b. 2 marks

In general, students could sketch the given quadratic function accurately, showing the x and y intercepts. The axis of symmetry was frequently omitted and, if included, not labelled. Students can improve their marks on graphing questions like this by drawing a single symmetrical curve.

Question 8

a. 1 mark

Well completed by most students.

b. 2 marks

Most students could find the y intercept accurately by substituting $x = 0$ into the equation. Many students had difficulty in solving the equation to find the y intercept. If students could resolve the division and divide -2 by $\frac{1}{4}$ accurately, then take the cube root of -8, they could find the correct x intercept.

c. 2 marks

Most students could sketch a cubic function that exhibited the correct shape. Students are reminded that all significant parts of the graph, including the intercepts and the point of inflection, need to be labelled.

Question 9

a. 1 mark

Some students found it hard to articulate why the relation was not a function.

b. 1 mark

Generally well done. The most common error was incorrect bracket types.

Part 1 – Criterion 6

Question 10

a. 2 marks

This question was not done well by many students. Some students were able to simplify the logarithms to integers and then solve easily for b. Other students who could successfully use the subtraction law for logarithms and then resolve their answer to an integer could then provide a correct answer.

b. 3 marks

This question was not completed well by many students. The main source of error was the inability to complete prime decomposition accurately. All expressions needed to be decomposed to a base of 3 before using the index laws to simplify and determine the value of b.

Question 11

a. 2 marks

In general, this question was well completed by students who could transform the square root of 5^3 to a fractional index. Most students were able to reduce 25^x to 5^{2x} and then solve for x.

b. 2 marks

This question was not completed well. Many students incorrectly assumed that the second and third terms of the equation were the one term, $\log_3(x-1)$, and this is where most of the errors came from in this question. Many students did not use the log laws correctly.

c. 2 marks

This question was not completed well by students. The first source of error was failing to rearrange the equation and recognise that $\sin x = \frac{\sqrt{2}}{2}$. The next source of error was not identifying that sine is positive in the first and second quadrants. Most candidates recognised that $x = \frac{\pi}{4}$ but then did not provide the other possible solution of $\frac{3\pi}{4}$.

Question 12

2 marks

Most students completed this question successfully and were able to recognise the horizontal asymptote of $y = 5$ was the value of b in the given equation. Students then used the point $(0, 3)$ with varying degrees of success to find the value of a.

Question 13

a. 1 mark

Many students did not understand the question with two main erroneous types of answers. The first type was to give $\cos(180° - 30°) = \cos(150°)$ as their answer and the other was to attempt to use the Pythagorean Identity. Of those students who did recognise that they needed to find the value of $\cos(30°)$, many did not take into account that this angle was in the second quadrant and that cos will be negative.

b. 2 marks

This question was not done well. Many students did not recognise that the angle was in the fourth quadrant, meaning Sine is negative. Some could find the angle of $\frac{\pi}{3}$.

Part 1 – Criterion 7

Question 14

a. 1 mark

This question was completed well.

b. 2 marks

Most students could differentiate the expression if they could accurately expand the brackets.

c. 2 marks

Students could perform differentiation well, but some struggled to simplify $\frac{7}{4} \times 8$ to 14. Most students knew to reduce the power of the index by 1 but some had difficulty doing this with fractions.

Question 15

3 marks.

This question was poorly done by most students. There were many types of errors in this question. Predominantly students found it difficult to resolve the fraction before differentiating. Successful candidates identified $5t^3$ as a common factor. Once this was applied, and the numerator multiplied by the denominator of t^{-5}, differentiation could then be performed. Many students did not express their answer with positive indices.

Question 16

a. 3 marks

Reasonably well done. A common error was to equate the derivate function to zero instead of substituting $x = 1$ into the derivate function to find the gradient, m.

b. 3 marks

Most students recognised that the gradient, $m = 18$. Few could then equate the gradient of 18 to the derivate function from item a), $f'(x) = 6x + 6$. Many students attempted to use the point $(1,7)$ from item a) which led to an incorrect equation.

Question 17

2 marks

For students who understood that $f'(x) > 0$ means a positive gradient, this question was completed well. Most students could explain that the gradient is positive when $x < 0$ and when $0 < x < 3$. Some students found it hard to explain and there were some errors with mathematical symbols and notation.

Part 1 – Criterion 8

Question 18

a. 1 mark

Well done by most students.

b. 2 marks

Many students did not understand this question. Some found the probability of having no purple balloons, and others the probability of having at least one purple balloon, which are both incorrect.

Question 19

a. 2 marks

The Venn Diagram was completed accurately by most students.

b. 1 mark

This question was well done by most students.

c. 2 marks

There were a number of errors in this question. Some students found $\Pr(B|P)$ instead of $\Pr(P|B)$. Students who used the conditional probability formula had more success.

d. 2 marks

This question was poorly done by most students. Whilst students could state the correct formula for independence, many used n instead of the probability.

e. 1 mark

Students did not appear to understand this question. Many students confused mutually exclusive with independence.

Question 20

a. 2 marks

The Probability Table was completed well by most students. There were some numerical errors, but the majority of candidates understood that the probabilities needed to sum to 1.0.

b.

i. 1 mark

Most students could extract the answer for this question successfully from the table in item a).

ii. 2 marks

The probability of A or B’ was not calculated accurately by most students. Many students omitted subtracting the intersection of A and B’.

Section B

General Comments

This was a section which rewarded students who had a fluent ability to use their calculator, while students who relied on working by hand struggled to complete the questions in an accurate and timely fashion. Students are reminded that any question which is worth more than one mark needs to be accompanied by working. Students who used exact values throughout their working were more likely to be successful than those who rounded their answers earlier in the question. Additionally, while no questions **required** use of a diagram for working, students who used appropriate diagrams to assist them with their thinking were more likely to be successful.

Part 1 – Criterion 4

Question 21

a. 2 marks

Most students who attempted this question realised they needed to construct some form of algebraic equation. Students who constructed one equation for the number of animals and another equation using two legs for the ducks and four legs for the cows were often successful in solving these simultaneously. Using the calculator proved helpful in avoiding algebraic errors and saving time. Setting out the information provided algebraically was required for full marks.

b. 2 marks

A number of students interpreted the number 72 as the amount of area rather than the perimeter, which gave an incorrect answer of 36. Including a diagram of the two pens was an excellent method of breaking the question down. Students who interpreted the pens as sharing a fence line and reached an answer of 105.8m^2 for the area were fully rewarded.

Question 22

2 marks

This question was generally well done by students. Students who chose to use their calculator to factorise the expression given were unable to receive full marks as the question specifically asked for algebraic working to be shown. Markers were looking either for $8a^3$ to be expressed as $(2a)^3$ or some other line of working to award full marks.

Question 23

a. 2 marks

Most students were familiar enough with this style of question to know that they needed to determine the value of the discriminant without it being explicitly requested. Some students used their calculator to attempt to solve the quadratic equation and hence stated that no real solutions existed, which was an acceptable approach but required some supporting explanation to receive full marks. Students are reminded that there is no need to state rational/irrational where no solutions exist, although no penalty was applied this time if they did so.

b. 2 marks

A number of students ignored the k values in the first two terms, including only $(k + 4)$ in their discriminant. Students are reminded that the values of a and b are the entire coefficients of the x^2 and x terms respectively, not just the integer coefficients. Students were required to explicitly discard the $k = 0$ solution to receive full marks.

Question 24

2 marks

Factorising the coefficient of x^2 out of the expression enabled many students to progress in completing the square. However, a number of students then forgot to expand the 2 back in to obtain the correct k value and lost half a mark for doing so. Students who ignored the coefficient of x^2 and attempted to complete the square were not well rewarded.

Question 25

a. 2 marks

This question was well done by the majority of candidates. Minor mistakes were in rounding incorrectly or not applying the square root to M.

Students are reminded to read the question carefully as many forgot to include units, likely because the units were mentioned at in the stem of Q25, rather than in 25a.

b. 2 marks

A number of students struggled to apply the inverse operation of square rooting in their first line of working. As standard for a two-mark question, a line of working was required for full marks.

Question 26

a. 2 marks

This question was well done by most candidates. Minor errors involved stating one of the solutions as $x = -3$ rather than $x = 3$.

b. 2 marks

Students who substituted the correct values of a, b and c into the quadratic formula as given on the information sheet and used their graphics calculator to simplify the solution were generally very successful in avoiding any mistakes. As the question did not specify whether the answer was to be in exact or decimal form, either form was fully rewarded. Students who chose the decimal form were more likely to receive full marks. Many students who chose to express their answer in exact form struggled to simplify $\frac{6 \pm \sqrt{20}}{4}$ and so lost half a mark.

Part 2 – Criterion 5

Question 27

a. 2 marks

Students generally did well on this question. Some successfully rearranged the equation but failed to state the values of m and c as required by the question. Common errors included failing to divide the 6 by 3.

b. 3 marks

Successful candidates in this section stated the gradient of the perpendicular line then substituted into the slope intercept form or gradient point form. Errors included using the same gradient as 27a) for the perpendicular line but including a negative sign.

Question 28

a. 1 mark

Well done by most students. Common errors included incorrect brackets for the interval notation and reversing the negative infinity and number value.

b. 2 marks

Successful candidates in this section started with $g(x) = a(x+3)(x-1)$. A common error was assuming the y-coordinate of the turning point was -7. Students who used the form $y = a(x-h)^2 + k$ needed to use simultaneous equations to find the answer and were rarely successful. Students needed to state their final answer to receive full marks. Students were not required to expand the final answer pleasingly and very few chose to do so.

c. 2 marks

Several candidates successfully found the coordinates of the turning point but missed adding the range.

d. 1 mark

This was generally well done by students.

Question 29

2 marks

A common error was starting with the cubic equation in the form $y = a(x-h)^3 + k$. Students taking this approach were rarely successful. Errors also often occurred in establishing the value of a with some successfully reaching $-9a = 5$, then adding 9 to both sides, instead of dividing by -9. Pleasingly, most students recognised the repeated factor and included this in their equation.

Question 30

2 marks

For this question, working of some description was required for full marks. Markers were not strict on the form that working had to take. Students who wrote down a new equation after each translation were generally more successful.

Question 31

a. 1 mark

Well done by most students. A small number of students seemed confused by the idea that this was a replica, giving answers in metres rather than centimetres.

b. 1 mark

As for 31a).

c. 3 marks

Students who drew a diagram for this question or annotated the given diagram were more likely to be successful on this question. Students who let $x = 10$ were not well rewarded with marks. No algebraic working was required to obtain full marks, so students with strong calculator skills were more likely to be successful.

Part 3 – Criterion 6

Question 32

a. 1 mark

A common error was using the ratio as the angle, e.g. $\sin\left(\frac{2.9}{4.5}\right) = \theta$.

Many students did not recognise the triangle as a right triangle and used the sine rule. While this could work, some of these students rounded too early leading to an inaccurate answer.

b. 2 marks

Generally well done. The most common error was not including the 4.5cm in the final answer. Students who used rules for right angled triangles were not rewarded.

Question 33

a. 1 mark

Mostly done well. Common errors were not giving units in the answer and not using brackets when entering the exponent in the calculator. Some students also though initial meant $t = 1$.

b. 1 mark

As above. Students were required to round their value for full marks as you cannot have half a fly.

c. 1 mark

This question was generally well done. While most students assumed doubling from the initial value, students who assumed doubling from $t = 5$ were also fully rewarded. Rounding of the time value was not required for full marks.

Question 34

a. 2 marks

Generally done well.

b. 1 mark

Very few students stated the equation of the asymptote, $y = -4$. Many simply stated -4, or the new equation after the translation. Many interpreted the question as a reflection in the x axis.

Question 35

a. 2 marks

Most students attempted this question and achieved partial marks but very few achieved full marks. Many students did not leave the answer as exact values, and many were clearly unfamiliar with a form of this question where a quadrant was not given, so the answer was positive or negative. Many students used right angled triangles to determine ratios and not identities and so lost half a mark. Students who attempted $\sin^{-1}\left(\frac{7}{10}\right) = 44.42° \therefore \cos(44.42) = 0.714$ were not well rewarded.

b. 1 mark

As for 35a).

Question 36

a. 1 mark

Generally done well. Errors commonly occurred when dividing 2π by $\frac{1}{2}$ to find the period.

b. 2 marks

Students who graphed the equation with a calculator answered this well. Common errors were not paying attention to the domain and period that had been determined in item a).

Question 37

a. 2 marks

A common error was to not label the asymptote on the graph. Another error was not labelling the end point or clearly locating the end point on the graph. Marks were also awarded for the shape of the function as it approached the asymptote.

b. 1 mark

Generally done well. Some students did not round the answer to 1 decimal place.

c. 2 marks

Mostly done well by those attempting the problem. A few students found the two wind speeds but were unable to give the difference correctly. Some students did not round to the nearest metre. No algebraic working was required for full marks.

Part 4 – Criterion 7

Question 38

3 marks

Whilst it is understandable some students attempt to save space by writing $2x + 2h \ldots + x^2/h$, they are encouraged not to do so, as only the final term is being divided by the h when expressed as such. They should use brackets if writing in this form.

Common errors included incorrect positive and negative signs after expanding. Students are reminded to use the limit notation correctly. It needs to be on the right side of the equals sign and is not a fraction. Some students included a vinculum between the limit and $h \to 0$. Some students also incorrectly expanded $(x + h)^2$ to $x^2 + h^2$. Students who just wrote the answer were only awarded half a mark.

Question 39

a. 2 marks

Both 39a) and b) were poorly done, with around two thirds of students substituting into the original equations (instead of the differentiated). No marks were awarded for this. Given that the answer of 16km/s was non-sensical for a train, no marks were deducted for wrong or lack or units. Similarly, no marks were deducted for students who crossed out the correct solution and attempted something else, though there were few of these.

b. 1 mark

As for 39a).

Question 40

a. 1 mark

Generally correctly answered.

b. 2 marks

Many students attempted to find the derivative rather than the average rate of change. Students should also remember to include units in questions like this.

c. 2 marks

This question was poorly done with students struggling to expand the $(t^2 + 1)^2$ before finding the derivate. Students who used their calculator to find the derivative were generally more successful. Many students also tried to solve $A(t) = 68$ rather than $A'(t) = 68$ and were not well rewarded for it.

Question 41

2 marks

Most students failed to include the derivative of the horizontal section of the function. A substantial number also didn't draw the derivative passing through the origin. Students were required to include open endpoints for full marks.

Question 42

a. 1 mark

Students are reminded that even though literacy is not being assessed, proper grammar and syntax are still important in communicating understanding. Many students lost marks on this question because they did not communicate effectively, and markers were left uncertain as to their meaning. Students who drew a diagram were generally successful.

b. 3 marks

Students who used their calculator to assist on this question were generally more successful. Students are reminded that they are not expected to show quadratic factorisation by hand in a calculus section. Students did need to show some calculus working to justify the nature of the stationary point for full marks. The second derivative and a sign table were both appropriate methods. When students used a sign table, they were not expected to state the values of the gradient, just the sign.

Question 43

3 marks

Many students struggled with this question. Common mistakes were substituting both points into the derivative, and incorrectly finding the derivative to the negative power. Students should also be reminded that their calculator can solve simultaneous equations, eliminating the risk of algebraic mistakes.

Part 5 – Criterion 8

Question 44

a. 1 mark

Generally well done. A few candidates attempted to find Pr(A').

b. 1 mark

Correctly answered by most students.

c. 2 marks

A majority of students were unable to find the value for Pr(AnB') for the numerator. Students who used a Venn Diagram or probability table were more likely to be successful.

Question 45

a. 1 mark

Correctly answered by most students.

b. 2 marks

Few students remembered the short cut here of finding 28-1. Students who used combinations were much more likely to make small mistakes. The small number of students who multiplied the combinations instead of adding were not well rewarded as the number was implausibly large.

Question 46

3 marks

Generally answered correctly. Common mistakes involved confusing when to use addition and multiplication appropriately and incorrectly stating $\frac{1}{4} \times \frac{1}{4} = \frac{1}{8}$.

Question 47

a. 1 mark

   Generally well done.

b. 1 mark

   Generally well done.

c. 2 marks

   A common mistake was calculating the probability of Joe buying 2 or more plants rather than more than 2 plants from Nursery A. A fully simplified fraction, or a decimal answer was required for full marks.

Question 48

a. 3 marks

   Candidates who did well in this question included all probabilities (as percent or decimal) and labels. Labels should be located correctly at the ends of each branch.

b. 2 marks

   A common incorrect answer was 5/9. A smaller number of students found the correct answer by examining the complementary event. Very few students recognised the shortcut that that the sum of DW, DD and DL was 0.2, and most expressed these individually, but still got the correct answer.

c. 1 mark

   Many candidates missed the statement in the question that the chances in the second round are irrespective of the first round. This fact greatly simplified the question.

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External Assessment 2024

Marking Scheme.

Attach your candidate label here

MATHEMATICS METHODS – FOUNDATION

MTM315117

Section A

Pages: 24
Questions: 20
Information Sheet: 1

Preparation time for this exam: 15 minutes

Suggested working time: 80 minutes

Instructions:

Calculators are not allowed to be used in this section.

Section A will be collected after 80 minutes.

- There are **five (5)** parts to this section.
- Answer **all** questions and **all** items within each question.
- Write your answers in the spaces provided in this exam paper.
  - Spare diagrams have been provided at the end of each part.
    Indicate in the box provided if you have used the spare diagram.
- The exam is **three (3) hours** in length. The suggested working time for this section is **approximately 80 minutes**.
- During the first 80 minutes you may move onto Section B, but you **cannot** use your calculator until told by your supervisor(s).
- The Mathematics Methods – Foundation Information Sheet can be used throughout the exam.
- All answers must be written in **English**.

Marker use	
C4	/ 16
C5	/ 16
C6	/ 16
C7	/ 16
C8	/ 16

Additional Exam Instructions

- You **must** make sure your answers address the listed criteria.
- For questions worth **one (1) mark**, you **are not required** to show workings. Markers will look at the presentation of the answer(s) and at the argument(s) leading to the final answer(s).
- For questions worth **two (2)** or more marks you **are required** to show relevant workings.
- Marks will be allocated:
  - according to the degree to which workings convey a logical line of reasoning, and
  - for suitable justifications and explanations of methods and processes when requested.

Criteria

You **must** make sure your answers address:

- Criterion 4 manipulate algebraic expressions and solve equations
- Criterion 5 understand linear, quadratic and cubic functions
- Criterion 6 understand logarithmic, exponential and trigonometric functions
- Criterion 7 use differential calculus in the study of functions
- Criterion 8 understand experimental and theoretical probabilities and of statistics.

Guide to Exam Structure

		Parts	Questions available	Questions to answer	Suggested working time	Marks available
Section	A	Part 1	4	4	16 minutes	16 marks
		Part 2	5	5	16 minutes	16 marks
		Part 3	4	4	16 minutes	16 marks
		Part 4	4	4	16 minutes	16 marks
		Part 5	3	3	16 minutes	16 marks
Totals			**20**	**20**	**80 minutes**	**80 marks**
Section	B	Part 1	6	6	20 minutes	20 marks
		Part 2	5	5	20 minutes	20 marks
		Part 3	6	6	20 minutes	20 marks
		Part 4	6	6	20 minutes	20 marks
		Part 5	5	5	20 minutes	20 marks
Totals			**28**	**28**	**100 minutes**	**100 marks**
Totals			**48**	**48**	**180 minutes (3 hours)**	**180 marks**

Part 1

Marker use

- Answer **all** questions in this part.
- This part assesses **Criterion 4**.

Question 1

Solve each of the following for x:

a) $x^2 - 9 = 0$

$(x-3)(x+3)=0$

$\therefore x=3$ or $x=-3$

0.5 0.5

/1

b) $x^2 + x - 12 = 0$

$(x+4)(x-3)=0$ 1.0

$\therefore x=-4$ or $x=3$

0.5 0.5

/2

c) $\frac{7x}{3} - \frac{x}{2} = 2(x+1)$

/2

$\frac{14x-3x}{6} = 2x+2$

$11x = 12x+12$ 1.0

$-x = 12$

$\therefore x=-12$ 1.0

Question 2

Marker use

Expand the following expressions:

a) $(x + 6)(3 - x)$ /1

$= 3x - x^2 + 18 - 6x$ 1.0

$= -x^2 - 3x + 18$

b) $(2x - 3)^3$ using Pascal's Triangle or the Binomial Theorem. /3

$= 1(2x)^3(-3)^0 + 3(2x)^2(-3)^1 + 3(2x)^1(-3)^2 + 1(2x)^0(-3)^3$

1
1 1
1 2 1
1 3 3 1

$= 8x^3 - 36x^2 + 54x - 27$

-0.5 for any error

Question 3

Simplify the following expressions, giving your answer in positive index form:

a) $\frac{3b^2c \times 2c^3}{2b^4c}$ /1

$= \frac{6b^2c^4}{2b^4c}$

$= 3b^{-2}c^3$ 0.5 $= \frac{3c^3}{b^2}$ 0.5

b) $\frac{(2e^{-2}f)^2 \times 5ef^{-3}}{2(ef^2)^2}$ /2

$= \frac{4e^{-4}f^2 \times 5ef^{-3}}{2e^2f^4}$

$= \frac{20e^{-3}f^{-1}}{2e^2f^4}$ 1.0

$= 10e^{-5}f^{-5}$ 0.5

$= \frac{10}{e^5f^5}$ 0.5

Question 4

Marker use

a) Show that $(x+1)$ is a factor of $f(x) = x^3 - 5x^2 + 2x + 8$

$f(-1) = (-1)^3 - 5(-1)^2 + 2(-1) + 8$ 0.5

$= -1 - 5 - 2 + 8$

$= 0$ 0.5

/1

b) Hence, solve $x^3 - 5x^2 + 2x + 8 = 0$

/3

$$\begin{array}{r|l} & x^2 - 6x + 8 \\ \hline x+1 & x^3 - 5x^2 + 2x + 8 \\ - & x^3 + x^2 \\ \hline & -6x^2 + 2x \\ - & -6x^2 - 6x \\ \hline & 8x + 8 \\ - & 8x + 8 \\ \hline & 0 \end{array}$$

2.0

$= (x+1)(x^2 - 6x + 8)$

$x \quad -2$

$x \quad -4$

$\therefore (x+1)(x-2)(x-4) = 0$ 0.5

$\therefore x = -1$ or $x = 2$ or $x = 4$. 0.5

Total P1

/16

Part 2

Marker use

- Answer **all** questions in this part.
- This part assesses **Criterion 5**.

Question 5

a) Determine the equation of the function shown: /2

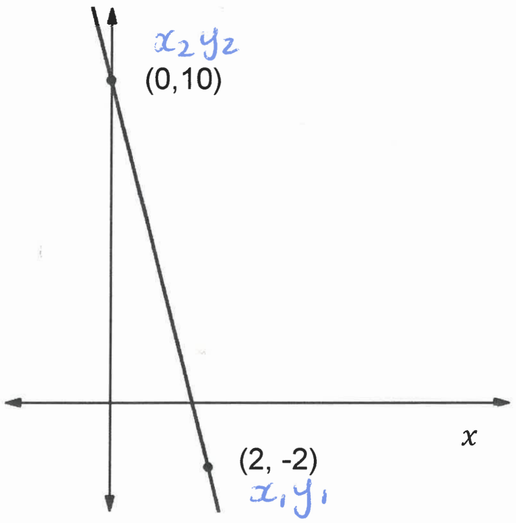


Figure 1: Graph of a function.

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{10 - -2}{0 - 2}$ $\quad c = 10$

$= \dfrac{12}{-2}$ $\quad \therefore m = -6$ 1.0

$y = mx + c$

$\therefore y = -6x + 10$ 1.0

b) Hence find the x intercept of the function. /1

$-6x + 10 = 0$ 0.5

$-6x = -10$

$x = 10/6$ 0.5

$\therefore x = 5/3$

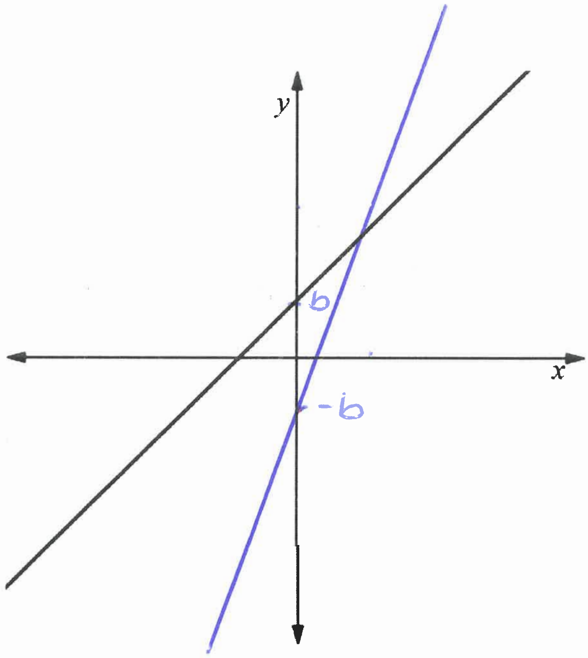
Question 6

Marker use

The graph shown below (Figure 2) has equation $y = ax + b$ where $a, b \in \mathbb{R}$

Using the same axes, sketch the graph of $y = 2ax - b$

/2



0.5 positive

0.5 x2 gradient

0.5 same distance (b and -b)

0.5 if negative

Spare diagram used (X) ☐

Figure 2: Axes for sketching answer to Question 6.

Question 7

Marker use

For the function $y = x^2 + 6x + 8$

a) Determine the x and y intercepts. /2

when $y=0$ $x^2+6x+8=0$ when $x=0$ $y=(0)^2+6(0)+8$

$(x+2)(x+4)=0$ 0.5 $\therefore y=8$ 0.5

$\therefore x=-2$ or $x=-4$

0.5 0.5

b) Sketch the graph of the function on Figure 3, labelling all intercepts and the axis of symmetry (the turning point is not required). /2

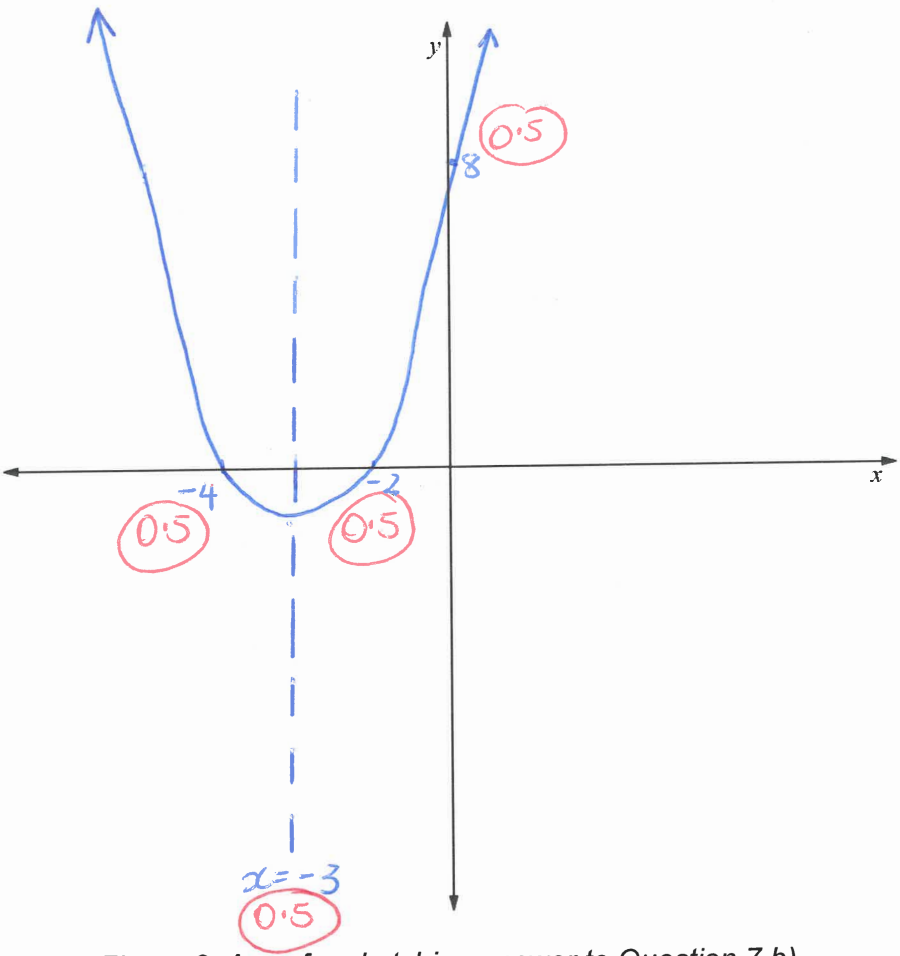


Figure 3: Axes for sketching answer to Question 7 b).

Spare diagram used (X) ☐

Question 8

Marker use

For the function $y = \frac{1}{4}(x+1)^3 + 2$

a) State the co-ordinates of the inflection point.

$(-1, 2)$ 0.5 0.5

/1

b) Determine the x and y intercepts.

When $y=0$: $\frac{1}{4}(x+1)^3 + 2 = 0$

$\frac{1}{4}(x+1)^3 = -2$

$(x+1)^3 = -8$ 0.5

$x+1 = \sqrt[3]{-8}$

$x+1 = -2$

$\therefore x = -3$ 0.5

When $x=0$: $y = \frac{1}{4}(0+1)^3 + 2$

$= \frac{1}{4} + 2$

$= 2\frac{1}{4}$ 1.0

$\therefore y = \frac{9}{4}$

/2

c) Sketch the graph of the function on Figure 4, labelling all intercepts and the inflection point.

/2

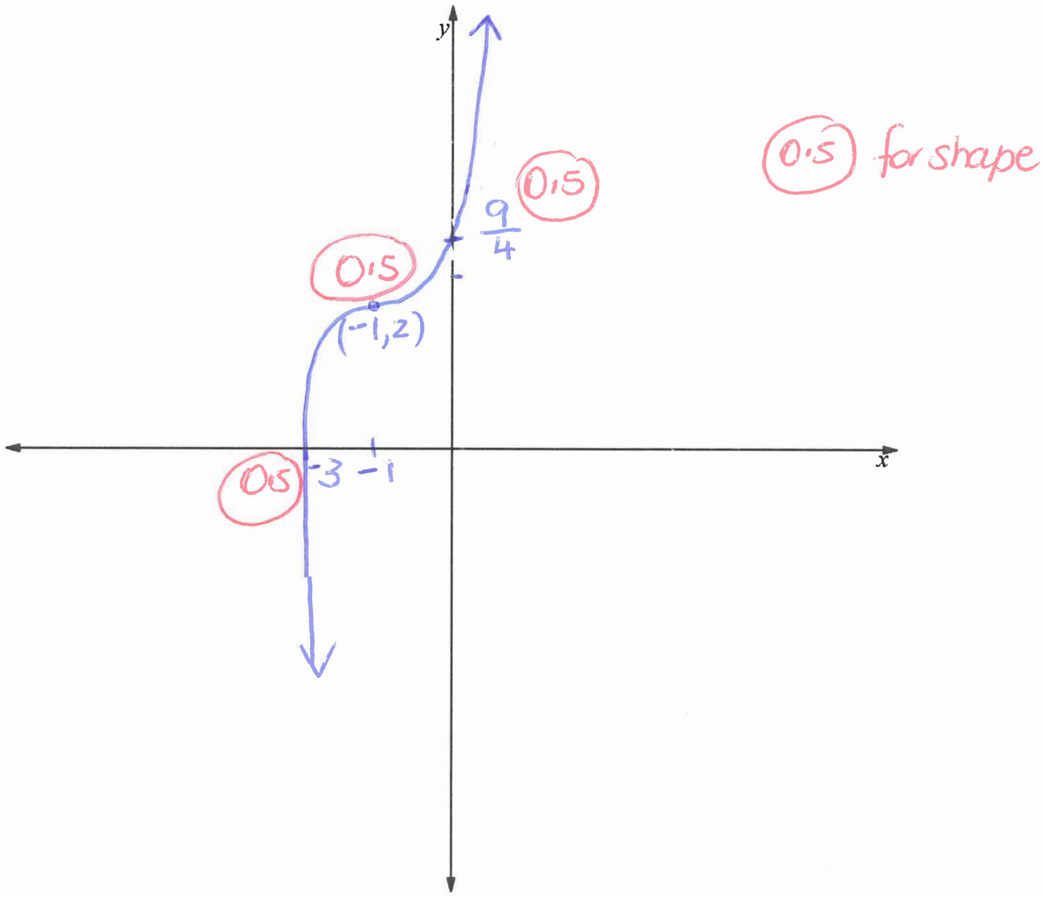


Figure 4: Axes for sketching answer to Question 8 c).

Spare diagram used (X) ☐

Question 9

Marker use

For the relation shown in Figure 5:

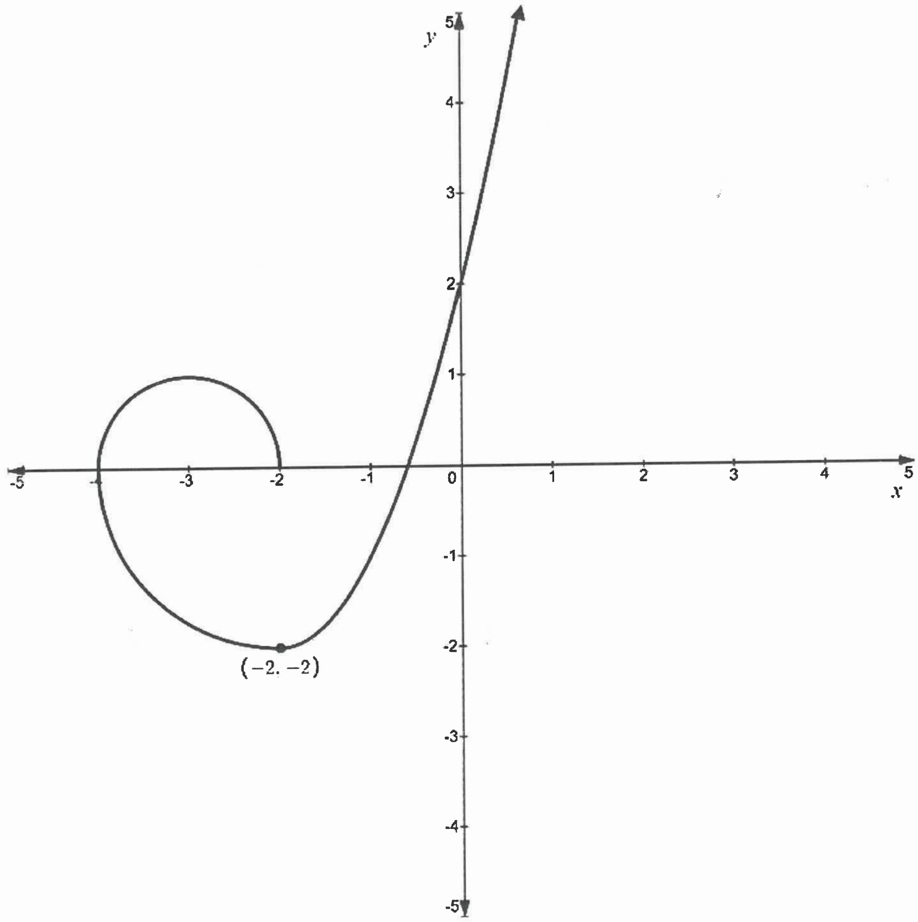


Figure 5: Graph showing relation.

a) State why it is not a function.

More than one y value for some x values OR does not pass the vertical line test.

/1

b) State the range of the relation.

$[-2, \infty)$ OR $y \geq -2$

0.5 0.5

/1

Total P2

/16

Part 3

Marker use

- Answer **all** questions in this part.
- This part assesses **Criterion 6**.

Question 10

Solve the following equations for b:

a) $b = (\log_2 8 - 2\log_2 2) \times \log_4 16$ /2

$b = (\log_2 8 - \log_2 4) \times \log_4 4^2$

$= \log_2 \frac{8}{4} \times 2\log_4 4$ (1.0)

$= \log_2 2 \times 2$

$= 1 \times 2$

$\therefore b = 2$ (1.0)

b) $\frac{3^{4b} \times 27^{1-b}}{9^{b+2}} = \frac{1}{3}$ (0.5) /3

$\frac{3^{4b} \times 3^{3-3b}}{3^{2b+4}} = 3^{-1}$ (0.5) (0.5)

$3^{b+3} = 3^{2b+4} \times 3^{-1}$ (0.5)

$3^{b+3} = 3^{2b+3}$ (0.5)

$b+3 = 2b+3$

$-b = 0$

$\therefore b = 0$ (0.5)

Question 11

Solve the following equations for x:

a) $\sqrt{5^3} \times 5^x = 25^x$ /2

(0.5) $5^{3/2} \times 5^x = 5^{2x}$ (0.5)

$5^{3/2 + x} = 5^{2x}$

$3/2 + x = 2x$ (0.5)

$\therefore x = 3/2$ (0.5)

Question 11 continues

Question 11 continued

Marker use

b) $2\log_3 x - \log_3 x - 1 = \log_3 4$

$\log_3 x^2 - \log_3 x - \log_3 4 = 1$

$\log_3 \frac{x^2}{x} - \log_3 4 = 1$

$\log_3 x - \log_3 4 = 1$

$\log_3 \frac{x}{4} = 1$ (1.0)

$\frac{x}{4} = 3^1$

$\therefore x = 12$ (1.0)

/2

c) $\sin(x) - \frac{\sqrt{2}}{2} = 0$, where $0 \le x \le 2\pi$

/2

$\sin x = \frac{\sqrt{2}}{2}$ sin is positive in the 1st and 2nd Quadrants

θ $\pi - \theta$

(0.5) $\therefore x = \frac{\pi}{4}$ or $x = \pi - \pi/4$ (0.5)

$\therefore x = \frac{\pi}{4}$ or $x = \frac{3\pi}{4}$

(0.5) (0.5)

Question 12

Determine the equation of the graph shown in Figure 6, that has form $y = a \times 2^x + b$

/2

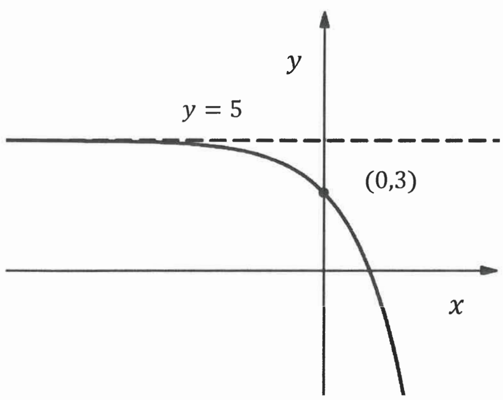


Figure 6: Graph showing equation.

Asymptote at $y = 5$ $\therefore b = 5$

$y = a \times 2^x + 5$ (0.5)

using $(\overset{x}{0}, \overset{y}{3})$

$3 = a \times 2^0 + 5$

$3 = a + 5$

$\therefore a = -2$ (1.0)

$\therefore y = -2 \times 2^x + 5$ (0.5)

Question 13

Marker use

Evaluate each of the following as an exact value:

a) $\cos(180° - 30°)$

2nd quadrant, cos is negative

$= -\cos 30°$ $= -\sqrt{3}/2$

0.5 0.5

/1

b) $\sin\left(\frac{5\pi^c}{3}\right)$

4th quadrant, sin is negative 0.5

$= -\sin(2\pi - 5\pi/3)$ 0.5

$= -\sin \pi/3$ 0.5

$= -\frac{\sqrt{3}}{2}$ 0.5

/2

Total P3

/16

Part 4

Marker use

- Answer **all** questions in this part.
- This part assesses **Criterion 7**.

Question 14

Find the derivative of each of the following:

a) $y = 4x^5 - 3x + 5$

$/_1$

$dy/dx = 20x^4 - 3$ 0.5 0.5

b) $y = (2x + 3)(x - 4)$

$/_2$

$y = 2x^2 - 8x + 3x - 12$

$y = 2x^2 - 5x - 12$ 1.0

$dy/dx = 4x - 5$ 1.0

c) $f(x) = 8x^{\frac{7}{4}} - 3x^{\frac{2}{3}}$

$/_2$

$f'(x) = 7/4 \times 8x^{3/4} - 2/3 \times 3x^{-1/3}$

$= 14x^{3/4} - 2x^{-1/3}$ 0.5 0.5 0.5 0.5

Question 15

Find $h'(t)$ for the following function. Express your answer with positive indices.

$/_3$

$$h(t) = \frac{15t^3 - 20t^4}{5t^8}, \qquad t \neq 0$$

$h(t) = \frac{5t^3(3-4t)}{5t^8}$

$= \frac{3-4t}{t^5}$

$= (3-4t) \times t^{-5}$

$= 3t^{-5} - 4t^{-4}$ 1.5

$h'(t) = -15t^{-6} + 16t^{-5}$ 1.0

$\therefore h'(t) = \frac{-15}{t^6} + \frac{16}{t^5}$ 0.5

Question 16

Marker use

Find the equation of the tangent to the curve $f(x) = 3x^2 + 6x - 2$

a) At the point $(1, 7)$

$f'(x) = 6x + 6$ 1.0

at $x = 1$ $f'(x) = 6(1) + 6$

$\therefore m = 12$ 1.0

$y - y_1 = m(x - x_1)$

$y - 7 = 12(x - 1)$

$y - 7 = 12x - 12$

$\therefore y = 12x - 5$ 1.0

/3

b) Parallel to the line $y = 18x - 10$

$m = 18$ 0.5

$\therefore f'(x) = 18$

$\therefore 6x + 6 = 18$ 0.5

$6x = 12$

$\therefore x = 2$ 0.5

when $x = 2$ $y = 3(2)^2 + 6(2) - 2$

$= 12 + 12 - 2$

$\therefore y = 22$ 0.5

$\therefore$ point at $(2, 22)$

$y - y_1 = m(x - x_1)$

$y - 22 = 18(x - 2)$

$y - 22 = 18x - 36$

$\therefore y = 18x - 14$ 1.0

/3

Question 17

Marker use

/2

For what values of x is $f'(x) > 0$ for the function $f(x)$ shown in Figure 7, that has a stationary point of inflection at $x = 0$?

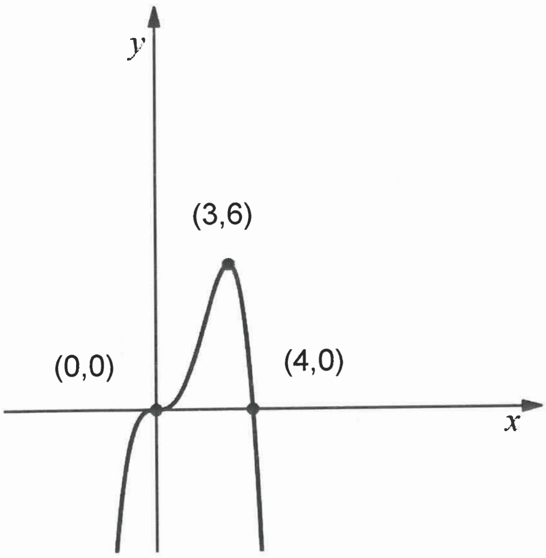


Figure 7: Graph of a function.

when $f'(x) > 0$ gradient is positive.

when $x < 0$ and $0 < x < 3$.

1.0 1.0

Total P4

/16

Part 5

Marker use

- Answer **all** questions in this part.
- This part assesses **Criterion 8**.

Question 18

Ziggy the clown was holding 3 yellow balloons, 6 green balloons and 2 purple balloons before he accidentally let a randomly coloured balloon float away.

a) What is the probability the balloon was yellow?

/1

0.5

$Pr(y) = 3/11$ 0.5

A while later, Ziggy let a second randomly coloured balloon float away.

b) What is the probability that Ziggy still has both purple balloons?

/2

$= 9/11 \times 8/10$ 1.0

$= 72/110$ 1.0

$= 36/55$

Question 19

Marker use

In a crew of 20 pirates, 6 own a parrot and 9 wear bandanas. 7 pirates do not own a parrot or wear bandanas.

a) Complete the Venn diagram in Figure 8, showing this information.

/2

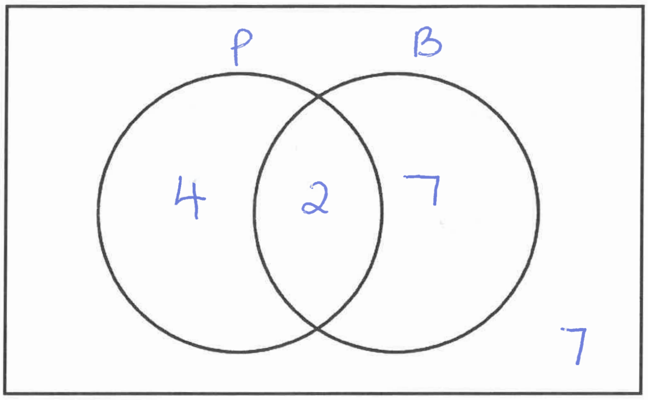


Figure 8: Venn diagram for annotation to answer Question 19 a).

Spare diagram used (X) ☐

b) What is the probability that a pirate owns a parrot and wears a bandana?

/1

$Pr(P \cap B) = 2/20 = 1/10$

c) **Given that** a pirate is wearing a bandana, what is the probability they also own a parrot?

/2

$Pr(P|B) = \frac{Pr(P \cap B)}{Pr(B)} = 2/20 \div 9/20 = 2/9$

d) Determine mathematically if the probability that a pirate wears a bandana is **independent** of owning a parrot.

/2

independent if $Pr(P \cap B) = Pr(P) \times Pr(B)$

$2/20 = 6/20 \times 9/20$

$2/20 \neq \frac{54}{400}$ $\therefore$ NOT independent

e) Owning a parrot and wearing a bandana is **mutually exclusive** on a different pirate crew. Explain what this means.

/1

NO pirate owns a parrot and a bandanna.

Question 20

Marker use

a) Complete the probability table (Table 1) below.

/2

-0.5 for each error

	$\Pr(B)$	$\Pr(B')$	
$\Pr(A)$	0.4	0.1	0.5
$\Pr(A')$	0.16	0.34	0.5
	0.56	0.44	1.0

Table 1

Spare diagram used (X) ☐

b) Hence determine the following:

i. $\Pr(A' \cap B)$

$= 0.16$ 1.0

/1

ii. $\Pr(A \cup B')$

$= Pr(A) + Pr(B') - Pr(A \cap B')$

$= 0.5 + 0.44 - 0.1$ 1.0

$= 0.84$ 1.0

/2

Total P5

/16

TASMANIAN
ASSESSMENT, STANDARDS
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External Assessment 2024

Solutions

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MATHEMATICS METHODS – FOUNDATION

MTM315117

Section B

Pages: 24
Questions: 28
Information Sheet: 1

Suggested working time: 100 minutes

Instructions:

Calculators are allowed to be used in this section.

- There are **five (5)** parts to this section.
- Answer **all** questions and **all** items within each question.
- Write your answers in the spaces provided in this exam paper.
  - o Spare diagrams have been provided at the end of each part. Indicate in the box provided if you have used the spare diagram.
- The exam is **three (3) hours** in length. The suggested working time for this section is **approximately 100 minutes**.
- During the first 80 minutes you may move onto Section B, but you **cannot** use your calculator until told by your supervisor(s).
- The Mathematics Methods – Foundation Information Sheet can be used throughout the exam.
- All answers must be written in **English**.

Marker use	
C4	/ 20
C5	/ 20
C6	/ 20
C7	/ 20
C8	/ 20

Part 1

Marker use

- Answer **all** questions in this part.
- This part assesses **Criterion 4**.

Question 21

a) A farmer has 41 animals made up of ducks and cows. The animals have 136 legs. How many ducks and how many cows does the farmer own?

/2

$41 = d + c \quad -①$

$136 = 2d + 4c \quad -②$

$① \times 2$

$82 = 2d + 2c$

$-(136 = 2d + 4c)$

$-54 = -2c$

$\therefore c = 27$

$41 = d + 27$

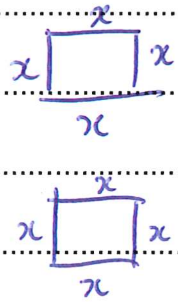
$d = 14$

14 ducks

27 cows

b) The farmer has 72 metres of fencing to make **two** square pens of **equal** size. What will the **area** of each pen be?

/2



$72 = 8x$

$\therefore x = 9$

Area $= 9 \times 9 = 81 m^2$ each

Question 22

Factorise $8a^3 - (3b)^3$ showing algebraic working.

/2

$= (2a)^3 - (3b)^3$

$= (2a - 3b)([2a]^2 + [2a][3b] + [3b]^2)$

$= (2a - 3b)(4a^2 + 6ab + 9b^2)$

Question 23

Marker use

a) Determine the number and type of solutions for the equation $0 = -4x^2 + \frac{7}{2}x - 1$

$\Delta = b^2 - 4ac$

$\therefore \Delta = \left(\frac{7}{2}\right)^2 - 4 \times -1 \times -4$

$= \frac{49}{4} - 16$

$= -\frac{15}{4} = -3.75 < 0 \therefore$ no real Solutions

/2

b) For what value(s) of k does the following equation have one solution?

$$5kx^2 - 6kx + (k+4) = 0, \qquad k \neq 0$$

$\Delta = (-6k)^2 - 4 \times (5k)(k+4)$

$\therefore \Delta = 36k^2 - 20k^2 - 80k$

$= 16k^2 - 80k$

One Solution exists where

$\Delta = 0$

$\therefore 0 = 16k^2 - 80k$

$k = 5, 0$ ← Discard as $k \neq 0$

$\therefore$ One Solution exists where $k = 5$

/2

Question 24

Use completing the square to express $2x^2 + 6x + 2$ in the form $a(x-h)^2 + k$

$= 2(x^2 + 3x + 1) \qquad \left(\frac{3}{2}\right)^2 = \frac{9}{4}$

$= 2\left(\left[x^2 + 3x + \frac{9}{4}\right] - \frac{9}{4} + 1\right)$

$= 2\left(\left[x + \frac{3}{2}\right]^2 - \frac{5}{4}\right)$

$= 2\left(x + \frac{3}{2}\right)^2 - \frac{5}{2}$

/2

Question 25

Marker use

The average molecular speed (metres/seconds) of gas can be found using the formula:

$$\mu = \sqrt{\frac{3RT}{M}}$$

a) Calculate the average molecular speed of a gas particle when $R = 8.314$, $T = 288$, and $M = 0.064$, correct to **two (2)** decimal places. /2

$\mu = \sqrt{\frac{3 \times 8.314 \times 288}{0.064}}$

$= \sqrt{\frac{7183.296}{0.064}}$

$= \sqrt{112,239}$

$= 335.02$ m/s

b) Rearrange the formula to make M the subject. /2

$\mu^2 = \frac{3RT}{M}$

$\therefore M = \frac{3RT}{\mu^2}$

Question 26

Solve the following equations:

a) $(2x + 1)(x - 2)(3 - x) = 0$ /2

$x = -\frac{1}{2}, 2, 3$

b) $0 = 2x^2 - 6x + 2$ by using the quadratic formula. /2

$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4 \times 2 \times 2}}{2 \times 2}$

$\therefore x = \frac{6 \pm \sqrt{36 - 16}}{4}$

$\therefore x = \frac{3}{2} \pm \frac{\sqrt{20}}{4}$

$\therefore x = \frac{3}{2} \pm \frac{2\sqrt{5}}{4}$

$\therefore x = 3/2 \pm \frac{\sqrt{5}}{2}$

OR $\therefore x = 2.618, 0.382$ (to 3 d.p.)

Total P1 /20

Part 2

Marker use

- Answer **all** questions in this part.
- This part assesses **Criterion 5**.

Question 27

The linear equation $3y - 2x = 6$ can be written in the form $y = mx + c$

a) Determine the values of m and c

$3y = 2x + 6$

$\therefore y = \frac{2}{3}x + 2$

$\therefore m = 2/3 \ , \ c = 2$

/2

b) Find the equation of the line perpendicular to $3y - 2x = 6$ which intersects at the point $\left(2, \frac{10}{3}\right)$

/3

$m_{perp} = -\frac{3}{2}$

$y = -\frac{3}{2}x + c$

$\therefore \frac{10}{3} = -\frac{3}{2}(2) + c$

$\therefore c = 19/3$

$\therefore y = -\frac{3}{2}x + \frac{19}{3}$

$\boxed{\text{OR}}$

$y - \frac{10}{3} = -\frac{3}{2}(x - 2)$

$\therefore y = -\frac{3}{2}x + 3 + \frac{10}{3}$

$\therefore y = -\frac{3}{2}x + 19/3$

Question 28

Marker use

The function $g(x)$ is graphed below (Figure 9).

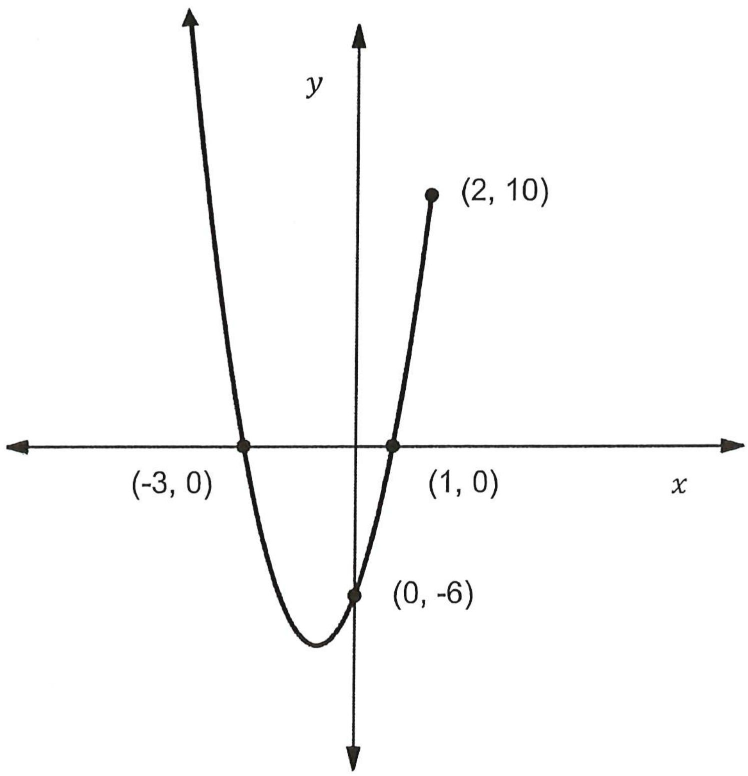


Figure 9: Graph showing function $g(x)$.

a) State the domain of $g(x)$

$x \in (-\infty, 2]$

/1

b) Determine the equation of $g(x)$

$g(x) = a(x+3)(x-1)$

let $x = 0$, $y = -6$

$\therefore -6 = a(3)(-1)$

$\therefore a = 2$

$\therefore g(x) = 2(x+3)(x-1)$

/2

c) Find the co-ordinates of the turning point of $g(x)$ and hence state its range.

$x_{TP} = \frac{-3+1}{2} = -1$

$g(-1) = 2(2)(-2)$

$= -8$

$\therefore$ TP @ $(-1, -8)$

Range is $[-8, \infty)$

/2

d) Describe a transformation which, when applied to $g(x)$, would mean the function touches the x axis only once.

Translated 8 units up

$\boxed{\text{or}}$ Translated > 10 units down

/1

Question 29

Marker use

Determine the equation of the cubic graph which passes through the point $(2, 5)$ and is shown in Figure 10.

/2

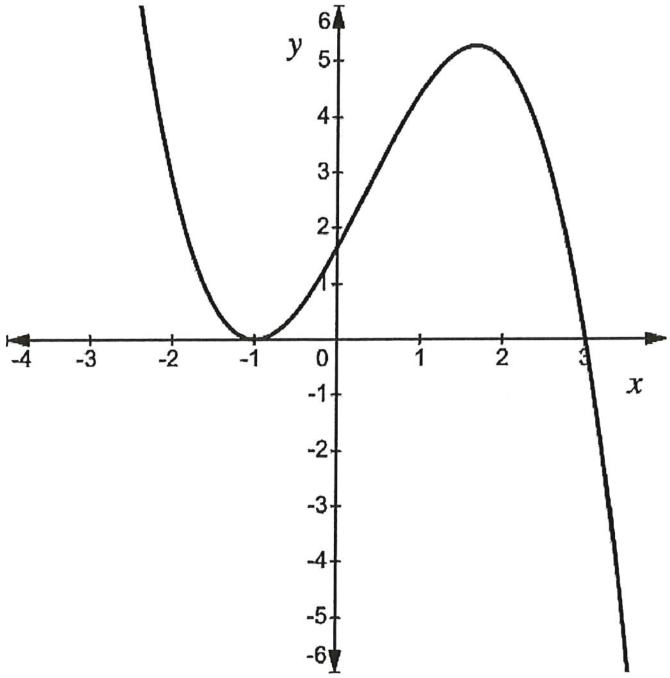


Figure 10: Graph of a cubic function.

$y = a(x+1)^2(x-3)$

$5 = a(3)^2(-1)$

$-9a = 5$

$a = -\frac{5}{9}$

$\therefore y = -\frac{5}{9}(x+1)^2(x-3)$

Question 30

The following transformations are applied to $y = x^3 + 1$:

- Translated down 2 units $\rightarrow y = x^3 - 1$
- Translated right 3 units. $\rightarrow y = (x-3)^3 - 1$

/2

State the co-ordinates of the new inflection point.

POI @ $(3, -1)$

Question 31

Marker use

Students are involved in making a replica of the Gateway Arch – the tallest human-made archway located in the United States.

The replica (shown in Figure 11) has the equation $h = -\frac{1}{10}x^2 + 4x$ where h cm is the height of the arch above ground level and x cm is the horizontal distance from the left-hand end.

Figure 11: Replica of the Gateway Arch.

a) What is the height of the replica?

By CAS, $h_{max} = 40cm$

/1

b) What is the distance between the ends of the replica?

$x_{int} = 0, 40$

$40 - 0 = 40$ cm

/1

c) The middle section of the arch is to be painted silver. This section will span a horizontal distance of 10cm as shown in Figure 12. Determine what height above ground the painted section will be.

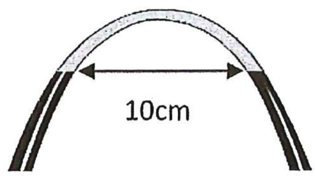
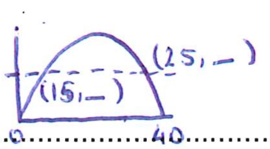


Figure 12: Middle section of the arch.

/3



$x_{TP} = 20$

$20 + 5 = 25$

$20 - 5 = 15$

$h(15) = 37.5$

At x = 15 cm and 25cm, height = 37.5cm

Total P2

/20

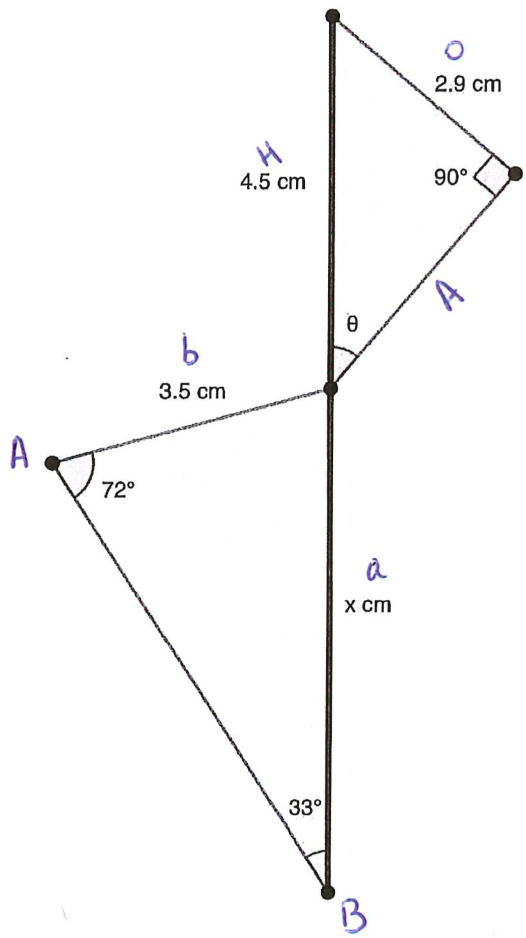
Part 3

Marker use

- Answer **all** questions in this part.
- This part assesses **Criterion 6**.

Question 32

Figure 13 shows a glass ornament made of two triangles on a vertical rod.



a) Determine the angle θ

$\sin(\theta) = \frac{2.9}{4.5}$ /1

$\sin^{-1}\left(\frac{2.9}{4.5}\right) = \theta$

$\therefore \theta = 40.12°$

b) Determine the length of the vertical rod.

$\frac{x}{\sin(72)} = \frac{3.5}{\sin(33)}$ /2

$\therefore x = \frac{3.5\sin(72)}{\sin(33)} = 6.11\text{ cm}$

Rod length $= 6.11 + 4.5 = 10.61$ cm

Figure 13: Diagram of glass ornament on a vertical rod.

Question 33

Marker use

The number of fruit flies, N, in a colony after t days of observation is modelled by

$$N = 30 \times 2^{0.072t}$$

a) How many fruit flies were present when the colony was initially observed?

At $t=0$ $N=30$ flies

$/_1$

b) How many fruit flies were present after 5 days?

At $t=5$ $N = 30 \times 2^{0.072 \times 5}$

$= 38.5027$

$\therefore$ 38 flies were present

$/_1$

c) How many days does it take for the colony to double?

let $N=60$

$60 = 30 \times 2^{0.072t}$

$\therefore t = 13.89$

$\therefore$ 14 days

$\boxed{OR}$

let $N=76$

$t = 18.66$ or 19 days

$/_1$

Question 34

For the exponential function $f(x) = 3^x - 5$,

a) State the domain and range of $f(x)$

$x \in \mathbb{R}$

$y \in (-5, \infty)$

$/_2$

b) State the equation of the horizontal asymptote after $f(x)$ is reflected in the y axis and translated up 1 unit.

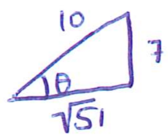
No change

$y = -4$

$/_1$

Question 35

Given that $\sin(\theta) = \frac{7}{10}$, use basic identities to find the exact value(s) of:



a) $\cos(\theta)$

$\sin^2(\theta) + \cos^2(\theta) = 1$

$\therefore (\frac{7}{10})^2 + \cos^2(\theta) = 1$

$\therefore \cos^2(\theta) = \frac{51}{100}$

$\therefore \cos(\theta) = \pm\frac{\sqrt{51}}{10}$

$/_2$

b) $\tan(\theta)$

$\tan(\theta) = \pm\frac{7}{\sqrt{51}}$ or $\frac{7\sqrt{51}}{51}$

$/_1$

Question 36

Marker use

For the function $y = 10\sin\left(\frac{1}{2}x\right)$,

a) Determine the period.

Period $= \frac{2\pi}{1/2} = 4\pi$

/1

b) Use Figure 14 to sketch the graph of the function over the domain $-2\pi \le x \le 2\pi$

/2

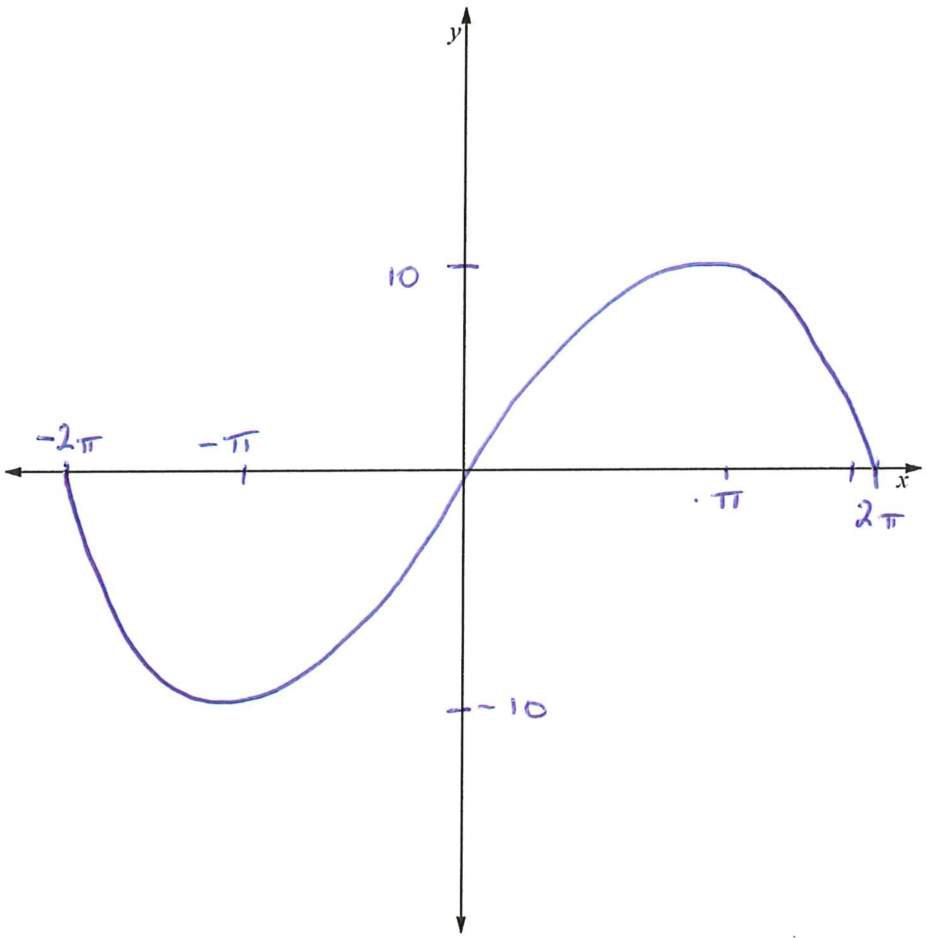


Figure 14: Axes for sketching answer to Question 36 b).

Spare diagram used (X) ☐

Question 37

Marker use

On a 10°C day, the temperature feels like T°C when the wind is travelling at speeds of w metres/sec and can be roughly modelled by the logarithmic function $T = \frac{58}{5} - \frac{27}{5}\log_{10}(w)$

a) Use Figure 15 to sketch a graph of the function for wind speeds of zero to 60 meters/sec showing any key features.

/2

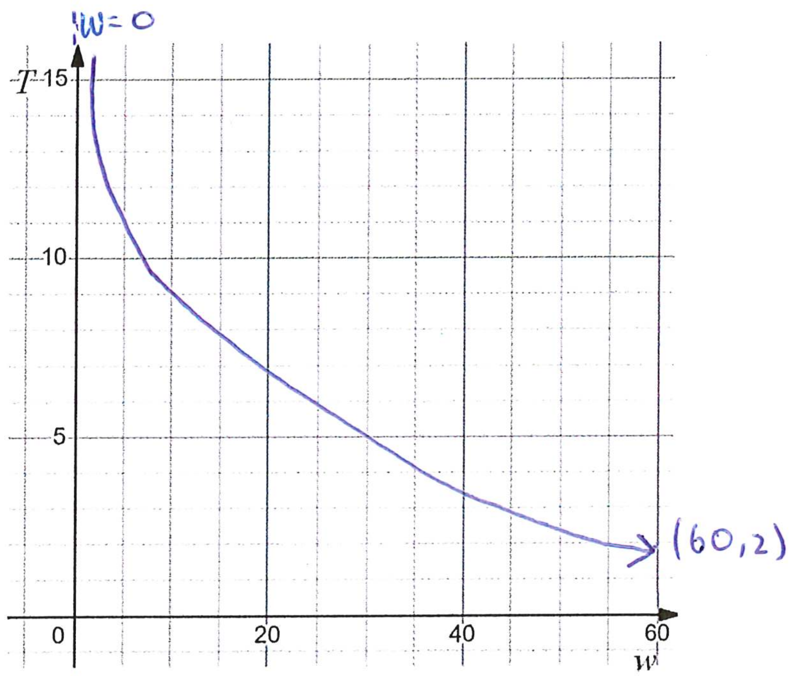


Figure 15: Graph for sketching answer to Question 37 a).

Spare diagram used (X) ☐

b) Determine the temperature it feels like when the wind speed is 32 metres/sec, correct to **one (1)** decimal place.

/1

$T = \frac{58}{5} - \frac{27}{5}\log_{10}(32) = 3.472$ or $3.5°C$ (to 1d.p.)

c) How much faster is the wind travelling when it feels like 3°C compared to 6°C? Give your answer to the nearest metre/sec.

/2

When $T = 3$, $w = 39.137$

When $T = 6$, $w = 10.89$

$39.137 - 7.398 = 28$ m/s faster

Total P3

/20

Part 4

Marker use

- Answer **all** questions in this part.
- This part assesses **Criterion 7**.

Question 38

Use first principles to determine the derivative of $f(x) = 2x - x^2$

/3

$f(x+h) = 2(x+h) - (x+h)^2$

$= 2x + 2h - x^2 - 2xh - h^2$

$\therefore f'(x) = \lim_{h \to 0} \frac{\cancel{2x} + 2h - \cancel{x^2} - 2xh - h^2 - \cancel{2x} + \cancel{x^2}}{h}$

$= \lim_{h \to 0} \frac{2\cancel{h} - 2x\cancel{h} - h^2}{\cancel{h}}$

$= \lim_{h \to 0} 2 - 2x - h$

As $h \to 0$

$f'(x) = 2 - 2x$

Question 39

A train moves in a straight line so that its displacement s km relative to O at time t seconds is given by $s(t) = -\frac{3}{2}t^2 + 16t$

a) Find the initial velocity of the train.

/2

$S'(t) = -3t + 16$

$S'(0) = 16$ km/sec

b) Find the velocity of the train when $t = 4.5$ seconds.

/1

$S'(4.5) = 2.5$ km/sec

Question 40

Marker use

The number of ants, A, attracted to a jar of honey t hours after its lid was left open can be modelled by the equation $A(t) = 0.25(t^2 + 1)^2 + 1.75$

a) Determine the initial numbers of ants. /1

$A(0) = 0.25(1)^2 + 1.75 = 2$ ants

b) Determine the average rate of change in the number of ants over the first **two (2)** hours. /2

$A(2) = 0.25(2^2+1)^2 + 1.75 = 8$

$AROC = \frac{8-2}{2-0} = 3$ Ants/hour

c) When is the number of ants increasing by 68 ants per hour? /2

$A(t) = 0.25t^4 + 0.5t^2 + 2$

$\therefore A'(t) = t^3 + t$

let $A'(t) = 68$

$\therefore 68 = t^3 + t$

$\therefore t = 4$ hours

Question 41

The graph of $f(x)$ is shown below (Figure 16). Sketch $f'(x)$ using the same set of axes. /2

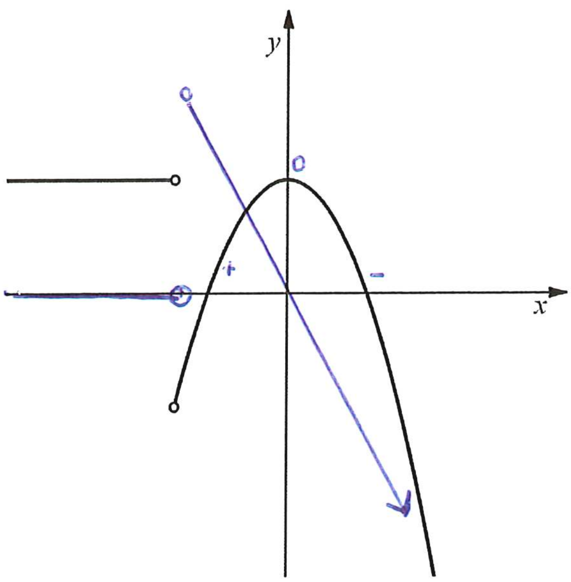


Figure 16: Axes for sketching answer to Question 41.

Spare diagram used (X) ☐

Question 42

Marker use

The curve $y = x^3 + 6x^2 - 15x + 4$ has two stationary points, one of which is a local maximum located at $(-5,\ 104)$

a) Explain what is meant by a local maximum compared to a global/absolute maximum.

/1

Local max is TP, but not necessarily max value of function

b) Use calculus to find the location of the other stationary point and justify its nature.

/3

$\frac{dy}{dx} = 3x^2 + 12x - 15$

let $\frac{dy}{dx} = 0$

$\therefore x = -5, 1$

Sub $x = 1$ into y

$\therefore y = 1 + 6 - 15 + 4$

$= -4$

$\frac{d^2y}{dx^2} = 6x + 12$

let $x = 1$

$\therefore \frac{d^2y}{dx^2} = 18 > 0$

$\therefore (1, -4)$ is local min

OR

x	0	1	2
$\frac{dy}{dx}$	-15	0	21
	\	—	/

$\therefore (1, -4)$ is local max

Question 43

The function $f: \mathbb{R}\backslash\{0\} \to R, f(x) = ax^3 + bx^{-1}$ passes through the co-ordinate $(1, 1.5)$ and has $f'\left(\frac{1}{\sqrt{2}}\right) = 4$. Find the values of a and b

/3

$f(1) = 1.5$

$\therefore 1.5 = a(1)^3 + b(1)^{-1}$

$\therefore a + b = 1.5 \rightarrow$ Equation ①

$f'(x) = 3ax^2 - bx^{-2}$

$4 = 3a\left(\frac{1}{\sqrt{2}}\right)^2 - b\left(\frac{1}{\sqrt{2}}\right)^{-2} \rightarrow$ Equation ②

By CAS $a = 2,\ b = -\frac{1}{2}$

Total P4

/20

Part 5

Marker use

- Answer **all** questions in this part.
- This part assesses **Criterion 8**.

Question 44

Two events A and B are such that $\Pr(A) = 0.20$, $\Pr(B) = 0.40$ and $\Pr(A \cap B) = 0.05$

a) Determine $\Pr(B')$

$Pr(B') = 1 - Pr(B) = 1 - 0.4 = 0.6$

/1

b) Determine $\Pr(A \cup B)$

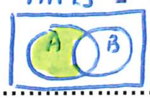
$Pr(A \cup B) = Pr(A) + Pr(B) - Pr(A \cap B)$

$= 0.2 + 0.4 - 0.05 = 0.55$

/1

c) Determine $\Pr(A|B')$

$A \cap B' = A - A \cap B$



$Pr(A|B') = \dfrac{Pr(A \cap B')}{Pr(B')}$

$= \dfrac{0.2 - 0.05}{0.6} = 0.25$

/2

Question 45

A campanologist (bell ringer) has eight different bells to ring at a wedding. How many different combinations of bells can she ring if she chooses:

a) Exactly **two (2)** bells.

${}^8C_2 = 28$

/1

b) At least **one (1)** bell.

${}^8C_1 + {}^8C_2 + {}^8C_3 + {}^8C_4 + {}^8C_5 + {}^8C_6 + {}^8C_7 + {}^8C_8$

$= 8 + 28 + 56 + 70 + 56 + 28 + 8 + 1 = 255$

OR $2^8 - 1 = 255$

/2

Question 46

Marker use

A dart thrown at the target shown has a 50% chance of striking A, a 25% chance of striking B and a 25% chance of striking C. What is the probability that if two darts are thrown, they will both land on the same letter?

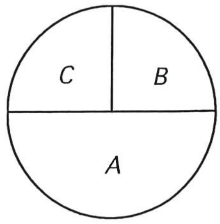


Figure 17: Diagram showing different sections of a target.

/3

$Pr(AA + BB + CC) = \frac{1}{2} \times \frac{1}{2} + \frac{1}{4} \cdot \frac{1}{4} + \frac{1}{4} \times \frac{1}{4}$

$= \frac{1}{4} + \frac{1}{16} + \frac{1}{16} = 0.25 + 0.0625 + 0.0625$

$= 3/8 = 0.375$

Question 47

Joe is going to buy 4 plants for their garden from 2 local nurseries. They discover that Nursery A has 8 different plants and Nursery B has 3 different plants.

a) How many different combinations of plants are possible if Joe buys the 4 plants from Nursery A? /1

$^{8}C_4 = 70$

b) How many different combinations of plants are possible if Joe buys the 4 plants using any of the 2 local nurseries? /1

$^{8+3}C_4 = {}^{11}C_4 = 330$

c) Calculate the probability that Joe buys **more than** 2 plants from Nursery A. /2

~~2A and 2B~~ $= \frac{^{8}C_3 \times {}^{3}C_1 + {}^{8}C_4 \times {}^{3}C_0}{330}$

or 3A and 1B

or 4A and 0B $= \frac{56 \times 3 + 70}{330}$

$= \frac{119}{165}$ or 0.72

Question 48

Marker use

Ash and Emerson are having an archery competition. They play two matches. There is always a 38% chance that Ash wins against Emerson and a 20% chance that they draw, irrespective of who wins or loses their first match.

a) Represent this information on the tree diagram below (Figure 18). Do not list or calculate the individual outcomes. /3

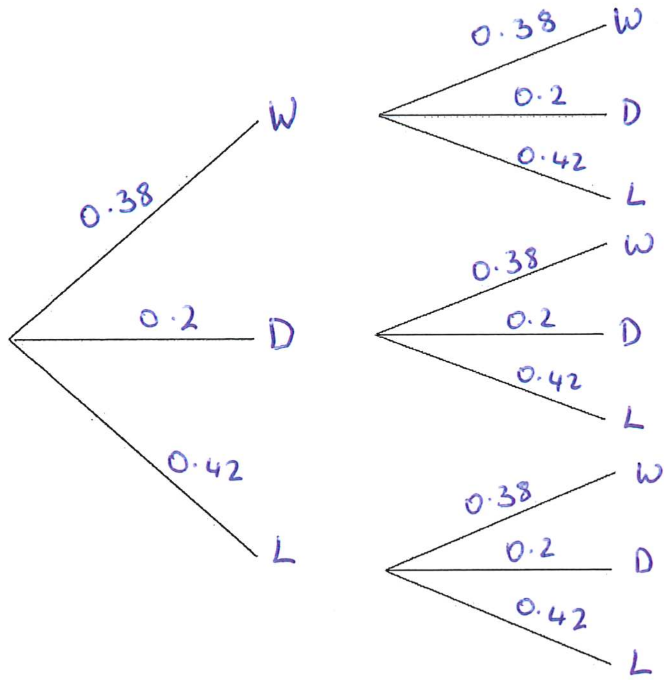


Figure 18: Tree diagram to answer Question 48 a).

Spare diagram used (X) ☐

b) Determine the probability that they draw at least once. /2

$0.38 \times 0.2 + 0.2 + 0.42 \times 0.2$

$0.076 + 0.2 + 0.084 = 0.36$ or $\frac{9}{25}$

c) **Given that** Ash wins the first match, what is the probability Emerson wins the second match? /1

0.42

Total P5 /20