

2025 ASSESSMENT REPORT

MTM315117 MATHEMATICS METHODS – FOUNDATION

Section A

Part 1

Question 1

Well done by most students. Students should remember to give a fully simplified final answer by combining the two x terms.

Question 2

- a) Many students were unable to deal with the negative index in the denominator. When dividing two terms with the same base, the indices are subtracted. Because the index on the denominator is negative already, this created a double negative situation.
 $5 - (-2) = +7$.
- b) A large number of students (close to a majority) ignored the fact that the 8 was also under the square root. Both $2^{\frac{1}{2}}x^2$ and $\sqrt{2}x^2$ were acceptable answers. Many forgot to simplify $\frac{4}{2}$ to 2 in the final index answer. Students who remembered the rule $\sqrt[n]{a} = a^{\frac{1}{n}}$ and converted everything to index form first did well. This formula is on the formula sheet, so students who can't remember what to do should check their formula sheet.
Students are advised to memorise their powers ($2^3 = 8$, $3^4 = 81$, $5^3 = 125 \dots$).

Question 3

- a) Students are encouraged to read the question in full. Some students left their answer in factorised form and hence, did not solve the equation for x .
- b) Key to this question was recognising that this equation followed a difference of squares pattern. Some students got mixed up with perfect squares and wrote it as $(2x - 3)^2$, resulting in only partial marks being awarded. Students who solved by rearranging (instead of factorising) received partial marks.
- c) Many students struggled with the denominator. Successful students either put each term over 10, or multiplied by 5 in one step, then 2 in the second step. Students should remember that when multiplying to cancel a denominator, all terms must be multiplied by that same value, as many students ignored the 3 term. Many students also struggled with simplifying $-\frac{28}{7}$. Those students who set their work out clearly in columns were more likely to get the correct answer.

- d) Most students were able to make some attempt at using elimination or substitution. Of those who made errors, those who used substitution generally got closer to the correct answer. There were many errors in using the elimination method and many students did not understand that in some situations the equations need to be subtracted to eliminate the variable, while in others they can be added. Students needed to simplify fractions for full marks. Unless otherwise stated, it is expected that solving a simultaneous equation requires finding both an x and y solution. Some students stopped after finding x .

Question 4

- a) Working or thinking needed to be shown to achieve full marks in this question. Successful students used division methods or expanded the expression and compared coefficients.
- b) Generally well done. Pleasingly, many students who could not complete 4a still completed 4b and were successful. Remember that "hence" means using what you have just done. That was a hint to use the working from the previous question to help with this one. Some students found a different factor and used division again to find a different quotient. This was still correct, but involved unnecessary work.

Part 2

Question 5

- a) Well done. But many students left their answer as $-\frac{6}{4}$. Answers should be fully simplified. Decimal form was acceptable.
- b) For full marks, the two lines needed to look parallel. Students who used a ruler were much more likely to achieve acceptably parallel lines. Marking the x -intercept was required for full marks.
Around 10% of students drew perpendicular (right angle) lines.
Some students wrote (0, 6) instead of (6, 0). The x -coordinate is always first.
- c) Some students misread the question and found the equation of the original line, which was partially rewarded. Working needed to be shown for full marks. Stating that y -intercept was 9, $\therefore c = 9$ was not sufficient unless it was supported by working. There were errors in fraction work i.e. $-\frac{3}{2} \times 6 = -\frac{18}{12}$ and in multiplying 3×6 .

Question 6

Students are encouraged to consider whether the points they find are logical. Some found the turning point at (-2, -18) and to make this fit, they changed the shape of the graph or changed their intercepts. If a value does not make sense, students should check working again.

The shape needs to be a parabola. Graphs which come to a point (V-shape), become vertical (U-shape) or are not symmetrical are not correct. The graph also had to come at least halfway up the positive half of the y -axis.

Some students lost time by not recognising the information that was easily accessible in each form. Some students let $x = 0$ in the second equation to find the y -intercept when it was quickly accessible through the first equation.

Question 7

- a) A common error was $48a = 6 \therefore a = 8$ instead of $\frac{1}{8}$. Many students had trouble simplifying $\frac{6}{48}$ into $\frac{1}{8}$. A small number of students attempted to substitute an x -intercept in to solve for a . This led to an incorrect answer. Those who picked the wrong form (i.e. $y = a(x - h)^3 + k$) were not successful and not well rewarded as the graph did not have a point of inflection. The final answer had to be written out for full marks. The factorised form should be given and the answer not expanded.
- b) Many students wrote the answer as $x \in (-3,5)$ or $y \in (-6,6)$ for which they received partial marks as $(5, 6)$ is not an endpoint. Students needed to identify which answer was the domain and range and use the correct brackets for full marks.
- c) Students needed to include an endpoint of $(-3,6)$ with an open circle for full marks. Some students reflected in the y -axis by changing the sign of the intercepts ($3 \rightarrow -3$), which led to an incorrect answer. When reflected in the x -axis, the value of the x -intercepts do not change.

Question 8

- a) Very few students achieved full marks on this question, but most received partial marks. The effect of the value of b was most confusing to students. The effect of b can be found by considering its effect on the formula for the x -coordinate of the TP ($x_{TP} = -\frac{b}{2a}$). Students need to be exact in their drawing of the graph; the graph had to clearly pass through the origin for full marks.
- b) There were a variety of responses for this question and many imprecise answers. Students are encouraged to memorise the exact definition of a function as in this answer, one misplaced word can change the meaning of the sentence. i.e. "for each x -value there is a y -value" does not fully explain functions.

"Passes the vertical line test" helps the student identify whether the graph is a function or not, but does not explain *why* the graph is a function and is not considered a full answer by the marking team. A good strategy may be to start by writing "passes vertical line test" then attempting to provide a full definition of a function. This ensures students will at least receive partial marks.

"Passes straight line test" was not an acceptable answer.

- c) Generally completed correctly. When completing a question like this, students are encouraged to make it very clear that the relation fails the vertical line test.

Part 3

Question 9

- a) Common errors included multiplying indices instead of adding, or multiplying bases together (base stays the same). Students made many errors with fraction addition.
- b) Most students changed the logarithm to index form correctly but struggled to work with the negative index. Students did not need to state both the positive and negative version of the solution for full marks in this particular question, but it is encouraging to see that some students could recognise that the base of a logarithm cannot be negative and then explicitly discard that solution.

Question 10

- a) Students that efficiently solved this question first moved the 10 and 5 to the other side of the equation *before* changing the log to index form. Some students put the logarithm in the form $\log_3(x^5) = 10$ and then converted to index form ($3^{10} = x^5$) but were rarely successful in then solving for x . At least one step of working was needed for full marks.
- b) This question was challenging for most students. Common errors included reflection over the asymptote or reflection in the x -axis, instead of the y -axis. For full marks students needed to label points and remember to translate the graph and asymptote. Most students were able to successfully perform a translation of two units right. A suggested strategy in solving these types of questions is to sketch the two transformations separately in the working space (first the translation, then the reflection on a second axis) before sketching the final answer on the axis provided.

Question 11

- a) Generally well understood. Common errors included using $\frac{\pi}{2}$ as the period instead of π .
- b) Many students could not determine x -coordinate of Q from the grid on the graph. Some students did not use the exact value table on their formula sheet to evaluate $\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$. Final answers should be simplified from $\frac{4\sqrt{3}}{2}$ to $2\sqrt{3}$. Pleasingly, most students who got to that point stated their final answer as a coordinate.

Question 12

- a) Almost all students were able to apply the formula. Many students could not simplify $\frac{18\pi}{180}$ correctly.
- b) To correctly solve this question, students needed to remember that $\cos(\theta)$ corresponds to the x -coordinate of a point on the unit circle and $\sin(\theta)$ the y -coordinate. Common errors included using 18° or $\frac{\pi}{10}$ as the value of θ , or substituting $\frac{3}{10}$ as θ rather than $\sin(\theta)$.

Students who correctly used a right-angled triangle and Pythagoras' Theorem to find the value of $\cos(\theta)$ were fully rewarded. No marks were deducted for not acknowledging the quadrant the answer was in as that was accounted for in the next question.

- c) Most students understood the x value had to be negative for quadrant 2 and the y value to be positive $\frac{3}{10}$.

Part 4

Question 13

- a) Generally well done.
- b) Generally well done, but some students attempted to find the derivative before expanding.
- c) Some students were unsure of how to convert a root into a fractional index correctly. Writing $x^{\frac{4}{7}}$ instead of $x^{\frac{7}{4}}$ was quite common. Many were also unable to subtract the fractional indices correctly. A few read the first term as the eighth root of x but then proceeded to differentiate correctly and were appropriately rewarded.

Question 14

Many students lost a mark here for incorrect notation such as not including the limit or putting it in the wrong place (it should be after the equal sign and not included in the final answer). Some students did not use $f'(x)$ notation and did not expand the bracket following the subtraction sign correctly.

Question 15

- a) Generally well done, although some students substituted the value into the original $f(x)$, rather than the derivative.
- b) Some students incorrectly used $f(x)$ to determine the gradient value, while others mistakenly took the 6 from $f'(x) = 6x - 7$ to be the gradient. In terms of the point to be used, many students used $x = 1$ (from (a)), while others used $x = 2$ and then $y = 0$. Once they had a point and gradient, most students could find the equation to a line.

Question 16

Some students struggled with the non-monic factorisation leading to incorrect x values. Many students found it difficult to find the corresponding y value for $x = \frac{1}{3}$.

Part 5

Question 17

- a) Generally well done.
- b) Many students either added the two fractions, or were not able to multiply correctly. Common errors included $\frac{1}{6} \times \frac{1}{6} = \frac{2}{12}$ or $\frac{1}{12}$

Question 18

For full marks, students needed to be clear about what was incorrect and not just make an observation. For example, to get full marks they must state “It is not possible to have a negative”, rather than “it is negative”. It was also important to point out that the total of all probabilities must = 1, rather than “the total is greater than 1” as it would also be incorrect if the total was less than 1.

Question 19

- Students had difficulty determining what values went in the various regions of the Venn diagram resulting in multiple incorrect variations. Many students assigned the 48 where 35 should be, but those 48 in the question include both the tourists who *only* went to a national park ($NP \cap M'$) **and** those who went to both.
- Generally well done.
- Some students wrote the conditional probability in the wrong order $\Pr(NP|M')$ or wrote a probability statement unrelated to the question e.g. M and NP. Other students used the letters A and B, without defining what they referred to and often made subsequent errors as a result.
- Students had difficulty determining an appropriate non-random method of selecting tourists appropriate to the problem. The best answers involved selecting tourists exiting the museum or national park or carrying hiking shoes or backpacks. Some students appeared to have misread the question and did not notice the word ‘not’ and so attempted to describe random techniques.

Question 20

- Many students were able to write the ${}^{11}C_4$ statement but did not calculate the final value or made an error in doing so. Students should practice these types of calculations without a calculator. Some students wrote statements the wrong way around, ${}^4C_{11}$.
- Many students misinterpreted the question and did not consider that 4 selections had to be made in total, rather than just the 3 referred to in this part of the question. The other common mistake was adding the numbers rather than multiplying. Students should remember that if you are picking a bat **and** marsupial the options are multiplied, if you are picking bat **or** marsupial the options are added. Some students also lost marks for attempting to find the probability rather than just the number of combinations.
- A number of students wrote only two of the three combinations possible and missed that zero bats was a possibility. A number of students did not write combination statements for this question. Students who are struggling with this type of question are encouraged to write out the different options in plain language first before converting it to combinations notation. (e.g. 2 rodents and 1 bat and 1 marsupial or ...).

Section B

Part 1

Question 21

- a) This question was done poorly by most students. Many were unable to identify a common denominator of $3t^2$. Many students used their calculators and did not input the variables correctly.
- b) There were a lot of students who gained marks for this question by error carried forward from the previous questions. In general, it was completed well.
- c) Common errors in rearranging equations were seen in this question, with some students using incorrect inverse operations. Error carried forward was again used from part (a) in many instances.

Question 22

Answered well by many students. A common error was not putting brackets around the final two terms being cubed.

Question 23

- a) Many students were able to identify the mistakes, but not all indicated the necessary corrections.
- b) Tended to be well done by most students. Some students did not recognise the alternating + - of terms in the final answer.

Question 24

- a) Answered poorly by many students. Many students showed no working out and gave incorrect solutions and, at times, ignored zero as a solution. Common errors included interpreting $\sqrt{2x}$ incorrectly as $\sqrt{2}x$.
- b) Handled well by most students. Those that completed the square correctly were mostly able to go on and solve correctly. Error carried forward was common for those students unable to complete the square but were able to follow the steps for solving correctly.

Question 25

- a) Most students were successful in correctly forming the discriminant and then solving for k .
- b) Perhaps not well understood by many students. Common errors included only finding the constraint $k > -\frac{1}{3}$. Many students did not go on to find a specific value of k to give two rational intercepts i.e. make Δ a perfect square and solve for k .

Question 26

- a) Common errors included students trying to work backwards from $h^2 - 2h - 8 = 0$ to set up expressions for base and height. Students also used $(h + 2)$ as the base instead of $(h - 2)$.
- b) Done well by most students. Errors included using incorrect values for a, b and c in the quadratic formula and not justifying why “-2cm” should be ignored as a solution.

Part 2

Question 27

- a) Answered well by students.
- b) Completed well by most students. There were some errors when stating the gradient of the perpendicular line, and errors calculating the point (2,0).

Question 28

Reasonably well done. Common errors included not calculating end point (1,8), not using open / closed circles at end points or not showing end points, limited calculations in space provided and not clearly labelling intercepts on graph.

Question 29

- a) Answered well by students. Some students subtracted 3 from the h value when translating to find the new equation.
- b) Completed well by most students. Some errors in the direction of the graph.

Question 30

- a) Answered well by most students.
- b) Answered well by most students. Some students did not recognise they needed to find the local maximum.
- c) Answered well by most students. Some students were able to identify the y value of 200 but could not solve for x .
- d) Students were able to state h and k quite easily. Some stated h as (-30) . Some students incorrectly substituted or failed to use $(0,120)$ to find the value of a .
- e) Answered reasonably well by students.
- f) Answered generally well by students. Some students did not round to the nearest kilometre.
- g) Was not answered well by students. Students could calculate how far north Sally was based on the equation given but had difficulty calculating for Pete, particularly if they could not find an equation from part d.

Part 3

Question 31

- a) Completed well by most students.
- b) Completed well by most students.
- c) Done well by most students. A common mistake was errors in rearranging the equation to find x .

Question 32

- a) Done well by most students. Most errors occurred with not identifying the non-inclusive endpoint with an open circle.
- b) There was some confusion with square (inclusive) or round brackets. In a few instances, the non-inclusive endpoint error was not carried forward to the set notation.

Question 33

Not completed well by many students. A lot of errors in drawing a cosine graph including accuracy in where the graph cuts the x axis and where minimum and maximum points occur. Few axes were labelled.

Question 34

Completed well by students. Errors occurred by not determining the length AB correctly, not applying the Sine identity correctly, or when applying the Cosine rule – a substitution error by not identifying the length 'a' correctly.

Question 35

- a) Mostly done well. However, if a student did not realise \log of $1 = 0$, they failed to answer the rest of (a) and (b).
- b) Answered well if the student could form the equation in part (a).

Part 4

Question 36

- a) Generally well done by most students who could differentiate and then equate to zero to find the time.
- b) If time was found successfully in part (a), then this question was done well by most students.
- c) Many students did not include the “-“ sign to indicate movement downwards.
- d) The most common error in this question included not equating the -10 to the differentiated equation.

Question 37

- a) Most students left their answer as “-1” and did not change it in line with the y axis units of \$ thousand to give their answer as -\$1000 per year.

- b) Well done by most students who equated the derivative function to -1 .
- c) Many students did not complete this question. Of those who did, most could draw an accurate representation of the graph of the derivative.

Question 38

- a) Part (i) and (ii) were completed accurately by most students.
- b) Many students did not understand that the gradient would be negative.

Question 39

- a) Done well by most students.
- b) Many students did not differentiate. They did not appear to understand that a rate at a particular point is the same as an instantaneous rate.

Question 40

Many students did not recognise that the gradient parallel to the x axis is equal to zero. Many used $x = 2$ as the gradient.

Part 5

Question 41

- a) Done well by most students.
- b) Done well by most students.
- c) This question was not done as well as the previous parts. Many students did not seem to interpret the **not** “A and B” notation. Correct responses were able to see that this will be the complement of “A and B”.

Question 42

This question was done well by most students. Many students did not interpret the question as involving combinations. Some who did attempt to use combinations did not correctly apply the addition and multiplication laws to include all the combinations that satisfy the condition that “at most 1 is not properly sealed”.

Question 43

- a) Most students completed this section accurately.
- b) Done well by most students.
- c) Done well by most students.
- d) This question tended to be done well or was left blank. Some students did not multiply the 1,500,000 by both the probability of arriving by air and the probability of arriving at Hobart airport. There were a few place value issues with answers out by a power of ten.

Question 44

- a) Done well by most students.
- b) This question was done quite well. Some students included the correct denominator but did not apply the multiplication rule in the numerator and the correct combinations.
- c) A considerable number of students left this question blank. Some students applied the multiplication rule instead of the addition rule in the numerator of their calculation.

External Assessment 2025

Solutions

MATHEMATICS METHODS – FOUNDATION

MTM315117

Section **A**

Pages: 24

Questions: 20

Information Sheets: 1

Preparation time for this exam: 15 minutes

Suggested working time: 80 minutes

Instructions:

Calculators are not allowed to be used in this section.

Section A will be collected after 80 minutes.

- There are **five (5) parts** to this section.
- Answer **all** questions and **all** items within each question.
- Write your answers in the spaces provided in this exam paper.
 - Spare diagrams have been provided at the end of each part.
Indicate in the box provided if you have used the spare diagram.
- The exam is **three (3) hours** in length. The suggested working time for this section is **approximately 80 minutes**.
- During the first 80 minutes of the exam you may move onto Section B, but you **cannot** use your calculator until told by your supervisor(s).
- The Mathematics Methods – Foundation Information Sheet can be used throughout the exam.
- All answers must be written in **English**.

Marker use	
C4	/ 16
C5	/ 16
C6	/ 16
C7	/ 16
C8	/ 16

Additional Exam Instructions

- You **must** make sure your answers address the listed criteria.
- For questions worth **one (1)** mark, you **are not required** to show workings. Markers will look at the presentation of the answer(s) and at the argument(s) leading to the final answer(s).
- For questions worth **two (2)** or more marks you **are required** to show relevant workings.
- Marks will be allocated:
 - according to the degree to which workings convey a logical line of reasoning, and
 - for suitable justifications and explanations of methods and processes when requested.

Criteria

You **must** make sure your answers address:

- Criterion 4 manipulate algebraic expressions and solve equations
- Criterion 5 understand linear, quadratic and cubic functions
- Criterion 6 understand logarithmic, exponential and trigonometric functions
- Criterion 7 use differential calculus in the study of functions
- Criterion 8 understand experimental and theoretical probabilities and of statistics.

Guide to Exam Structure

		Parts	Questions available	Questions to answer	Suggested working time	Marks available
Section A	Part 1		4	4	16 minutes	16 marks
	Part 2		4	4	16 minutes	16 marks
	Part 3		4	4	16 minutes	16 marks
	Part 4		4	4	16 minutes	16 marks
	Part 5		4	4	16 minutes	16 marks
Totals			20	20	80 minutes	80 marks
Section B	Part 1		6	6	20 minutes	20 marks
	Part 2		4	4	20 minutes	20 marks
	Part 3		5	5	20 minutes	20 marks
	Part 4		5	5	20 minutes	20 marks
	Part 5		4	4	20 minutes	20 marks
Totals			24	24	100 minutes	100 marks
Totals			44	44	180 minutes (3 hours)	180 marks

Part 1

- Answer **all** questions in this part.
- This part assesses **Criterion 4**.

Question 1

Expand $(3x + 1)(x - 2)$.

$$= 3x^2 - 6x + x - 2$$

$$= 3x^2 - 5x - 2$$

/1

Question 2

Simplify the following expressions:

a) $\frac{3a^2b^{-2} \times 4a^3b^3}{2a^{-2}}$

$$= \frac{12a^5b^1}{2a^{-2}}$$

$$= 6a^7b$$

/1

b) $\sqrt{8x^5} \div (2\sqrt{x})$

$$= \frac{(8x^5)^{1/2}}{2x^{1/2}}$$

$$= \frac{8^{1/2} x^{5/2}}{2x^{1/2}}$$

$$= \frac{2^{3/2} x^{5/2}}{2x^{1/2}}$$

$$= 2^{3/2-1} \times x^{5/2-1/2}$$

$$= 2^{1/2} x^{4/2}$$

$$= \sqrt{2} x^2$$

/2

Question 3

Solve the following:

a) $x^2 + 9x + 14 = 0$

$$(x+7)(x+2) = 0$$

$$\therefore x = -7, -2$$

/1

b) $4x^2 - 9 = 0$ by factorising.

$$(2x)^2 - (3)^2 = 0$$

$$\therefore (2x+3)(2x-3) = 0$$

$$\therefore x = -\frac{3}{2}, \frac{3}{2}$$

/2

c) $\frac{6x-1}{5} + 3 = \frac{x}{2}$

$$\therefore 2(6x-1) + 30 = 5x$$

$$\therefore 12x - 2 + 30 = 5x$$

$$\therefore 7x = -28$$

$$\therefore x = -4$$

/2

d) $3x + y = 9$ and $-2x - 2y = 4$ using simultaneous methods.

Elimination Method

$$\textcircled{1} \times 2$$

$$\therefore 6x + 2y = 18$$

$$+ \quad -2x - 2y = 4$$

$$4x = 22$$

$$\therefore x = \frac{22}{4} = \frac{11}{2}$$

$$\therefore -2\left(\frac{11}{2}\right) - 2y = 4$$

$$\therefore -11 - 2y = 4$$

$$\therefore -2y = 15$$

$$\therefore y = \frac{15}{-2}$$

Substitution Method

$$y = 9 - 3x \textcircled{3}$$

$$\text{Sub } \textcircled{3} \text{ into } \textcircled{2}$$

$$\therefore -2x - 2(9 - 3x) = 4$$

$$\therefore -2x - 18 + 6x = 4$$

$$\therefore 4x = 22$$

$$\therefore x = \frac{22}{4} = \frac{11}{2}$$

sub into $\textcircled{1}$

$$\therefore 3\left(\frac{11}{2}\right) + y = 9$$

$$\therefore y = 9 - \frac{33}{2}$$

$$\therefore y = \frac{18}{2} - \frac{33}{2} = -\frac{15}{2}$$

/3

Sub x into $\textcircled{2}$

Question 4

$x^3 - 8x^2 + 11x + 20$ can be written as $(x - 4)(ax^2 + bx + c)$.

a) Determine the values of a, b and c .

$$\begin{array}{r}
 x^2 - 4x - 5 \\
 x - 4 \overline{) x^3 - 8x^2 + 11x + 20} \\
 \underline{-(x^3 - 4x^2)} \\
 -4x^2 + 11x \\
 \underline{-(-4x^2 + 16x)} \\
 5x + 20 \\
 \underline{-(-5x + 20)} \\
 \hline
 0
 \end{array}$$

$$= (x - 4)(x^2 - 4x - 5)$$

$$\therefore a = 1, b = -4, c = -5$$

b) Hence, solve $0 = x^3 - 8x^2 + 11x + 20$

$$(x - 4)(x^2 - 5x - 4) = 0$$

$$\therefore (x - 4)(x - 5)(x + 1) = 0$$

$$\therefore x = 4, 5, -1$$

/2

/2

Alternative strategies for (4a)

$$\begin{array}{r}
 1 \quad -8 \quad 11 \quad 20 \\
 4 \left\{ \begin{array}{l} \downarrow \\ 4 \quad -16 \quad -20 \end{array} \right. \\
 \hline
 1 \quad -4 \quad -5 \quad 0
 \end{array}$$

$$= (x - 4)(x^2 - 4x - 5)$$

$$\therefore a = 1, b = -4, c = -5$$

$$(x - 4)(ax^2 + bx + c) = ax^3 - 4ax^2 - 4bx + cx - 4c + bx^2$$

$$\begin{aligned}
 x^3 \text{ Terms: } ax^3 &= 1x^3 \\
 \therefore a &= 1
 \end{aligned}$$

$$\begin{aligned}
 x^2 \text{ Terms: } -4 \times 1x^2 + bx^2 &= -8x^2 \\
 \therefore -4 + b &= -8 \\
 \therefore b &= -4
 \end{aligned}$$

$$\begin{aligned}
 x^0 \text{ Terms: } -4c &= 20 \\
 \therefore c &= -5
 \end{aligned}$$

Total
P1
/16

Part 2

- Answer all questions in this part.
- This part assesses **Criterion 5**.

Question 5

a) Find the gradient of the line shown in Figure 1.

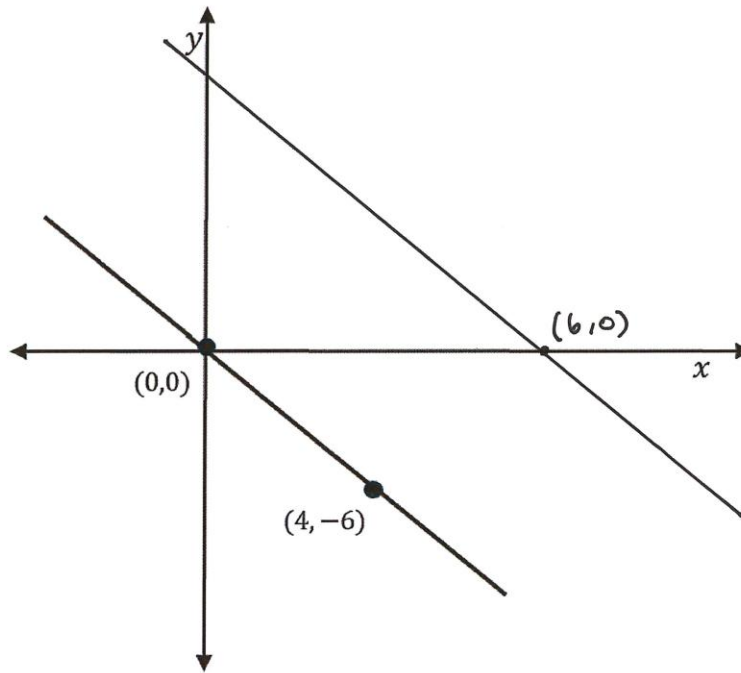


Figure 1

Spare diagram used (X)

/₁

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-6 - 0}{4 - 0} = -\frac{6}{4} = -\frac{3}{2}$$

b) On the same axes, sketch the linear function which has an x -intercept at $(6, 0)$ and is parallel to the line shown.

/₁

c) Find the equation of the linear function sketched in part b).

/₂

$$\begin{array}{l|l}
 y = -\frac{3}{2}x + c & \therefore 0 = -9 + c \\
 \text{let } x = 6, y = 0 & \therefore c = 9 \\
 \therefore 0 = -\frac{3}{2} \times 6 + c & \therefore y = -\frac{3}{2}x + 9
 \end{array}$$

Question 6

Marker use

$y = -10$

The function $f(x) = 2x^2 - 8x - 10$ can be written in the following ways:

$f(x) = (2x + 2)(x - 5) \rightarrow x = -1, 5$

$f(x) = 2(x - 2)^2 - 18 \rightarrow \text{TP@ } (2, -18)$

Sketch $f(x)$ using the axes provided in Figure 2, labelling any intercepts and the turning point.

/3

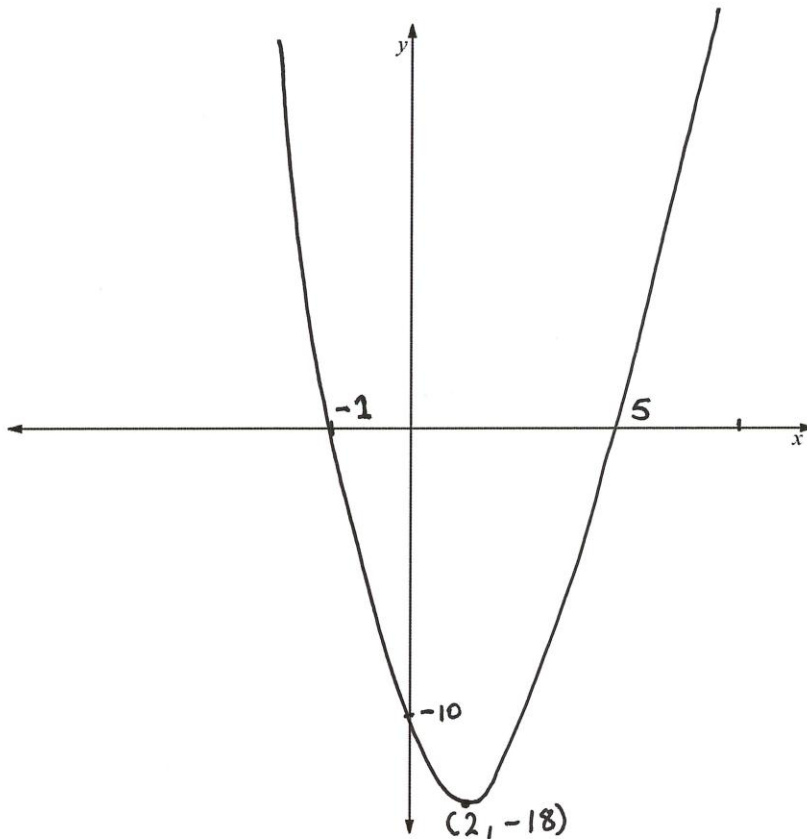


Figure 2

Spare diagram used (X)

Question 7

Marker use

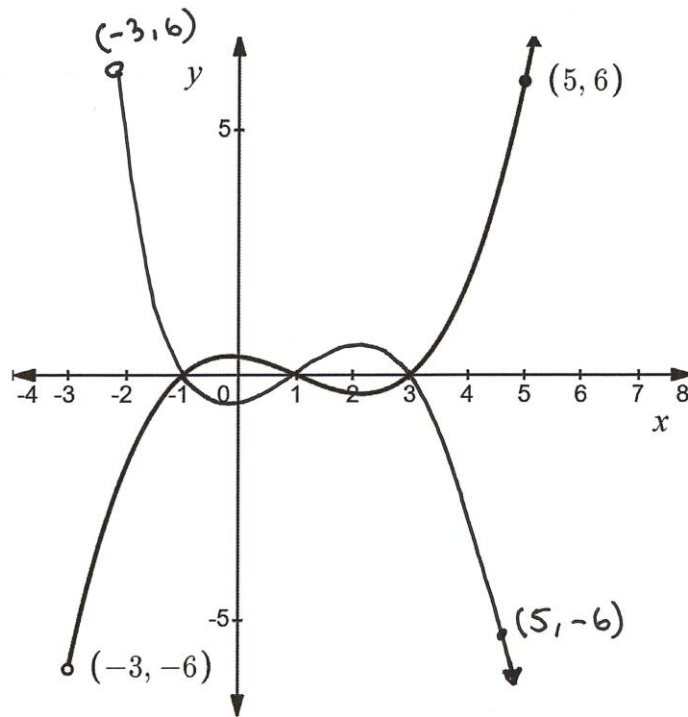


Figure 3

Spare diagram used (X)

- a) Find the equation of the cubic function shown in Figure 3.

$$y = a(x+1)(x-1)(x-3)$$

$$\text{let } x = 5, y = 6$$

$$\therefore 6 = a(5+1)(5-1)(5-3)$$

$$\therefore 6 = a(6)(4)(2)$$

$$\therefore 6 = 48a$$

$$\therefore a = \frac{6}{48} = \frac{1}{8}$$

$$\therefore y = \frac{1}{8}(x+1)(x-1)(x-3)$$

- b) State the domain and range.

$$x \in (-3, \infty)$$

$$y \in (-6, \infty)$$

- c) Using Figure 3, graph the cubic function after a reflection in the x -axis has been applied.

/2

/2

/1

Question 8

a) Sketch a possible graph for the quadratic function $y = ax^2 + bx + c$, where:

$$a < 0, b < 0, c = 0 \text{ and } \Delta > 0$$

using the axes provided in Figure 4.

$a < 0 \rightarrow \text{A}$

$c = 0 \therefore$ passes through origin $(0,0)$

$\Delta > 0 \therefore$ Two solutions

Let $a = -1, b = -1$

$$x_{TP} = -\frac{b}{2a} = -\frac{-1}{2(-1)}$$

$\therefore x_{TP} = \text{Negative}$

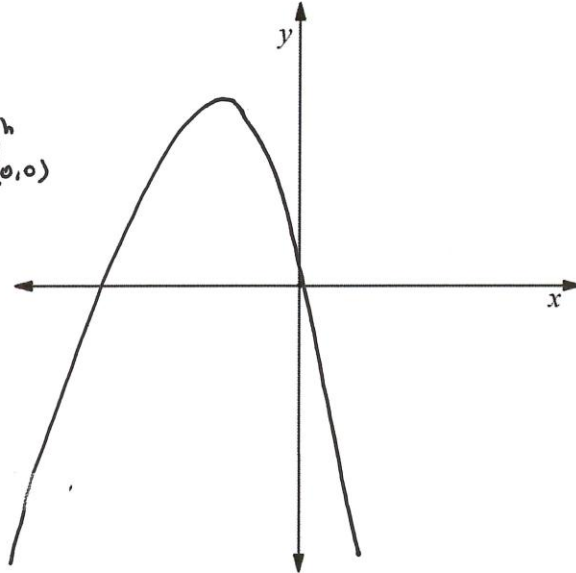


Figure 4

Spare diagram used (X)

b) Explain why the graph is a function.

- For each x -value there is at most one y -value
- Each x -value maps to one y -value

c) Sketch a relation which is **not** a function using the axes provided in Figure 5.

Needs to obviously fail vertical line test

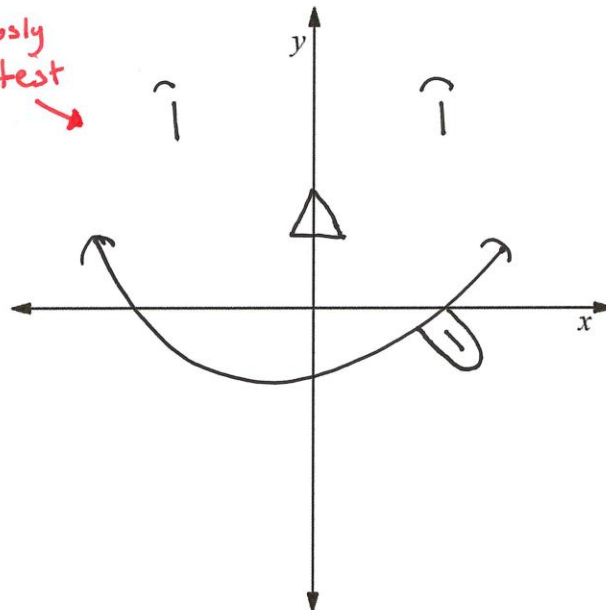


Figure 5

Spare diagram used (X)

/2

/1

/1

Total P2

/16

Spare Diagrams

Question 5

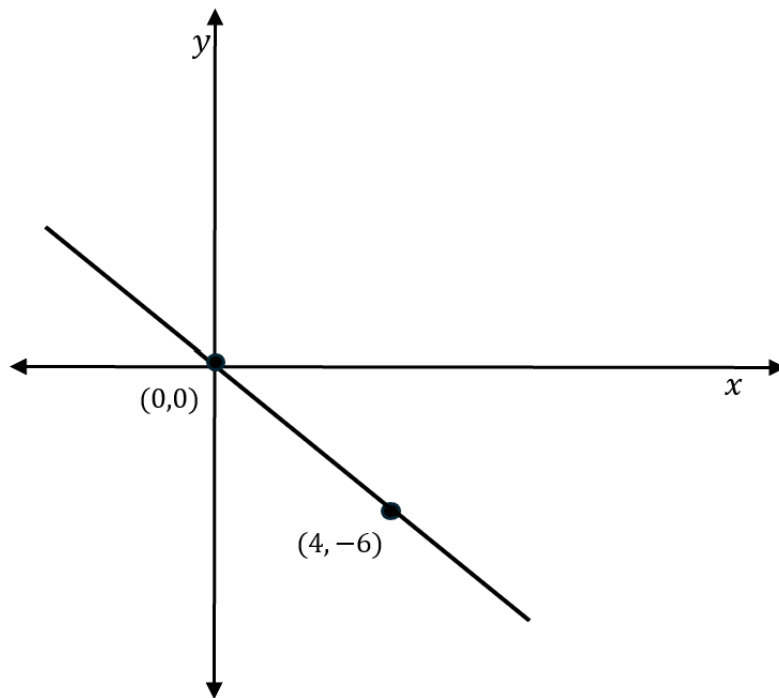


Figure 1

Question 6

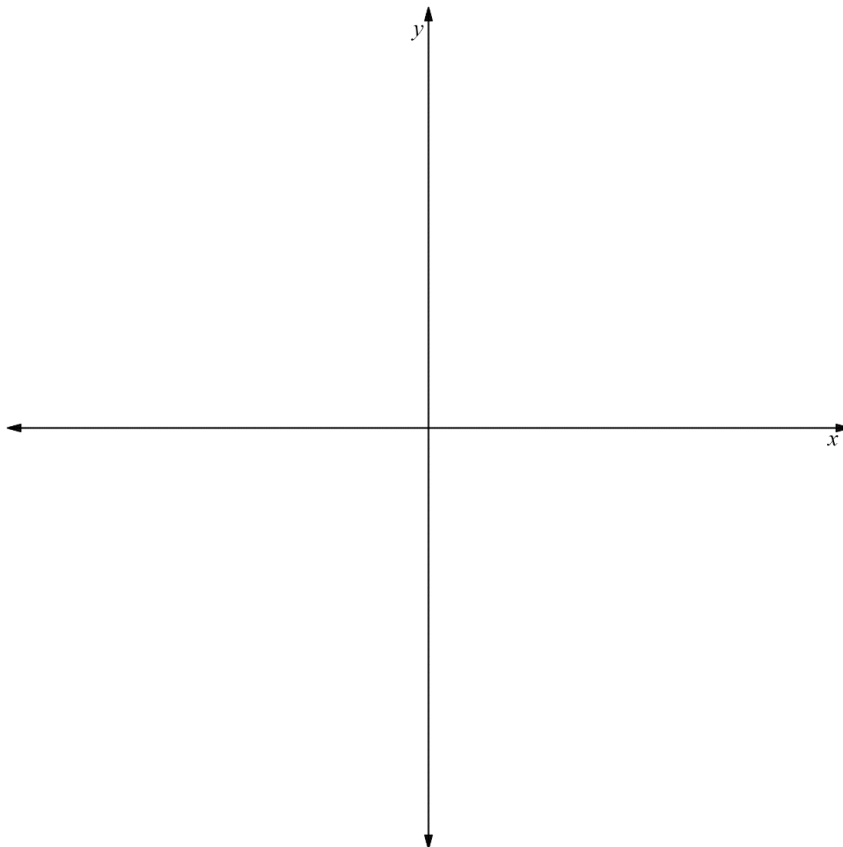


Figure 2

Spare Diagrams

Question 7

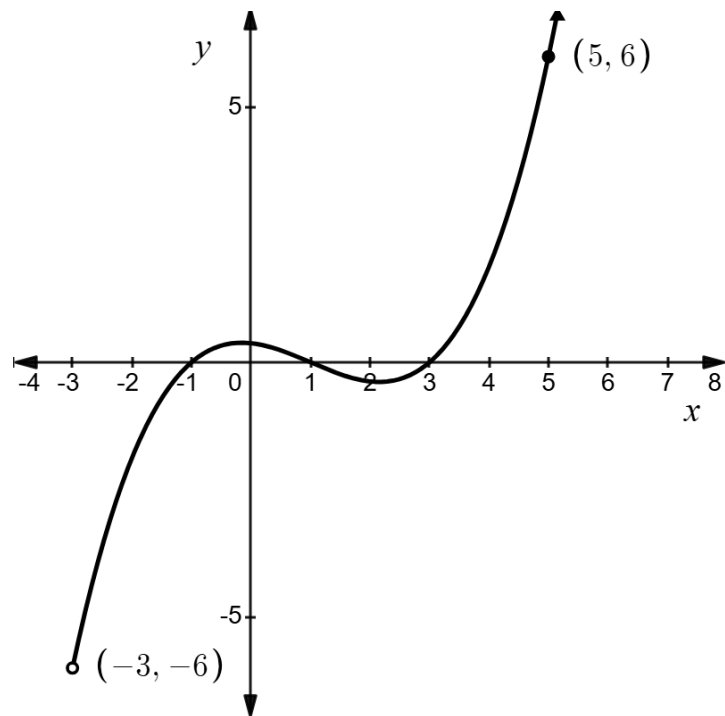


Figure 3

Question 8 a)

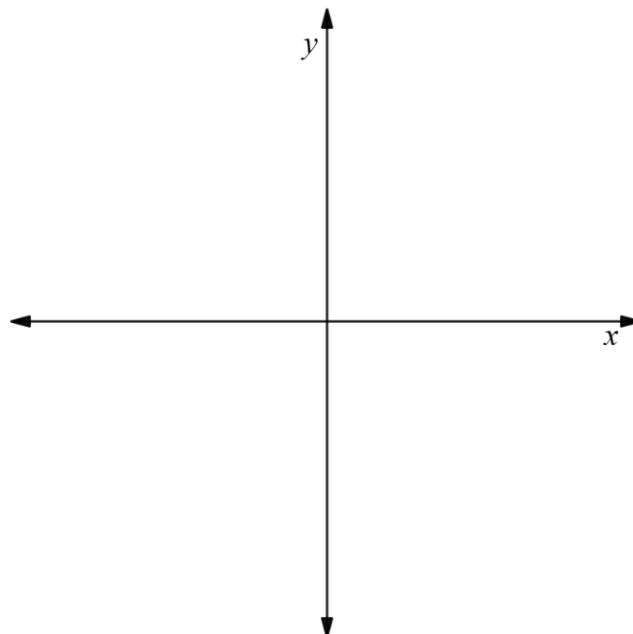


Figure 4

Spare Diagrams

Question 8 c)

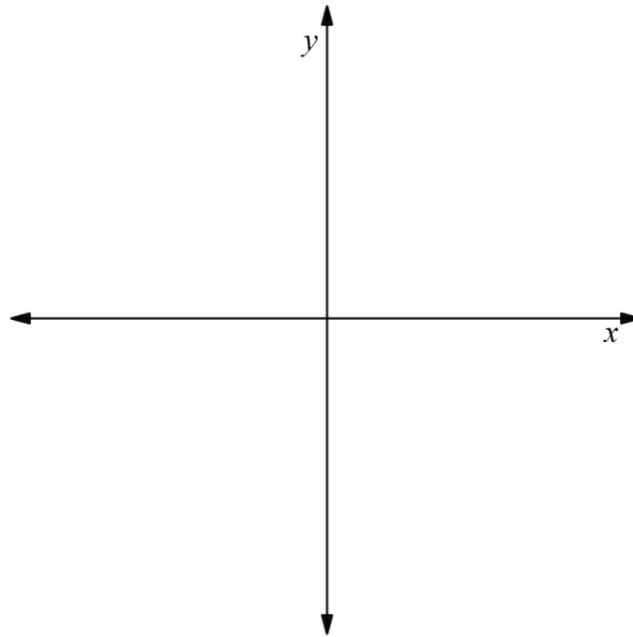


Figure 5

Part 3

- Answer all questions in this part.
- This part assesses **Criterion 6**.

Question 9

Solve the following for x :

a) $2^{x-5} \times 2^2 = \sqrt{2}$

$$2^{x-5+2} = 2^{1/2}$$

$$\therefore x-3 = \frac{1}{2}$$

$$\therefore x = 3.5 \text{ or } \frac{7}{2}$$

/2

b) $\log_x\left(\frac{4}{25}\right) = -2$

$$x^{-2} = \frac{4}{25}$$

$$\frac{1}{x^2} = \frac{4}{25}$$

$$1 = \frac{4x^2}{25}$$

$$\therefore 4x^2 = 25$$

$$\therefore x^2 = \frac{25}{4}$$

$$\therefore \sqrt{x^2} = \sqrt{\frac{25}{4}} = \pm \frac{5}{2}$$

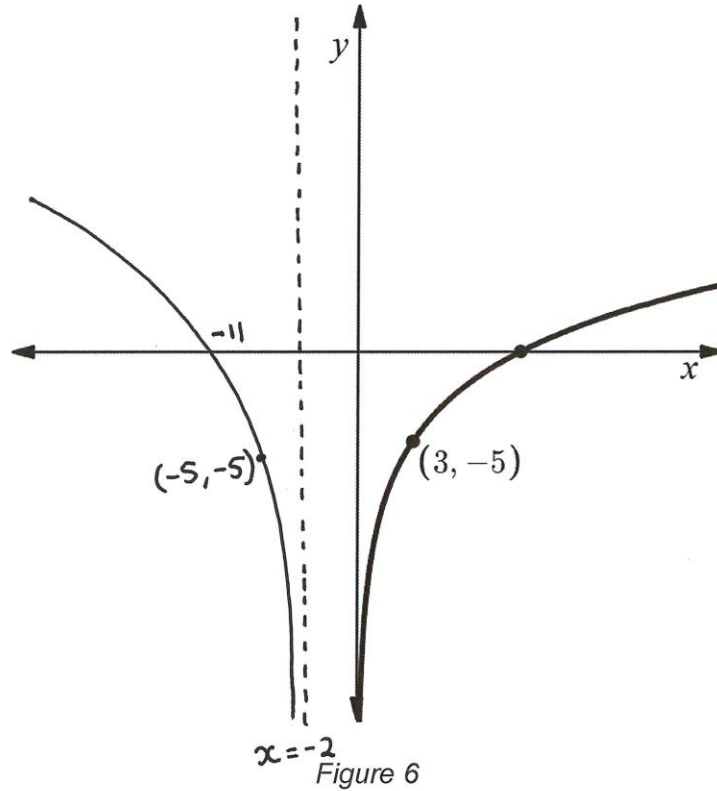
Discard negative solution
as base of log cannot
be negative

$$\therefore x = \frac{5}{2}$$

/2

Question 10

The graph of the logarithmic function $y = 5 \log_3(x) - 10$ is shown in Figure 6.



Spare diagram used (X)

a) Determine the x -intercept of the function.

$$0 = 5 \log_3(x) - 10$$

$$\therefore 5 \log_3(x) = 10$$

$$\therefore \log_3(x) = 2$$

$$\therefore x = 3^2$$

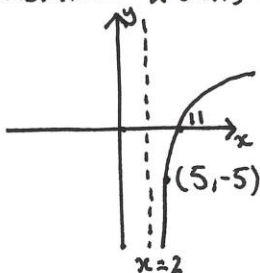
$$\therefore x = 9$$

/2

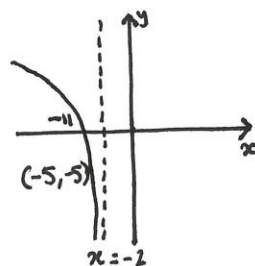
The function is translated 2 units to the right and then reflected in the y -axis.

b) Sketch the transformed function using Figure 6 above, showing the new locations of the asymptote and the points marked.

Step 1: Translation 2 units right



Step 2: Reflection



/2

Question 11

Marker use

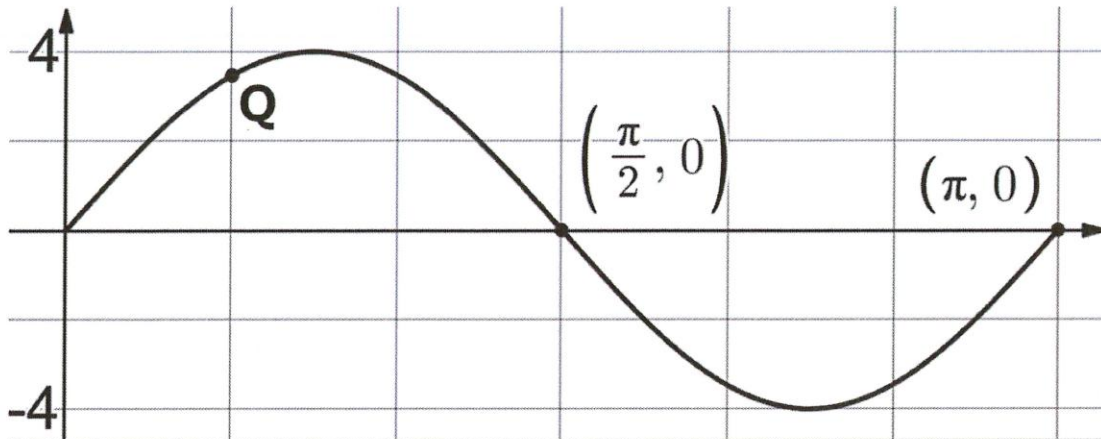


Figure 7

- a) Find the equation of the function shown in Figure 7, that has form $y = a \sin(nx)$.

/2

Amplitude = 4

$\therefore y = 4 \sin(nx)$

Period = $\pi = \frac{2\pi}{n}$

$\therefore n = 2$

$\therefore y = 4 \sin(2x)$

- b) State the exact co-ordinates of the Point Q.

/2

$x = \frac{\pi}{6}$

$\therefore y = 4 \sin\left(2 \times \frac{\pi}{6}\right)$

$\therefore y = 4 \sin\left(\frac{\pi}{3}\right)$

$\therefore y = 4 \times \frac{\sqrt{3}}{2}$

$\therefore y = 2\sqrt{3}$

\therefore Point Q is $\left(\frac{\pi}{6}, 2\sqrt{3}\right)$

Question 12

The unit circle is displayed in Figure 8.

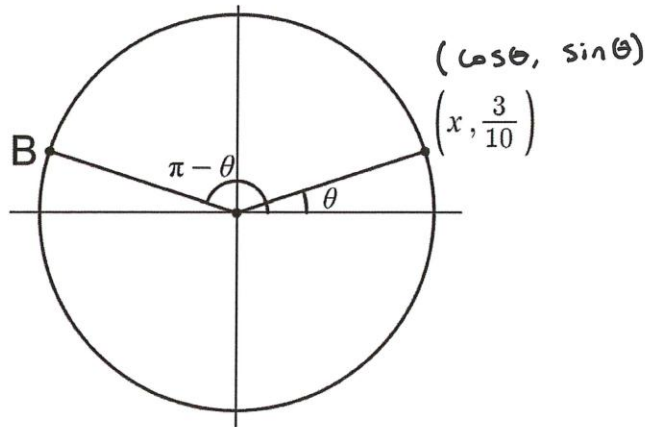


Figure 8

- a) The angle θ is approximately 18° . Convert 18° into radians.

$$18 \times \frac{\pi}{180} = \frac{18\pi}{180} = \frac{\pi}{10}$$

/1

- b) Use a basic identity to determine x , the exact value of $\cos(\theta)$.

$$\sin^2(\theta) + \cos^2(\theta) = 1$$

$$\therefore \left(\frac{3}{10}\right)^2 + \cos^2(\theta) = 1$$

$$\therefore \cos^2(\theta) = 1 - \frac{9}{100}$$

$$\therefore \cos^2(\theta) = \frac{91}{100}$$

$$\therefore \cos(\theta) = \pm \sqrt{\frac{91}{100}}$$

cos is positive in Q1

$$\therefore \cos \theta = +\sqrt{\frac{91}{100}} = +\frac{\sqrt{91}}{10}$$

/2

- c) State the exact co-ordinates of Point B.

cos is neg in Q2

sin is pos in Q2

$$\left(-\frac{\sqrt{91}}{10}, \frac{3}{10}\right)$$

/1

Total
P3

/16

Spare Diagram

Question 10

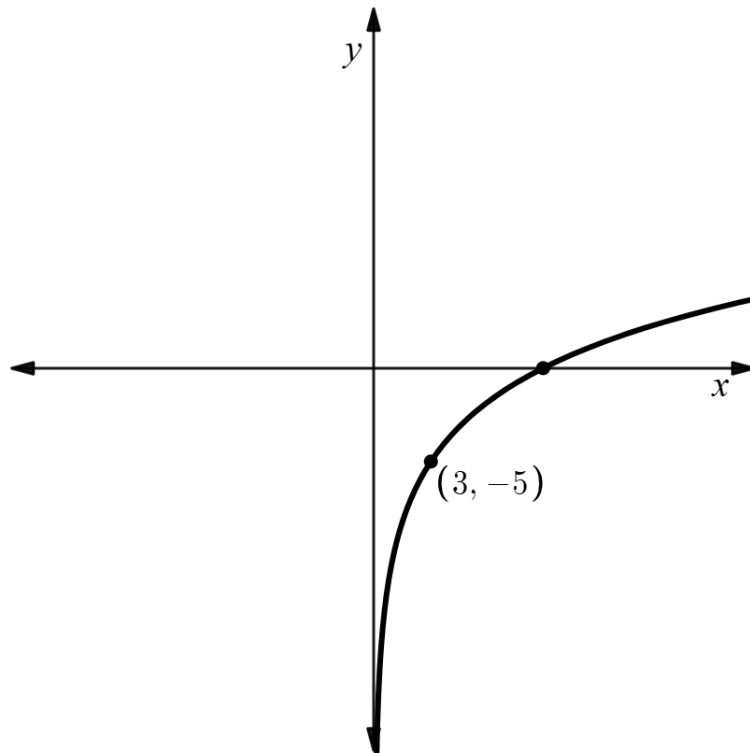


Figure 6

Part 4

Marker use

- Answer all questions in this part.
- This part assesses Criterion 7.

Question 13

Determine the derivative of each of the following:

a) $y = 4x^5 - 6x + 7 + 2x^{-4}$

$$\frac{dy}{dx} = 20x^4 - 6 - 8x^{-5}$$

/2

b) $y = 3x^2(5 - x)$

$$y = 15x^2 - 3x^3$$

$$\therefore \frac{dy}{dx} = 30x - 9x^2$$

/2

c) $y = 8\sqrt{x} + \sqrt[4]{x^7}$

$$y = 8x^{1/2} + x^{7/4}$$

$$\therefore \frac{dy}{dx} = 4x^{-1/2} + \frac{7}{4}x^{3/4}$$

/2

Question 14

Use first principles to differentiate $f(x) = 7x - 1$.

$$f(x+h) = 7(x+h) - 1$$

$$= 7x + 7h - 1$$

$$\therefore f'(x) = \lim_{h \rightarrow 0} \frac{7x + 7h - 1 - (7x - 1)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{7h}{h}$$

$$= \lim_{h \rightarrow 0} 7$$

$$\therefore f'(x) = 7$$

/2

Question 15

For the function $f(x) = 3x^2 - 7x + 5$

a) Find $f'(1)$.

$$f'(x) = 6x - 7$$

$$f'(1) = 6(1) - 7 = -1$$

/1

b) Determine the equation of the tangent of $f(x)$ at $x = 2$.

$$f(2) = 3(2)^2 - 7(2) + 5 = 3$$

\therefore point is $(2, 3)$

$$f'(2) = 6(2) - 7 = 5$$

\therefore gradient of tangent is 5

$$y = 5x + c$$

$$\therefore 3 = 5 \times 2 + c$$

$$\therefore c = -7$$

$$\therefore y = 5x - 7$$

/3

Question 16

Find the co-ordinates of the stationary points for the function $f(x) = x^3 + x^2 - x$.

Do not classify their nature.

$$f(x) = 3x^2 + 2x - 1$$

$$3x^2 + 2x - 1 = 0$$

$$3x^2 + 3x - x - 1 = 0$$

$$\therefore 3x(x+1) - 1(x+1) = 0$$

$$\therefore (x+1)(3x-1) = 0$$

$$\therefore x = -1, \frac{1}{3}$$

$$f(-1) = (-1)^3 + (-1)^2 - (-1) = -1 + 1 + 1 = 1$$

$$f\left(\frac{1}{3}\right) = \left(\frac{1}{3}\right)^3 + \left(\frac{1}{3}\right)^2 - \left(\frac{1}{3}\right)$$

$$= \frac{1}{27} + \frac{1}{9} - \frac{1}{3} = \frac{1}{27} + \frac{3}{27} - \frac{9}{27} = -\frac{5}{27}$$

$$\therefore \text{SP @ } (-1, 1) \text{ and } \left(\frac{1}{3}, -\frac{5}{27}\right)$$

/4

Total
P4

/16

Part 5

- Answer **all** questions in this part.
- This part assesses **Criterion 8**.

Question 17

An unbiased 6-sided die is rolled.

- a) State the probability of rolling an odd number.

$$\Pr(\text{odd}) = \frac{3}{6} = \frac{1}{2}$$

/1

- b) State the probability of rolling a '3' twice in a row.

$$\Pr(3 \text{ and } 3) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$$

/1

Question 18

	<i>B</i>	<i>B'</i>	
<i>A</i>	0.23	-0.11	0.12
<i>A'</i>	0.33	0.56	0.89
	0.56	0.45	1.01

Table 1

List **two (2)** reasons why Table 1 is not a valid probability table.

/2

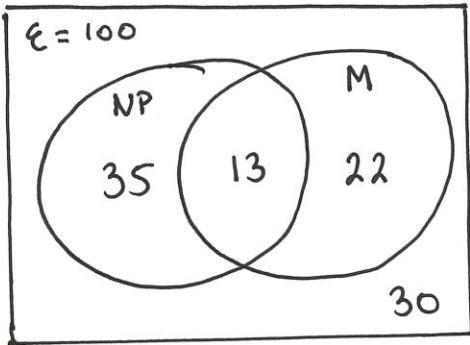
• Probabilities cannot be negative

• Probability total must equal 1

Question 19

When 100 random tourists leaving Tasmania were surveyed, 48 had been to a national park, 22 had not been to a national park but had been to a museum, and 13 had been to both a national park and a museum.

a) Show this information in a Venn diagram.



/2

b) If a surveyed tourist is randomly selected to win a prize, calculate the probability that they had been to a museum.

/1

..... $Pr(M) = \frac{35}{100} = 0.35$

c) Given that a tourist has been to a national park, calculate the probability that they have not been to a museum.

/2

..... $Pr(M'|NP) = \frac{35}{48}$

d) Use an example to explain a method of selecting tourists for the survey that would not be considered random.

/1

..... Only selecting those exiting a museum

Question 20

A wildlife research team wants to conduct a conservation study on 4 of Tasmania's 11 endemic mammals, which are grouped by species in Table 2.

Marsupials	Rodents	Bats
<ul style="list-style-type: none"> Bennetts Wallaby Eastern Barred Bandicoot Eastern Quoll Spotted-tailed Quoll Southern Bettong Tasmanian Devil Tasmanian Pademelon 	<ul style="list-style-type: none"> Broad-toothed Mouse Long-tailed Mouse 	<ul style="list-style-type: none"> Little Forest Bat Long-eared Bat

Table 2

How many ways can the team make this selection if:

- a) there are no restrictions on which endemic mammal species are studied.

$$\begin{aligned}
 {}^{11}C_4 &= \frac{11!}{4!(11-4)!} \\
 &= \frac{11 \times 10 \times 9 \times 8 \times 7!}{4 \times 3 \times 2 \times 1 \times 7!} \\
 &= \frac{11 \times 10 \times 9 \times 8}{3} = 110 \times 3 = 330 \text{ ways}
 \end{aligned}$$

/2

- b) they must study exactly 2 marsupials and 1 bat.

$$\begin{aligned}
 &2M1B1R \\
 &= {}^7C_2 \times {}^2C_1 \times {}^2C_1 \\
 &= 21 \times 2 \times 2 = 84 \text{ ways}
 \end{aligned}$$

/2

- c) they must study more rodents than bats. Answer as a combination expression in the form nC_r (no simplification required).

$$\begin{aligned}
 &2R \times 1B \times 1M + 2R \times 0B \times 2M + 1R \times 0B \times 3M \\
 &{}^2C_2 \times {}^2C_1 \times {}^7C_2 + {}^2C_2 \times {}^2C_0 \times {}^7C_2 + {}^2C_1 \times {}^2C_0 \times {}^7C_3
 \end{aligned}$$

/2

End of Section A

Total
P5
/16



TASMANIAN
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MATHEMATICS METHODS – FOUNDATION

MTM315117

Section **B**

Pages: 28

Questions: 24

Information Sheets: 1

Suggested working time: 100 minutes

Instructions:

Calculators are allowed to be used in this section.

- There are **five (5) parts** to this section.
- Answer **all** questions and **all** items within each question.
- Write your answers in the spaces provided in this exam paper.
 - Spare diagrams have been provided at the end of each part.
Indicate in the box provided if you have used the spare diagram.
- The exam is **three (3) hours** in length. The suggested working time for this section is **approximately 100 minutes**.
- During the first 80 minutes of the exam you may move onto Section B, but you **cannot** use your calculator until told by your supervisor(s).
- The Mathematics Methods – Foundation Information Sheet can be used throughout the exam.
- All answers must be written in **English**.

Marker use	
C4	/ 20
C5	/ 20
C6	/ 20
C7	/ 20
C8	/ 20

Additional Exam Instructions

- You **must** make sure your answers address the listed criteria.
- For questions worth **one (1)** mark, you **are not required** to show workings. Markers will look at the presentation of the answer(s) and at the argument(s) leading to the final answer(s).
- For questions worth **two (2)** or more marks you **are required** to show relevant workings.
- Marks will be allocated:
 - according to the degree to which workings convey a logical line of reasoning, and
 - for suitable justifications and explanations of methods and processes when requested.

Criteria

You **must** make sure your answers address:

- Criterion 4 manipulate algebraic expressions and solve equations
- Criterion 5 understand linear, quadratic and cubic functions
- Criterion 6 understand logarithmic, exponential and trigonometric functions
- Criterion 7 use differential calculus in the study of functions
- Criterion 8 understand experimental and theoretical probabilities and of statistics.

Guide to Exam Structure

		Parts	Questions available	Questions to answer	Suggested working time	Marks available
Section A	Part 1		4	4	16 minutes	16 marks
	Part 2		4	4	16 minutes	16 marks
	Part 3		4	4	16 minutes	16 marks
	Part 4		4	4	16 minutes	16 marks
	Part 5		4	4	16 minutes	16 marks
Totals			20	20	80 minutes	80 marks
Section B	Part 1		6	6	20 minutes	20 marks
	Part 2		4	4	20 minutes	20 marks
	Part 3		5	5	20 minutes	20 marks
	Part 4		5	5	20 minutes	20 marks
	Part 5		4	4	20 minutes	20 marks
Totals			24	24	100 minutes	100 marks
Totals			44	44	180 minutes (3 hours)	180 marks

Part 1

Marker use

- Answer **all** questions in this part.
- This part assesses **Criterion 4**.

Question 21

For the formula $M = \frac{1}{3t} + \frac{h-5}{t^2} - \frac{2}{t}$

- a) Combine the fractions so that M is equal to a single fraction.

$$M = \frac{t + 3(h-5) - 6t}{3t^2}$$
$$\therefore M = \frac{3h - 15 - 5t}{3t^2}$$

/1

- b) Determine the value of M when $h = 10.2$ and $t = 0.8$.

Answer correct to **one (1)** decimal place.

$$M = \frac{3(10.2) - 15 - 5(0.8)}{3(0.8)^2}$$
$$\therefore M = 6.0$$

/1

- c) Rearrange the formula to make h the subject.

$$M = \frac{3h - 15 - 5t}{3t^2}$$
$$3Mt^2 = 3h - 15 - 5t$$
$$3h = 3Mt^2 + 15 + 5t$$
$$\therefore h = \frac{3Mt^2 + 15 + 5t}{3}$$

/2

Question 22

Write $(\sqrt{3}x + 2y)(3x^2 - 2\sqrt{3}xy + 4y^2)$ as a sum of two cubes.

/1

$$(\sqrt{3}x)^3 + (2y)^3 \quad \text{OR} \quad 3\sqrt{3}x^3 + 8y^3$$

Question 23

A student has attempted to expand $(2x - 5)^4$ as shown:

$$1(2x)^4 + \overset{4}{\textcircled{2}}(2x)^3(-5) + 6(2x)^2(-5)\overset{2}{\textcircled{3}} + 4(2x)(-5)^3 \ominus 1(\overset{+}{\textcircled{5}})^4$$

a) Locate and correct the mistakes made in their working above.

/2

b) Hence, simplify the expansion of $(2x - 5)^4$.

/1

$$= 16x^4 - 160x^3 + 600x^2 - 1000x + 625$$

Question 24

Solve the following for x :

a) $3x^3 - 7\sqrt{2}x = 0$, answering correct to **two (2)** decimal places.

/2

$$x(3x^2 - 7\sqrt{2}) = 0$$

$$\therefore x = 0 \quad \text{OR} \quad x = 1.82 \quad \text{OR} \quad x = -1.82$$

b) $x^2 - 4x - 10 = 0$, by completing the square.

/3

$$(x^2 - 4x + 4) - 4 - 10 = 0$$

$$(x - 2)^2 - 14 = 0$$

$$(x - 2 - \sqrt{14})(x - 2 + \sqrt{14}) = 0$$

$$x = 2 + \sqrt{14} \quad \text{OR} \quad x = 2 - \sqrt{14}$$

Question 25

Marker use

Use the discriminant to determine a value of k such that the quadratic function

$$f(x) = -3x^2 + 2x + k \text{ has:}$$

- a) Exactly one x -intercept.

One intercept when $\Delta = 0$

$$(2)^2 - 4(-3)(k) = 0$$

$$4 + 12k = 0$$

$$12k = -4 \quad \therefore k = -\frac{1}{3}$$

/2

- b) Two rational x -intercepts.

When $\Delta > 0$, For rational roots Δ perfect square

$$4 + 12k > 0$$

$$4 + 12k = 1$$

$$4 + 12k = 4$$

$$4 + 12k = 9$$

$$12k > -4$$

$$12k = -3$$

$$12k = 0$$

$$12k = 5$$

$$k > -\frac{1}{3}$$

$$\therefore k = -\frac{1}{4}$$

$$\therefore k = 0$$

$$\therefore k = \frac{5}{12}$$

/2

Question 26

A triangle with an area of 4 cm^2 has a height 2 cm more than the length of its base.

- a) Show that the equation $0 = h^2 - 2h - 8$ describes the height of the triangle, $h \text{ cm}$, by using the formula $\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}$.

$$4 = \frac{1}{2} \times (h - 2) \times h$$

$$0 = \frac{1}{2}h^2 - h - 4$$

$$0 = h^2 - 2h - 8$$

/1

- b) Determine the height of the triangle by solving $0 = h^2 - 2h - 8$ with the quadratic formula.

$$h = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(-8)}}{2}$$

$$h = -2 \text{ or } h = 4$$

disregard

$$\therefore \text{height} = 4 \text{ cm}$$

/2

Total
P1

/20

Part 2

- Answer **all** questions in this part.
- This part assesses **Criterion 5**.

Question 27

For the line $8 - 2y = 4x$

- a) Determine the gradient.

$$\begin{aligned}
 -2y &= 4x - 8 \\
 y &= -2x + 4 \\
 \therefore m &= -2
 \end{aligned}$$

/1

- b) Determine the equation of the perpendicular line which intersects the line above at $x = 2$.

$$\begin{aligned}
 m_{\perp} &= \frac{1}{2} && \text{when } x=2, y = -2(2)+4 \\
 &&& \therefore y = 0 \\
 y - y_1 &= m(x - x_1) && \text{point at } (2, 0) \\
 y - 0 &= \frac{1}{2}(x - 2) \\
 y &= \frac{1}{2}x - 1
 \end{aligned}$$

/3

Question 28

Sketch the function $f: (-3, 1] \rightarrow \mathbb{R}$ where $f(x) = 3x^2 + 8x - 3$, using Figure 9.

Label the x and y intercepts and the end points.

x -intercepts $3x^2 + 8x - 3$ $f(-3) = 3(-3)^2 + 8(-3) - 3$
 $\begin{matrix} 3x & & -1 \\ x & \times & 3 \end{matrix}$ $= 0 \quad \therefore (-3, 0)$

$(3x-1)(x+3) = 0$ $f(1) = 3(1)^2 + 8(1) - 3$
 $x = \frac{1}{3} \text{ or } x = -3.$ $= 8 \quad \therefore (1, 8)$

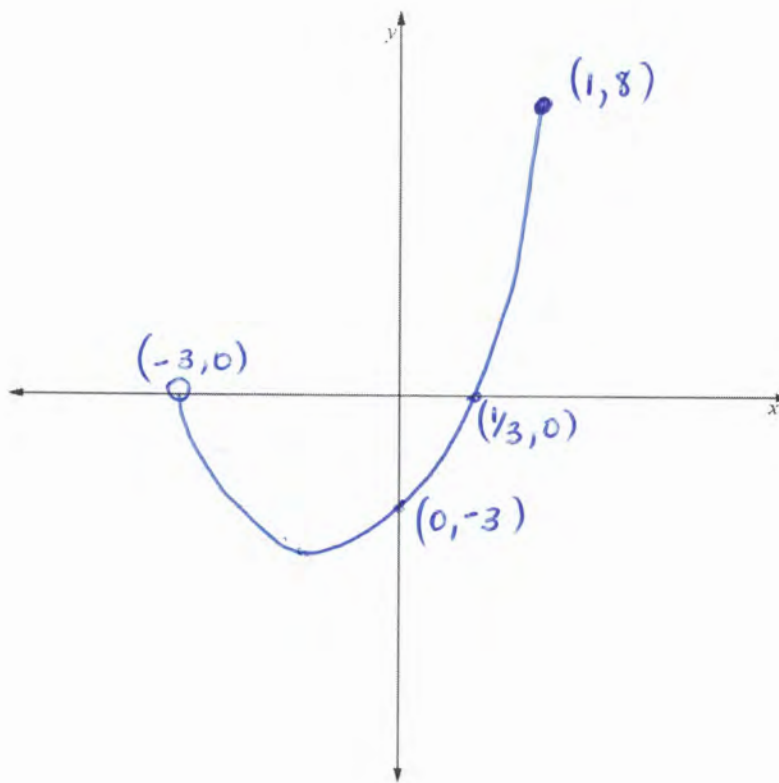


Figure 9

Spare diagram used (X)

Question 29

Marker use

The function $f(x) = (x - 2)^3$ is translated 3 units left and 1 unit down.

a) State the equation of the new function.

..... $f(x) = (x+1)^3 - 1$

.....

/1

b) Sketch the new function on the axes in Figure 10. Label any intercepts and the inflection point.

/2

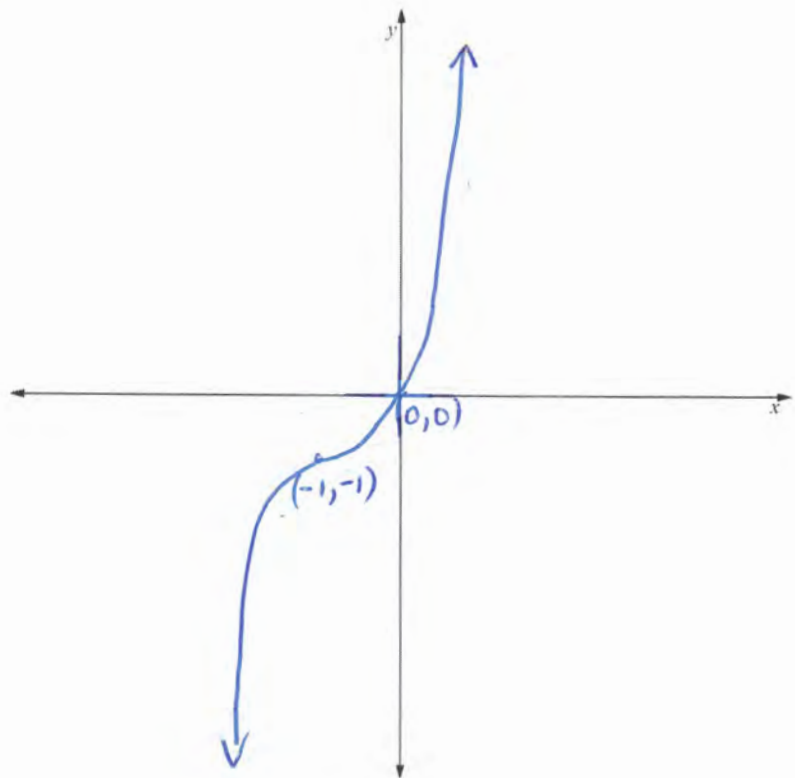


Figure 10

Spare diagram used (X)

Question 30

The map in Figure 11 shows the voyage of the ships captained by Scallywag Sally and Pirate Pete.

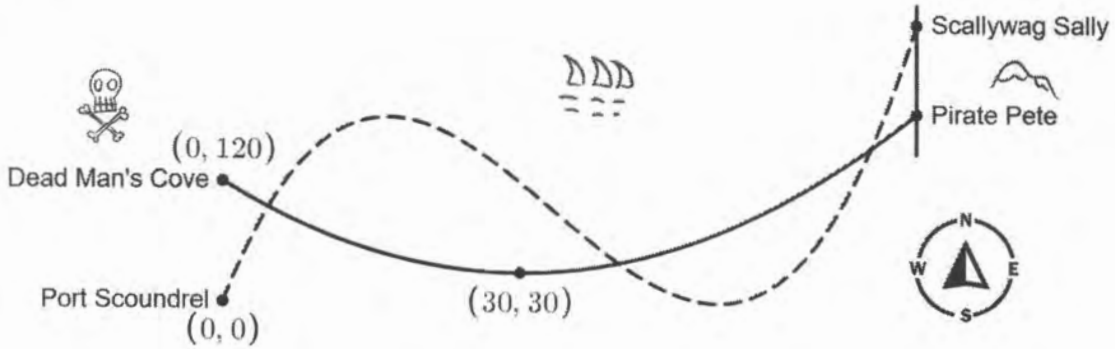


Figure 11

The voyage of Scallywag Sally is given by the equation $y = \frac{1}{100}x(x - 50)^2$, where she is y km north and x km east of her starting point at Port Scoundrel.

a) How far north is Sally when she has travelled 10 km east?

when $x=10$ $y=160$ $\therefore 160$ km east

/1

b) How far north (to the nearest km) does Sally get, before she heads south?

local maximum 185 km

/1

c) How far east (to the nearest km) is Sally when she is 200 km north?

when $y=200$, $x=67$ km

/1

Question 30 continues

Question 30 continued

Marker use

The voyage of Pirate Pete starts at Dead Man's Cove, located exactly 120 km north of Port Scoundrel, and can be described by the equation $y = a(x - h)^2 + k$.

The furthest south Pete reaches is the co-ordinate (30, 30).

d) Determine a , h and k .

$(h, k) = (30, 30)$ $120 = a(0 - 30)^2 + 30$
using $(0, 120)$ $\therefore a = \frac{1}{10}, h = 30, k = 30$

/2

e) How many times do the ships pass through exactly the same location?

3 times

/1

f) State the co-ordinates of one of these locations to the nearest km.

$(5, 95)$ OR $(40, 40)$ OR $(65, 155)$

/1

Both ships finish their journey when they hit the coast 70 km east of Port Scoundrel.

g) How much further north is Sally than Pete at this point?

at $x = 70$ Sally is 280km North Pete is 190km North
 \therefore Sally is 90km further North.

/2

Total
P2

/20

Spare Diagrams

Question 28

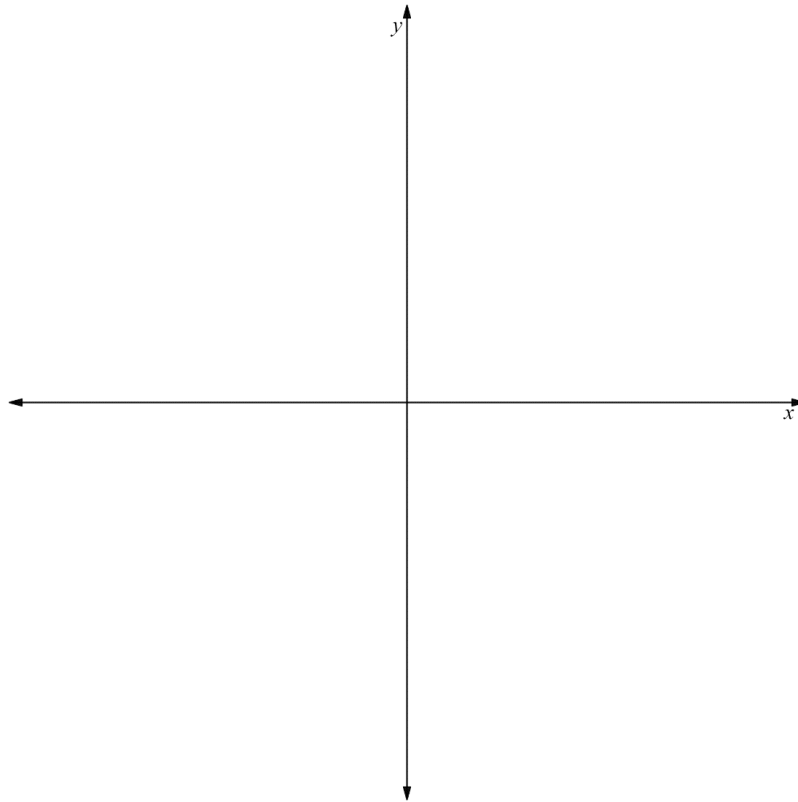


Figure 9

Question 29 b)

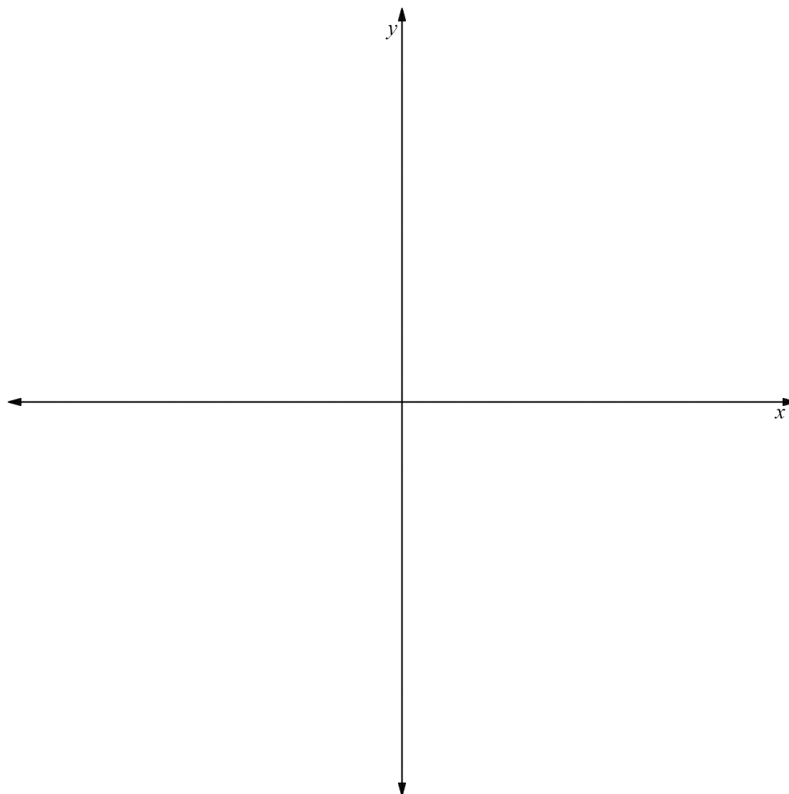


Figure 10

Part 3

Marker use

- Answer **all** questions in this part.
- This part assesses **Criterion 6**.

Question 31

For the function $y = \tan\left(\frac{x}{3}\right)$

a) Determine the period.

$$\pi / \frac{1}{3} = 3\pi$$

/1

b) State the range.

$$y \in (-\infty, \infty)$$

/1

c) What is the equation of the first positive asymptote?

$$\pi/2 = x/3$$

$$\therefore x = 3\pi/2$$

/1

Question 32

Marker use

- a) Sketch the graph of $y = -1 \times 2^{x+1} + 5$ for the domain $-\infty < x < 3$ using the axes in Figure 12. Clearly label any asymptotes, intercepts and endpoints.

/4

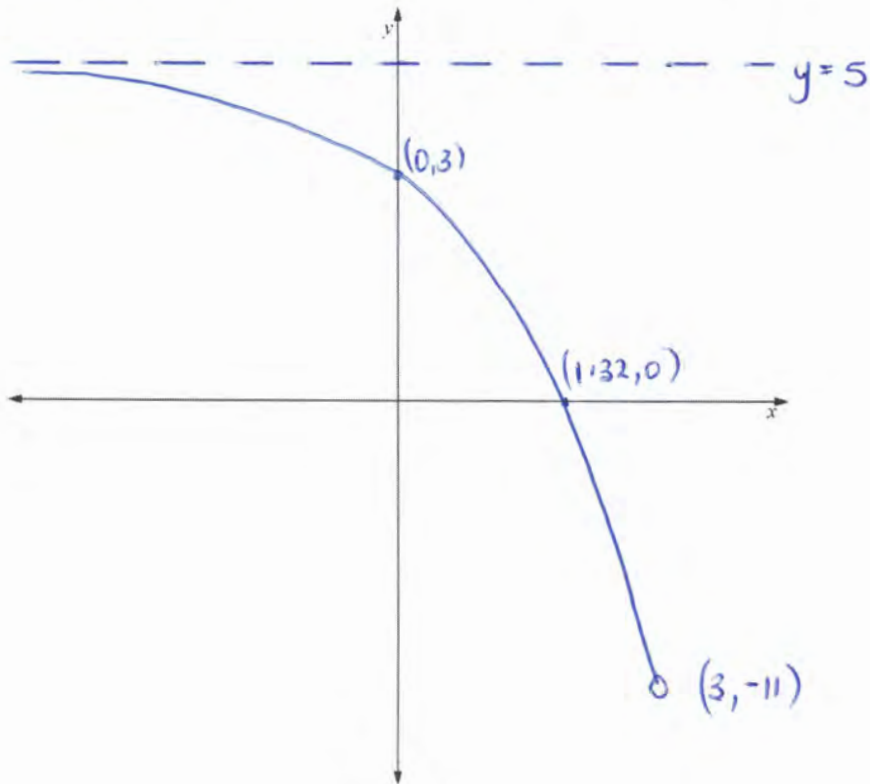


Figure 12

Spare diagram used (X)

- b) State the range.

..... $y \in (-11, 5)$

/1

Question 33

The height (in metres) of the tide above the average sea level can be modelled by a cosine function.

- High tide is 3 metres above average and occurs at midnight.
- Low tide is 3 metres below average and occurs 6 hours later.
- This pattern repeats every 12 hours.

Sketch a graph of the height of the tide over a 24-hour period (starting from midnight) using the axes in Figure 13.

Clearly indicate the height the tide reaches when it is above and below average, and what times these occur.

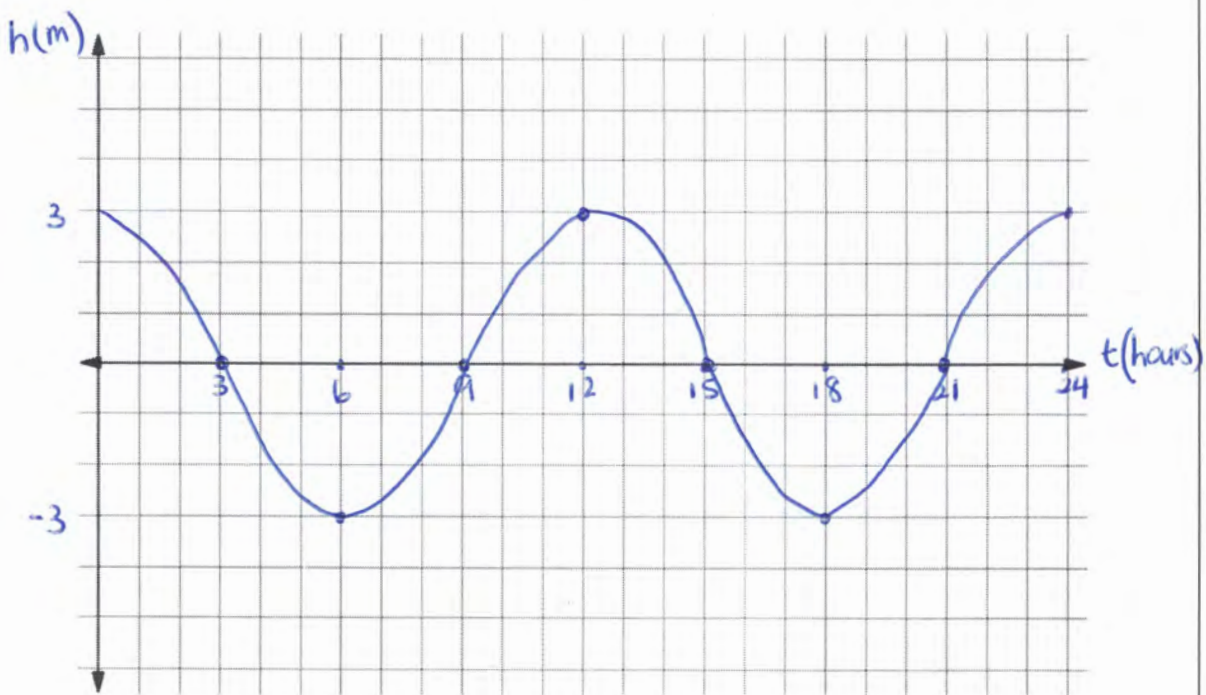


Figure 13

Spare diagram used (X)

Question 34

A shape with four sides of equal length, known as a rhombus, is shown in Figure 14. The perimeter is 120 cm and the length of the diagonal BD is 20 cm.

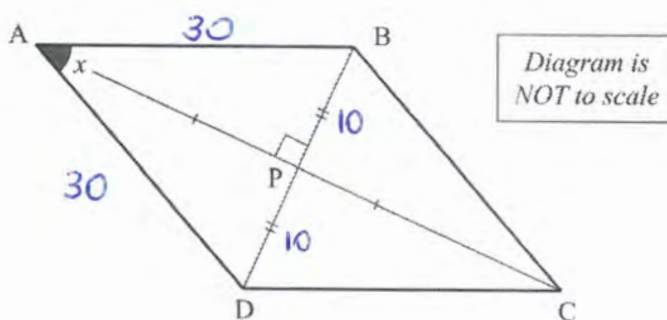


Figure 14

Find the size of x (the angle BAD) to the nearest degree.

.....

$\cos A = \frac{b^2 + d^2 - a^2}{2bd}$

.....

$A = \cos^{-1} \left(\frac{30^2 + 30^2 - 20^2}{2(30)(30)} \right)$

.....

$\therefore A = 39^\circ$

.....

.....

.....

/3

Question 35

The pOH scale is used to express how acidic a solution is using the formula

$$pOH = a \log_{10}(x) + b$$

where x represents the concentration of hydroxide ions in mol/litre.

- A concentration of 1 mol/litre gives a pOH of 14.
- A concentration of 10^{-3} mol/litre gives a pOH of 11.

a) Determine a and b .

using (1, 14) $14 = a \log_{10} 1 + b$

$$\therefore b = 14$$

using (10^{-3} , 11) $11 = a \log_{10} (10^{-3}) + 14$

$$\therefore a = 1$$

/3

b) A pOH level of 7 is considered neutral. What hydroxide ion concentration does this correspond to?

$$7 = \log_{10} x + 14$$

$$\therefore x = 10^{-7} \text{ moles/litre}$$

/2

Total
P3

/20

Spare Diagrams

Question 32 a)

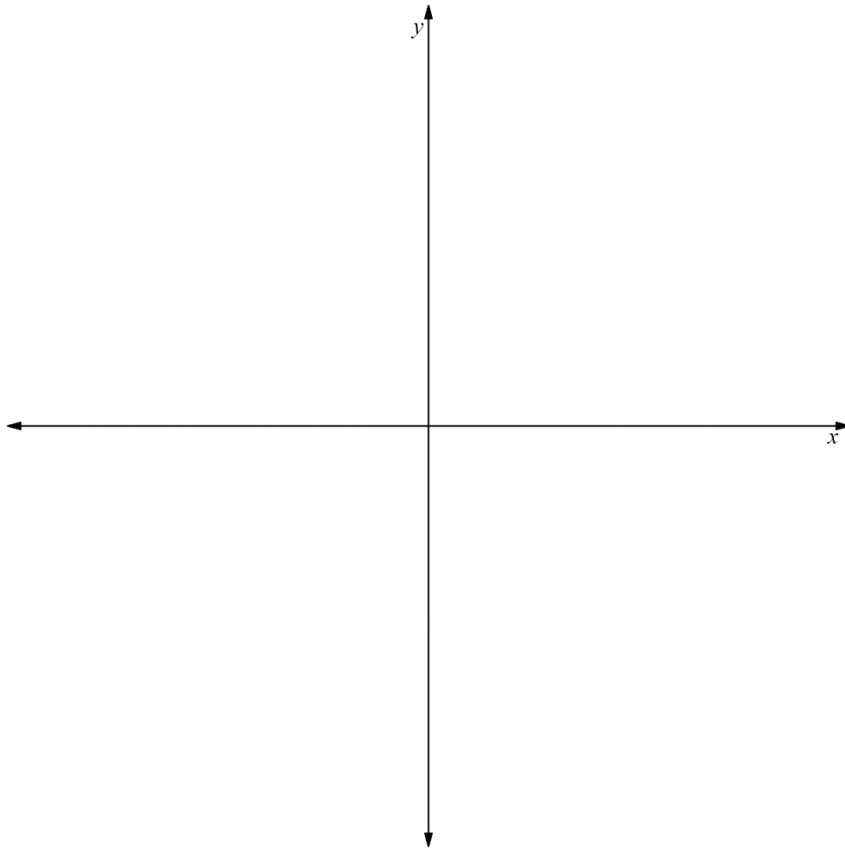


Figure 12

Question 33

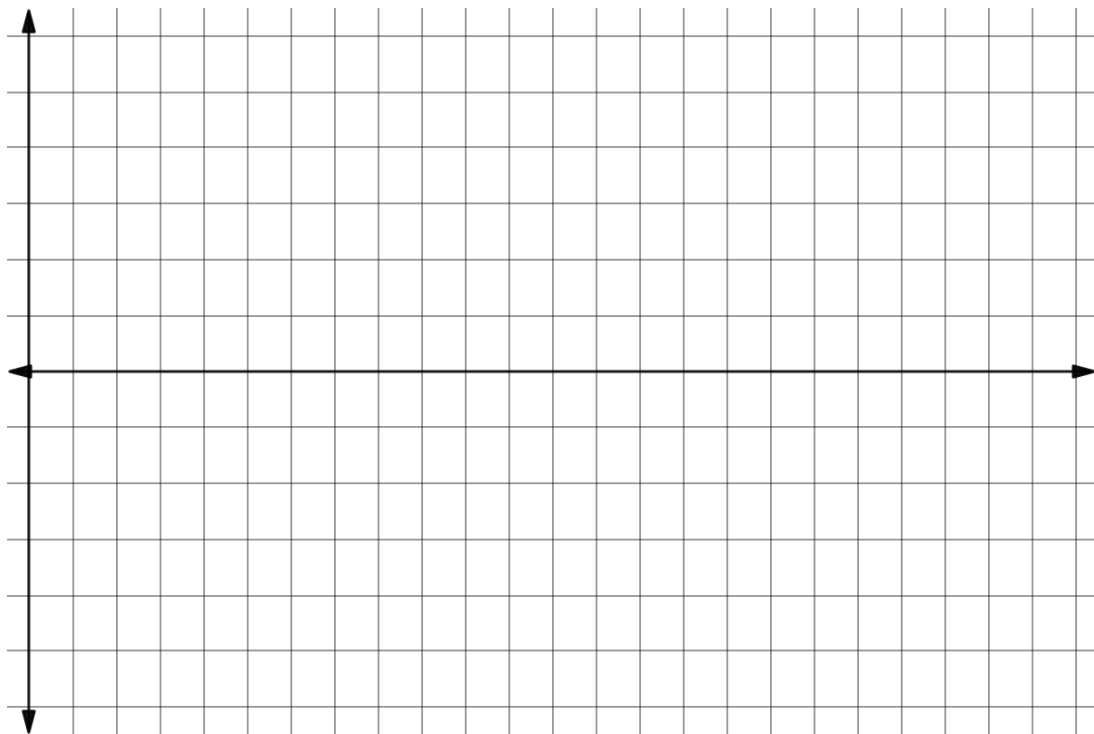


Figure 13

Part 4

Marker use

- Answer **all** questions in this part.
- This part assesses **Criterion 7**.

Question 36

The displacement of a spanner thrown vertically upwards is given by $H(t) = 15t - 5t^2$ where H is the height in metres, and t is the time in seconds after the spanner is thrown.

- a) Use calculus techniques to determine when the spanner reaches its maximum height.

$$H'(t) = 15 - 10t$$

$$\therefore 0 = 15 - 10t$$

$$10t = 15 \quad \therefore t = 1.5 \text{ seconds}$$

- b) What is the maximum height?

$$\text{When } t = 1.5 \quad H(t) = 15(1.5) - 5(1.5)^2$$

$$\therefore H(t) = 11.25 \text{ m}$$

- c) Determine the velocity of the spanner 2 seconds after being thrown.

$$\text{When } t = 2 \quad H'(t) = 15 - 10(2)$$

$$= -5 \text{ m/s}$$

- d) At what time is the spanner moving at a velocity of 10 m/s downwards?

$$-10 = 15 - 10t$$

$$10t = 25$$

$$\therefore t = 2.5 \text{ seconds}$$

/2

/1

/1

/1

Question 37

The profit, P thousand dollars, recorded by a shop over several years is graphed in Figure 15 and given by the equation $P = t^3 - 7t^2 + 15t - 4$.

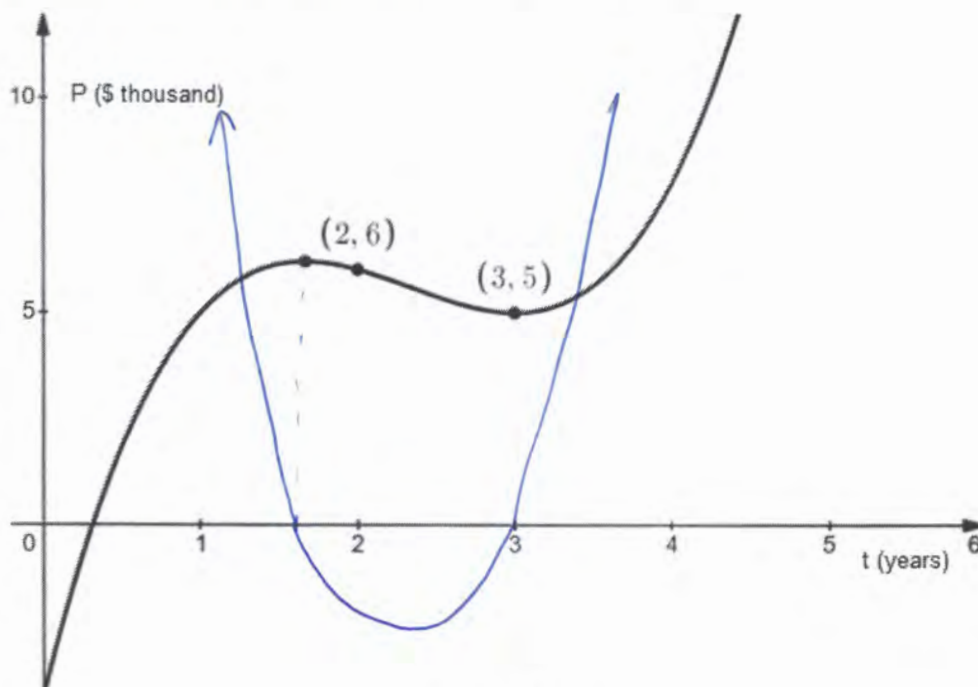


Figure 15

Spare diagram used (X)

- a) Determine the average rate of change of the company's profit from year 2 to year 3.

Ave. RoC = $\frac{5-6}{3-2}$
 = - \$1000 per year

/2

- b) At what time(s) is the company's instantaneous rate of change in profit the same as the average rate calculated in part a)?

$P'(t) = 3t^2 - 14t + 15$
 $-1 = 3t^2 - 14t + 15$
 $\therefore t = 2 \text{ seconds and } t = \frac{8}{3} \text{ seconds.}$

/2

- c) Sketch $\frac{dP}{dt}$ using the same axes as Figure 15.

/2

Question 38

Figure 16 is a graph of a function $f(x)$ that has range $f(x) \in (-\infty, 3]$.

a) State the nature of the co-ordinate:

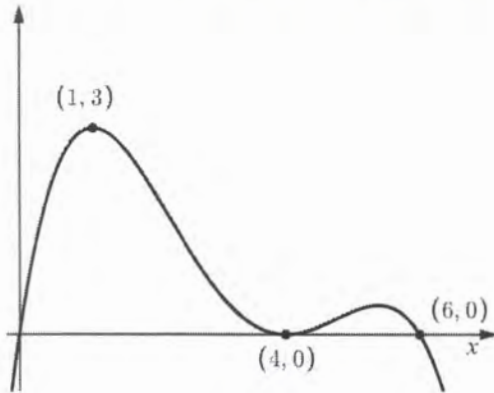


Figure 16

i. (1, 3)

Global maximum

/1

ii. (4, 0)

Local minimum

/1

b) True or false: $f'(2) < 0$?

True

/1

Question 39

The volume of water in a pond is given by $V(t) = \frac{1}{2}(t - 3)^3 + 100$, where V is the volume in litres and t is the time in days since measuring began.

a) State the initial volume of the pond.

$$V(0) = \frac{1}{2}(-3)^3 + 100$$

$$= 86.5 \text{ L}$$

/1

b) Determine the rate at which the volume is changing after 2 days.

$$V'(t) = \frac{3}{2}(t-3)^2$$

$$V'(2) = \frac{3}{2}(2-3)^2$$

$$\therefore V'(2) = 1.5 \text{ L/day}$$

/2

Question 40

Marker use

Use calculus techniques to find the equation of the tangent to $f(x) = 5x^2 - 20x + 30$ that is parallel to the x axis.

/3

$$f'(x) = 10x - 20$$

parallel to x axis, $m=0$.

$$0 = 10x - 20$$

$$\therefore x = 2$$

$$f(2) = 5(2)^2 - 20(2) + 30$$

$$\therefore f(2) = 10 \quad \text{point at } (2, 10)$$

$$y - y_1 = m(x - x_1)$$

$$y - 10 = 0(x - 2)$$

$$\therefore y = 10$$

Total
P4

/20

Spare Diagram

Question 37

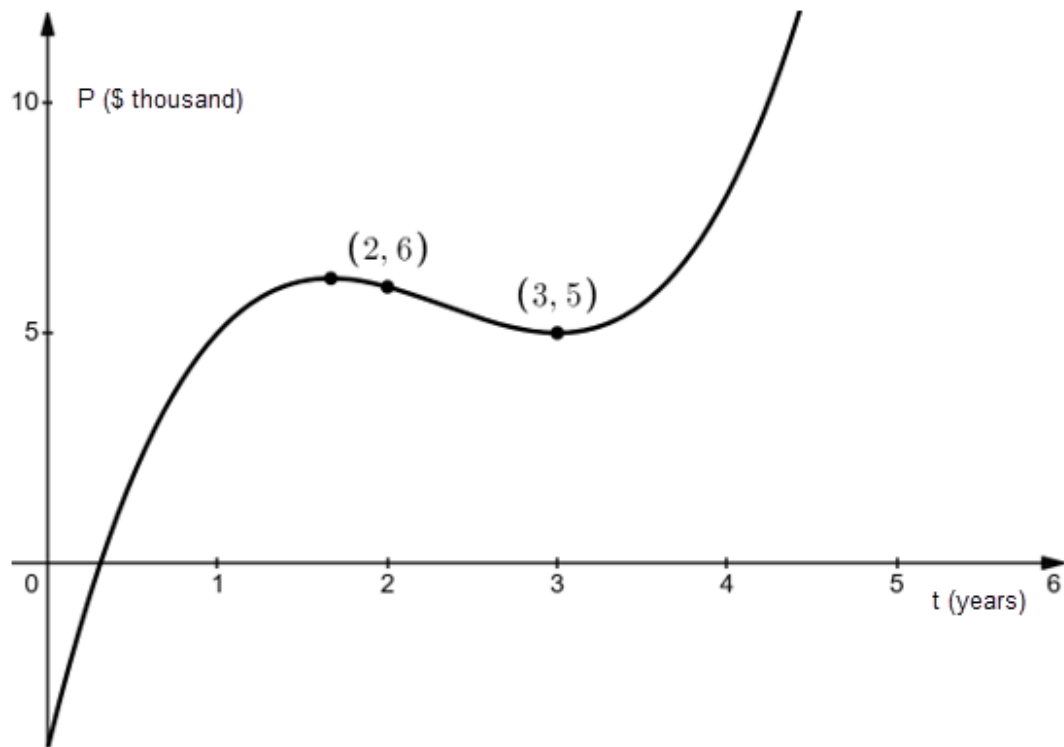


Figure 15

Part 5

Marker use

- Answer **all** questions in this part.
- This part assesses **Criterion 8**.

Question 41

Two events A and B are such that $\Pr(A) = 0.36$, $\Pr(B) = 0.28$ and $\Pr(A \cap B) = 0.15$.

- a) Determine if A and B are independent.

$$\Pr(A) \times \Pr(B) = 0.36 \times 0.28$$

$$= 0.1008$$

$$\neq 0.15.$$

\therefore Not independent.

/2

- b) Calculate $\Pr(A \cup B)$.

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$$

$$= 0.36 + 0.28 - 0.15 = 0.49$$

/1

- c) Calculate $\Pr((A \cap B)')$.

$$\Pr((A \cap B)') = 1 - \Pr(A \cap B)$$

$$= 1 - 0.15 = 0.85.$$

/1

Question 42

In my house, 4 of 12 jars of honey are not properly sealed.

If I select 3 jars at random, without replacement, calculate the probability that at most 1 is not properly sealed.

$$= \frac{{}^8C_3 \times {}^4C_0 + {}^4C_1 \times {}^8C_2}{{}^{12}C_3}$$

$$= \frac{168}{220}$$

$$= \frac{42}{55} \quad (\text{OR } 0.7636)$$

/3

Question 43

Approximately 72% of visitors to Tasmania arrived by air last year, with the rest arriving by sea. Of those who arrived by air, 62% arrived via Hobart Airport and 24% arrived via Launceston Airport. Of those who arrived by sea, 60% arrived on a cruise ship, with the rest arriving on a ferry.

a) Complete Figure 17 which presents the information above using a tree diagram.

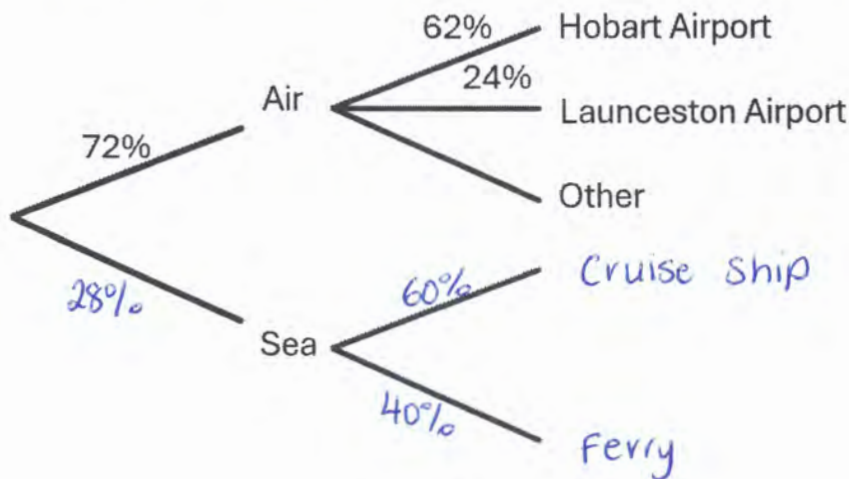


Figure 17

Spare diagram used (X)

/3

b) Given that a visitor arrived by sea, what is the probability that they arrived via ferry?

..... 40%

/1

c) Determine the probability that a visitor arrives by ferry or Launceston Airport.

..... $Pr(F \cup LA) = (0.28 \times 0.4) + (0.72 \times 0.24)$

..... = 0.2848

/2

d) Modelling suggests Tasmania will have around 1.5 million visitors this year. What number of these visitors are predicted to arrive via Hobart Airport?

..... $1500\ 000 \times 0.72 \times 0.62$

..... = 669 600

/2

Question 44

Ash is buying 3 bikes from a shop that stocks 12 mountain bikes. There are 6 different blue bikes, 4 different green bikes, and the remaining are different red bikes.

- a) How many different combinations of 3 bikes could Ash choose?

$${}^{12}C_3 = 220$$

/1

- b) Determine the probability of Ash choosing exactly 1 red bike.

$$\text{Pr (1 Red Bike)} = \frac{{}^2C_1 \times {}^{10}C_2}{{}^{12}C_3}$$

/2

$$= \frac{90}{220} = \frac{9}{22} \text{ or } 0.4091$$

- c) Determine the probability of Ash choosing 3 bikes of the same colour.

$$\text{Pr (3 same colour)} = \frac{{}^6C_3 + {}^4C_3}{{}^{12}C_3}$$

/2

$$= \frac{24}{220} = \frac{6}{55} \text{ or } 0.1091$$

Total
P5

/20

Spare Diagram

Question 43

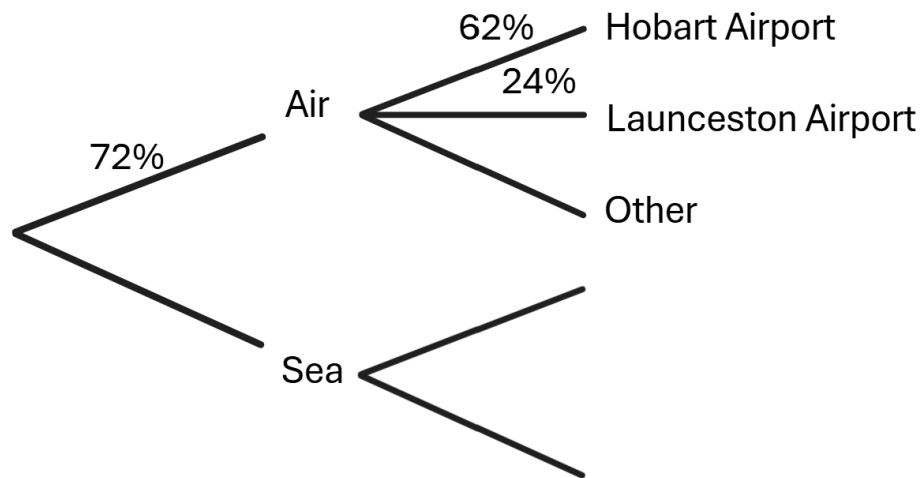


Figure 17

End of Exam



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