2023 ASSESSMENT REPORT

MTM415117 MATHEMATICS METHODS

The following cut-offs were determined by the Assessment Panel. All marks are out of 36.

Criterion	Threshold mark for a 'C'	Threshold mark for a 'B'	Threshold mark for an 'A'
Criterion 4: Function Study	16	24.5	30
Criterion 5: Circular Functions	15	25	32
Criterion 6: Differential Calculus	16	25.5	30.5
Criterion 7: Integral Calculus	13	23	29.5
Criterion 8: Probability	12	22	30

Function Study (Section A)

Question 1

Successful students used brackets to correctly expand the co-efficient of x and combined this with the pascal co-efficient. It should be noted that the wording of the question did not require students to have simplified their answer.

Question 2

a) This was handled well.

b) Less successful responses to this question concluded in either 'Yes' with working out that was contradictory, or 'No' with algebraic reasoning that should have led to a 'Yes' statement.

c) The accurate position of axis intercepts, asymptotes, and a logarithmic shape was assessed. Several students did not consider the position of the reflected asymptote.

d) This question proved to be challenging with many students giving the included domain rather than the excluded domain. Students who used the composite function rule of the inner functions range being a subset of the outer functions range often mixed up the domain and the range.

Question 3

a) This was handled well.

b) This was handled well.

c) Whilst there are a number of alternative sequences that lead to a correctly transformed function, many responses referred unnecessarily to dilations or did not have both translations and reflections in a valid order.

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d) Full marks were awarded for students who carried forward their erroneous response to item c) and thus determined an appropriate answer.

Circular Functions (Section A)

Question 4

This was handled well, with minor errors in simplifying fractions and with multiplication.

Question 5

a) This was generally well done.

b) A common misconception seen involved $\sin\left(\frac{3\pi}{2}+\frac{\pi}{6}\right)=\sin\left(\frac{3\pi}{2}\right)+\sin\left(\frac{\pi}{6}\right)$.

Question 6

a) This was handled well.

b) While some students left this question, others came up with thoughtful answers which demonstrated a deep understanding. Vocabulary involving middle or centre raised some concern.

c) Finding the value of a proved challenging. Substitution of an inflection point resulted in $0 = a(0)$ which was often subsequently ignored.

Question 7

Approaches which avoided using the general trigonometric equations were more successful.

Differential Calculus (Section A)

Question 8

a) This was handled well.

b) Some students didn't place brackets around each term when using the product rule, which is not mathematically correct.

Question 9

a) The derivative of $\ln(4x+1)$ is $\frac{4}{4x+1}$. Many students had $\frac{1}{4x+1}$.

b) This was not done well. The chain rule is required for the exponential. Many students did not put brackets around $(6x+1)$. The second part of the expression could be differentiated using the quotient rule or by turning the expression into $x^2(x+1)^{-1}$ and then using the product rule. It should be noted that the wording of the question did not require students to simplify their answer.

Question 10

a) This was handled well.

b) This was handled well, with minor errors.

Question 11

Whilst most students knew how to approach this question, the factor of 3 proved challenging and there was often poor notation or algebraic manipulation shown. Students could check their answer, as differentiating gave x^2.

Integral Calculus (Section A)

Question 12

a) Several potential errors marred many responses, such as the algebraic manipulation when simplifying $\frac{x^2+1}{x}$ to $x+\frac{1}{x}$ or $+\,c$ in the solution of a definite integral. The conceptual misunderstanding trying to distribute antidifferentiation was often seen. $\int(f(x)^n)(g(x)^m)\,dx \neq ((\int f(x)dx)^n)((\int g(x)dx)^m)$.

b) This was done reasonably well with some errors when dividing by $\frac{\pi}{3}$.

c) This was done reasonably well. Some responses did not divide the exponential by 2 when anti-differentiating or, more concerning, divided by $(2x-1)$.

Question 13

Observant students who noticed that the terminals were reversed for one of the definite integrals were able to achieve the correct result.

Question 14

a) Students needed to take an absolute value or multiply the integral by -1 to achieve the correct positive result for area. Many students struggled with using fractions or adding directed numbers.

b) The positive upper terminal (and the positively oriented region) meant this was handled more successfully.

Question 15

The responses to this question were polarised. Students who obtained the linear function was $y=\frac{h}{b}x$ had little trouble gaining full marks with a correct calculation of $\int_0^b \frac{h}{b}x\,dx$. But many students were significantly distracted by the right-angled triangle and gave $y=\sqrt{h^2+b^2}$ which meant they could make little further progress.

Probability (Section A)

Question 16

a) Most students indicated that there were two options – success or failure, but a relatively small portion of students added that he was doing two discrete trials and that the probability remained the same for each.

b) This was generally well done. Most students realised that it was an 'at least one' situation and worked through to the answer, while a smaller proportion added the probabilities for 1 and 2 successes.

Question 17

a) This was handled well. A few students used the symmetry of the distribution to find the answer, while most successfully applied the formula.

b) This was handled well. Students needed to show the subtraction of $E(X)^2$.

c) Many students were able to the values for two standard deviations on either side of the mean, but few went on to explain that 100% of the distribution was included in this range. A common error was to give an answer of 95% which is an approximation for a normal distribution, and not appropriate for this discrete scenario.

d) Most students were successful in stating the new mean. Only a relatively small number realised the variance was unchanged, instead giving answers of 101.2 or 12.

Question 18

Most students substituted the new information into the formula and, not being able to see a way forward, gained part marks. Stronger students realised that this meant the value of M was decreased by a factor of 10 and were able to calculate the new interval. Some arithmetic errors followed.

Question 19

Several students tried to calculate a numerical value for 20% and 40%. Successful students interpreted which parts of the graph represented these percentages and used the correct inequality signs with reference to the original graph.

Function Study (Section B)

Question 20

Students who graphed a symmetrical truncus and took care in accurately plotting the location of the y-intercept were rewarded. Less successful responses did not state the equations used to solve for the x-intercepts or gave no working if the graphics calculator was used.

Question 21

Most students used the co-ordinates $(1, 3)$ and $(5, 5)$ to set up two equations and went on to solve them simultaneously on the graphics calculator.

Question 22

a) Most students understood the instruction to solve 'algebraically'. A common approach used log laws to show that the left-hand side and right-hand side were equivalent when $x = 2$ was substituted.

b) Correctly disregarding $x = -3$ as a solution proved challenging. Common errors included simplifying the expressions on each side of the equation by cancelling out $x = -3$ and disregarding the solution by stating that you cannot have a negative value inside a logarithm (when in fact $\log(0)$ occurs which is undefined).

Question 23

a) Students who correctly set-up this question were rewarded if they used the graphics calculator to avoid algebraic manipulation of the function and included $f^{-1}(x)$ in their final answer.

b) Students generally understood that the domain and range of an inverse function are opposite to the original function. Most students stated the correct domain but the range proved more challenging.

Question 24

a) This was universally well done.

b) Students needed to apply transformations to the function one at a time and state their final answer in function notation. The few students who recognised what dilations were needed struggled with the difference between applications of x and y dilations.

Circular Functions (Section B)

Question 25

a) This was handled well.

b) Markers were looking for a smooth sinusoidal graph that carefully passed through midpoints and had no sharp maxima or minima.

c) This was handled well.

d) The horizontal translation proved challenging to find for many students.

Question 26

a) This proved very challenging with many responses indicating a clockwise rotation.

b) This was handled well.

c) Many students incorrectly applied the dilation by doubling the argument.

d) This was handled well using the response from item c). Students are advised to keep their calculator in 'Radians' mode.

Question 27

a) Many students were unable to gain full marks due to insufficient algebraic working.

b) This was handled well.

c) Students who identified that the x-intercepts needed to be located at $(3, 0)$ and $(-3, 0)$ were often then able to solve for the correct value. Many students used the point $(3, -3)$ which is not quite on the curve, and thus answered $k = 6$.

Differential Calculus (Section B)

Question 28

This was not handled well as students struggled to find a starting point or chose to guess a solution. Many students differentiated $y = \frac{16}{x}$ and got stuck equating it this to zero.

Question 29

This was well-attempted by most students; however, many didn't have the correct open or closed circle at the end of each line segment.

Question 30

a) Many students used the solve feature of their calculator to equate the two equations which resulted in $x = \frac{5}{2}$. Some students substituted $\frac{5}{2}$ into each function to show that the y −value was the same. Both approaches were accepted.

b) Most students found the derivative and then substituted $x = \frac{5}{2}$ to get the same gradient for each function. Some students used the solve command to find the solution when the derivatives were equal. Both approaches were accepted.

c) Both reasons were required for full marks. A significant number of students talked about the functions being inverses of each other which is irrelevant.

Question 31

a) This was generally well done. Some students forgot to include the correct units.

b) Students who found the derivative and solved for x when the derivative was zero were able to obtain the values $h = 0$ and $h = 8$. The nature of the stationary points was found either through a gradient table or use of the second derivative. Some students forgot to find the corresponding y-coordinate of each point and state their final answers.

c) The graph of the derivative was poorly drawn with little accuracy given for a question that provided a grid. Many students did not take notice of the domain specified in the question.

d) This was poorly done. The question asked for when the temperature was increasing, rather than when the derivative was increasing which results in the incorrect answer of $0 - 4$ hrs.

Integral Calculus (Section B)

Question 32

a) This was handled well with most students using absolute value signs and adding a constant.

b) Most students focused on the process that both A and B used. Very few mathematical explanations were given that articulated how the answers were equivalent with the apparent difference being accounted for by the constant.

Question 33

a) An error too frequently seen was $\int \frac{1}{x^2} dx = \ln(x^2)$ or a variant.

b) The use of the calculator helped students to obtain the correct answer. Interestingly, many students integrated correctly in this part but not the prior.

c) Students discussed the integrand decreasing or the definite integral increasing but few determined the correct response of approaching 1.

Question 34

Students who achieved well in this question had:

- An expression of definite integrals that would give rise to a correct evaluation. Common misconceptions here included considering the intersection of the arches as a triangle which leads to an answer incorrect by 0.1 units of area or using $\int \text{top}(x) - \text{bottom}(x) dx$ over the whole interval which leads to an answer with added areas below the axis.
- Antiderivatives for the integrands.
- The answer.

Question 35

a) Successful solutions used the product rule with the derivative $\frac{d}{dx}(x) = 1$ being clearly articulated.

b) There was an expectation that the fact given should be used in solutions which proved challenging as alternative approaches might have been more familiar.

Probability (Section B)

Question 36

This was generally handled well, with successful students providing a clear definition of the binomial distribution involved in the final part. Those who tried to calculate a solution using $Pr(X = x)$ occasionally left out the nCr term.

Question 37

This was generally handled well, with successful students providing a diagram or clear definition of the distributions involved and including units in their answer were appropriate. Some students calculated $Pr(X = 2)$ rather than $Pr(X \geq 2)$ or incorrectly used $p = 0.2$.

Question 38

Most students calculated the appropriate z-score and used this to determine the solution, whilst a small number of students answered using a solve-function on the CAS calculator and supported this with an explanation or diagram.

Question 39

a) This was handled well.

b) This question proved challenging. Successful students found the z-score, wrote down the interval, and from this were able to determine the highest possible level of confidence. Students needed to be careful with their rounding here.

MATHEMATICS METHODS (MTM415117)

2023 MARKING TOOL

Section A – No calculator

A1 – Function Study

Q1. $(2x+3)^3+(3x+2)^3$

$= 2^3x^3+3\times 2^2\times 3x^2+3\times 2\times 3^2x+3^3+3^3x^3+3\times 3^2\times 2x^2+3\times 3\times 2^2x\ +2^3$

$= 8x^3+36x^2+54x+27+27x^3+54x^2+36x+8$

$= 35x^3+90x^2+90x+35$

(2 marks)

Q2. (a) $f\big(g(x)\big)=f(-x)=\ln\,(-x+2)$

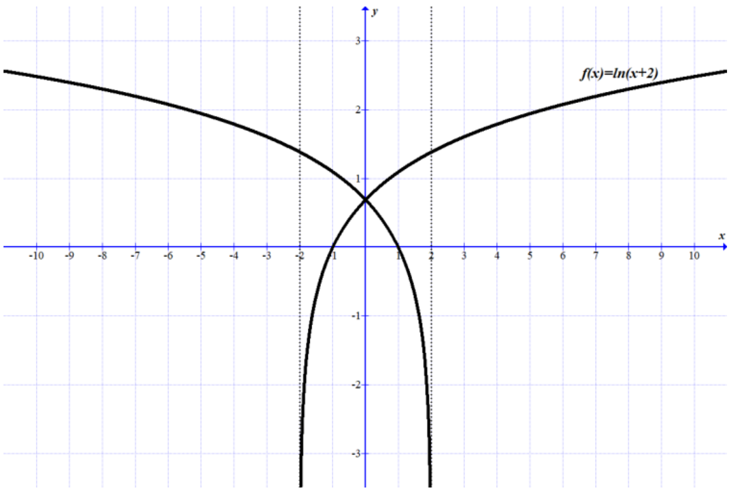
(1 mark)

(b) $f\big(g(1)\big)=\ln(-1+2)=ln1=0$

Yes, it is defined.

(1 mark)

(c)



(2 marks)

(d) We need $-2+x>0$ so that $x>2$. So $x\leq 2$ needs to be excluded from the domain.

(2 marks)

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Q3. (a) Let $y = \frac{a}{x+2} + b$

$b = -2$

Sub. $x = 1$ and $y = 2$

$2 = \frac{a}{1+2} - 2$

$a = 12$

So $y = \frac{12}{x+2} - 2$

(2 marks)

(b) $g(x) = a\sqrt{x} + c$

$c = 4$

$g(1) = 2 = a\sqrt{1} + 4$

$a = -2$

$g(x) = -2\sqrt{x} + 4$

(2 marks)

(c) translation 4 right

Reflection in *y*-axis

Reflection in *x*-axis

Translation down 4.

(2 marks)

(d) $h(x) = - - 2\sqrt{-x-4} - 4 - 4$

$h(x) = 2\sqrt{-x-4} - 8$

(2 marks)

A2 - Trigonometry

Q4. $\frac{7\pi}{6} \times \frac{180}{\pi} = 210°$

(1 mark)

Q5. (a) $\cos\left(\frac{5\pi}{6}\right) = \cos\left(\pi - \frac{\pi}{6}\right)$ 2[nd] quadrant

$= -\cos\left(\frac{\pi}{6}\right)$

$= -\frac{\sqrt{3}}{2}$

(1 mark)

(b) $\sin\left(\frac{3\pi}{2} + \frac{\pi}{6}\right)$ 4[th] quadrant

(2 marks)

$= -\cos\left(\frac{\pi}{6}\right)$

$= -\frac{\sqrt{3}}{2}$

Q6. $y = a\tan\big(n(x+b)\big) + c$

(a) Successive asymptotes are at $-\frac{\pi}{2}$ and $\frac{\pi}{6}$,

so period = $\frac{\pi}{6} - -\frac{\pi}{2} = \frac{2\pi}{3}$, normally π.

$\therefore \ n = \frac{\pi}{\frac{2\pi}{3}} = \frac{3}{2}$

(2 marks)

(b) $\tan(0) = 0$, so the graph could be shifted $\frac{\pi}{2}$ radians right as $x = \frac{\pi}{2}$ is a similar point on the graph.

(2 marks)

(c) We have $c = 1$, translated up by 1.

Sub. the point $(0, 1+\sqrt{3})$

$\therefore \ 1 + \sqrt{3} = \text{a}\tan\left(\frac{3}{2}\left(0 - \frac{\pi}{2}\right)\right) + 1$

$\therefore \ \sqrt{3} = \text{a}\tan\left(-\frac{3\pi}{4}\right)$

$\therefore \sqrt{3} = a \times 1$

$\therefore \ a = \sqrt{3}$

The equation is $y = \sqrt{3}\tan\frac{3}{2}\left(x - \frac{\pi}{2}\right) + 1$.

(4 marks)

Q7. $2\cos\left(x-\frac{\pi}{3}\right) = -\sqrt{3}$

$\therefore\ \cos\left(x-\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$

$\therefore\ x-\frac{\pi}{3} = 2n\pi \pm \arccos\left(-\frac{\sqrt{3}}{2}\right)$

$\therefore\ x = 2n\pi \pm \frac{5\pi}{6} + \frac{\pi}{3}$

For $x \in [-2\pi, 2\,\pi]$ we have:

$n = 0$: $x = -\frac{5\pi}{6} + \frac{\pi}{3} = \frac{3\pi}{6} = \frac{\pi}{2}$

$x = +\frac{5\pi}{6} + \frac{\pi}{3} = \frac{7\pi}{6}$

$n = 1$: $x = 2\pi - \frac{5\pi}{6} + \frac{\pi}{3} = \frac{9\pi}{6} = \frac{3\pi}{2}$

$x = 2\pi + \frac{5\pi}{6} + \frac{\pi}{3} = \frac{19\pi}{6} > 2\pi$

$n = -1$: $x = -2\pi - \frac{5\pi}{6} + \frac{\pi}{3} = -\frac{15\pi}{6} < -2\pi$

$x = -2\pi + \frac{5\pi}{6} + \frac{\pi}{3} = -\frac{5\pi}{6}$

Solutions are $-\frac{5\pi}{6}, -\frac{\pi}{2}, \frac{7\pi}{6}, \frac{3\pi}{2}$.

(4 marks)

A3 – Differential Calculus

Q8. (a) $f(x) = (3x^2-2)(4x+1) = 12x^3 + 3x^2 - 8x - 2$

$\therefore\ f'(x) = 36x^2 + 6x - 8$

(2 marks)

(b) $f(x) = (3x^2-2)(4x+1)$

$f'(x) = 6x \times (4x+1) + (3x^2-2) \times 4$

$f'(x) = 24x^2 + 6x + 12x^2 - 8$

$= 36x^2 + 6x + 8$

(2 marks)

Q9. (a) $= \frac{1}{4x+1} \times 4 + \cos(3x^2) \times 6x$

$= \frac{4}{4x+1} + 6x\cos(3x^2)$

(2 marks)

(b) $= 5e^{3x^2+x} \times (6x+1) + \frac{2x \times (x+1) - x^2 \times 1}{(x+1)^2}$

$= 5(6x+1)e^{3x^2+x} + \frac{2x(x+1)-x^2}{(x+1)^2}$

(3 marks)

Q10. (a) $g(x) = x^3 - 5x^2 + 3x + 9$

$$\frac{g(x_2) - g(x_1)}{x_2 - x_1} = \frac{10-4}{4-1} = \frac{6}{3} = 2$$

(1 mark)

(b) $g'(x) = 3x^2 - 8x + 1$

$g'(3) = 3 \times 3^2 - 8 \times 3 + 1$

$= 27 - 24 + 1 = 4$

So the instantaneous rate of change at $x = 3$ is 4.

(2 marks)

Q11. $f(x) = \frac{x^3}{3}$ $\qquad f(x+h) = \frac{(x+h)^3}{3} = \frac{x^3+3x^2h+3xh^2+h^3}{3}$

$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$

$= \lim_{h \to 0} \frac{\frac{x^3+3x^2h+3xh^2+h^3}{3} - \frac{x^3}{3}}{h}$

$= \lim_{h \to 0} \frac{\frac{x^3+3x^2h+3xh^2+h^3-x^3}{3}}{h}$

$= \lim_{h \to 0} \frac{\frac{3x^2h+3xh^2+h^3}{3}}{h}$

$= \lim_{h \to 0} \frac{\frac{h(3x^2+3xh+h^2)}{3}}{h}$

$= \lim_{h \to 0} \frac{(3x^2+3xh+h^2)}{3}$

$= \frac{3x^2 + 3x \times 0 + 0^2}{3}$

$= x^2$

(4 marks)

A4 – Integral Calculus

Q12. (a) $\int_1^4 \frac{x^2+1}{x} dx$

$= \int_1^4 \left(x + \frac{1}{x}\right) dx$

$= \left[\frac{x^2}{2} + \ln\ |x|\right]_1^4$

$= \frac{4^2}{2} + ln4 - \left(\frac{1^2}{2} - ln1\right)$

$= 8 + ln4 - \frac{1}{2}$

$= \frac{15}{2} + ln4$

(2 marks)

(b) $\int -\cos\left(\frac{\pi}{3}x\right) dx$

$= -\frac{\sin\left(\frac{\pi}{3}x\right)}{\frac{\pi}{3}} + C$

$= -\frac{3\sin\left(\frac{\pi}{3}x\right)}{\pi} + C$

(2 marks)

(c) $\int_{\frac{1}{2}}^1 e^{2x-1} dx$

$= \left[\frac{e^{2x-1}}{2}\right]_{\frac{1}{2}}^1$

$= \frac{e^{2\times1-1}}{2} - \frac{e^{2\times\frac{1}{2}-1}}{2}$

$= \frac{e^1}{2} - \frac{e^0}{2}$

$= \frac{e}{2} - \frac{1}{2}$

(2 marks)

Q13. $\int_a^b (3f(x) + kg(x))dx = -17$

$3\int_a^b f(x)dx + k\int_a^b g(x)dx = -17$

$3 \times 5 - k\int_b^a g(x)dx = -17$

$-k \times 8 = -32$

$k = 4$

(3 marks)

Q14. (a) $-\int_{-1}^{0}(-x^3 + x)dx$

$= -\left[-\frac{x^4}{4} + \frac{x^2}{2}\right]_{-1}^{0}$

$= -\left(0 - \left(-\frac{1}{4} + \frac{1}{2}\right)\right)$

$= \frac{1}{4}$ units^2

(2 marks)

(b) Area = $\int_0^2 (-x^2 + 2x)dx - \frac{1}{4}$

$= \left[-\frac{x^3}{3} + x^2\right]_0^2 - \frac{1}{4}$

$= \left(-\frac{8}{3} + 4\right) - 0 - \frac{1}{4}$

$= \frac{4}{3} - \frac{1}{4}$

$= \frac{13}{12}$ units^2

(2 marks)

Q15. (a) Line with gradient $\frac{h}{b}$ passing through the origin. Thus $y = \frac{h}{b}x$.

(1 mark)

(b) Area Triangle = $\int_0^b \left(\frac{h}{b}x\right) dx$

$$= \frac{h}{b}\int_0^b x dx$$

$$= \frac{h}{b}\left[\frac{x^2}{2}\right]_0^b$$

$$= \frac{h}{b}\left(\frac{b^2}{2} - 0\right)$$

$$= \frac{hb^2}{2b}$$

$$= \frac{1}{2}bh$$

(2 marks)

A5 - Probability

Q16. (a) The two tests are independent of each other and the probability of each test is the same so the binomial distribution is appropriate.

(1 mark)

(b) $X \sim Bi(2, 0.6)$

$P(X \geq 1) = 1 - P(X = 0) = 1 - 0.4^2 = 1 - 0.16 = 0.84$

(2 marks)

Q17. (a) $E(X) = -2 \times 0.1 + -1 \times 0.2 + 0 \times 0.4 + 1 \times 0.2 + 2 \times 0.1$

$= -0.2 - 0.2 + 0.2 + 0.2$

$= 0$

(1 mark)

(b)

x	-2	-1	0	1	2
x^2	4	1	0	1	4
$P(X = x)$	0.1	0.2	0.4	0.2	0.1

$var(X) = E(X^2) - E(X)^2$

$= 4 \times 0.1 + 1 \times 0.2 + 0 + 1 \times 0.2 + 4 \times 0.1 - 0^2$

$= 0.4 + 0.2 + 0.2 + 0.4$

$= 1.2$

(2 marks)

(c) $sd(X) = 1.095$, so $P(0 - 2 \times 1.095 < X < 0 + 2 \times 1.095)$

$= P(-2 \leq X \leq 2) = 1$

The whole distribution is within 2 sd's of the mean.

(2 marks)

(d) Y will have a mean of 100 and a variance the same as X, which is 1.2.

(2 marks)

Q18. Making n 100 times larger, will make the margin of error 10 times smaller.

So the 95% confidence interval will be

$$(0.6 - 0.0304, 0.6 + 0.0304) = (56.96\%, 63.04\%)$$

(3 marks)

Q19. (a) $P(0 \leq X \leq 0.5244) = 0.2$

(1 mark)

(b) $P(-0.5244 \leq X \leq 0.5244)$

(2 marks)

Section B – Calculator Allowed

B1 – Function Study

Q20. Zeros: Solve $0 = \frac{1}{(x-2)^2} - 3$

$\therefore \ (x-2)^2 = \frac{1}{3}$

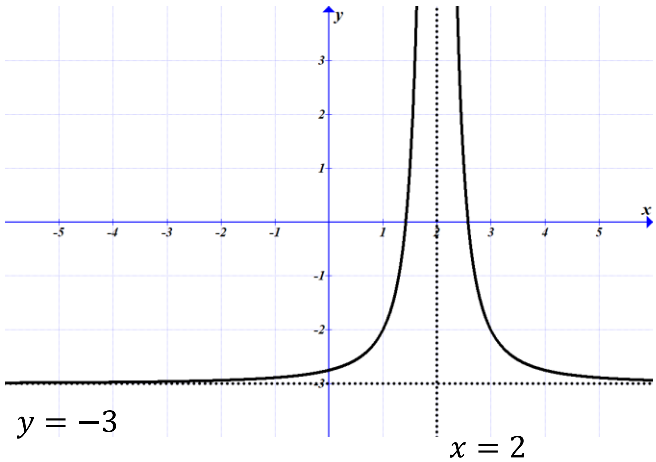
$\therefore \ (x-2) = \pm\frac{1}{\sqrt{3}}$

$\therefore \ x = 2 \pm \frac{1}{\sqrt{3}}$

Y-int: $t(0) = \frac{1}{(0-2)^2} - 3$

$= \frac{1}{4} - 3 = \frac{11}{4}$

VA at $x = 2$



HA at $y = -3$

(4 marks)

Q21. Since the VA is at $x = -3.$, $h = -3.$

Sub. the point $(1, 3)$

$\therefore\ 3 = a \times \log_2(1 + 3) + k$

$\therefore\ 3 = 2a + k$ - Equation 1

Sub. the point $(5, 5)$

$\therefore\ 5 = a \times \log_2(5 + 3) + k$

$\therefore\ 5 = 3a + k$ - Equation 2

Eqn 2 – Eq1 gives $a = 2$

$\therefore\ k = -1$

$\therefore\ y = 2 \times \log_2(x + 3) - 1.$

(4 marks)

Q22. (a) $\log_2(x + 3) + \log_2(x + 4) = \log_2(6x + 18)$

Sub. $x = 2$

LHS = $\log_2(2 + 3) + \log_2(2 + 4)$

$= \log_2(5 \times 6)$

$= \log_2(30)$

RHS = $\log_2(6 \times 2 + 18)$

$= \log_2(30)$

(1 mark)

(b) $\therefore\ \log_2(x + 3)(x + 4) = \log_2(6x + 18)$

$\therefore\ (x + 3)(x + 4) = 6x + 18$

$\therefore\ x^2 + 7x + 12 = 6x + 18$

$\therefore\ x^2 + x - 6 = 0$

$\therefore\ (x - 2)(x + 3) = 0$ (We know $(x - 2)$ is a factor)

$\therefore\ x = 2$ or $x = -3$

But $x = -3$ cannot be a solution since $\log_2(-3 + 3) = \log_2(0)$ is not defined.

(3 marks)

Q23. (a) $f(x) = \sqrt{3x-2} + 6$

Let $y = \sqrt{3x-2} + 6$

Swap x and y

$\therefore\ x = \sqrt{3y-2}+6$

$\therefore\ x - 6 = \sqrt{3y-2}$

$\therefore\ (x-6)^2 = 3y - 2$ squaring both sides

$\therefore\ y = \frac{(x-6)^2+2}{3}$

$\therefore\ f^{-1}(x) = \frac{(x-6)^2+2}{3}$

(3 marks)

(b) $Domain f^{-1} = [6, \infty)$, Range$f^{-1} = \left[\frac{2}{3}, \infty\right)$

(1 mark)

Q24. (a) $\text{f}(0) = \frac{e^0+e^{-0}}{2} = \frac{1+1}{2} = 1$

(1 mark)

(b) We need to translate down by 1,

Dilate by a factor of 2 in the *x* direction

Dilate by a factor of $\frac{1}{0.54}$ in the *y* direction

$R(2) = a \times \left(f\left(\frac{2}{2}\right) - 1\right)$

$\therefore\ 1 = a \times (1.54 - 1)$

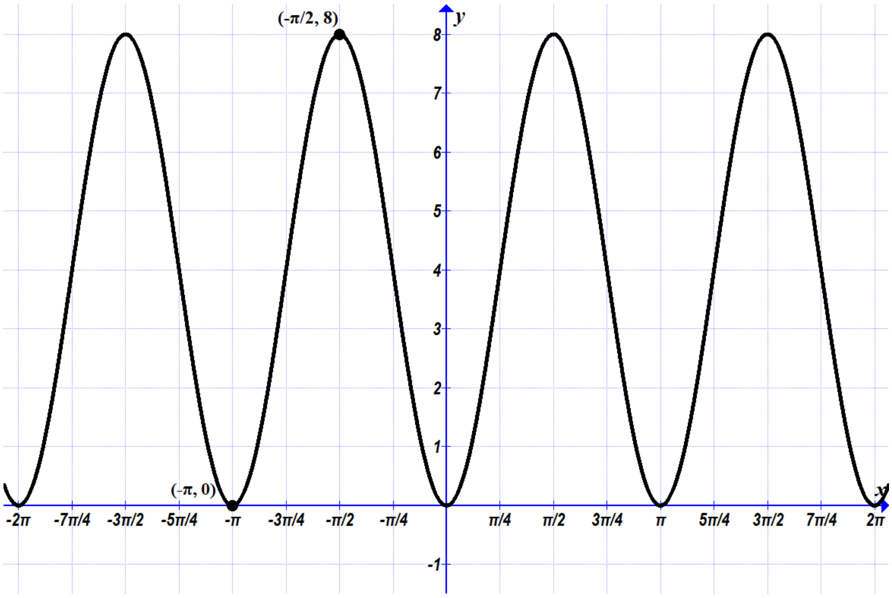
$\therefore\ a = \frac{1}{0.54}$

$\therefore\ R(x) = \frac{f\left(\frac{x}{2}\right)-1}{0.54}$ or $R(x) = 1.85\left(f\left(\frac{x}{2}\right) - 1\right)$

(3 marks)

B2 - Trigonometry

Q25. (a) and (b) below:



(1 mark)

(2 marks)

(c) The period of the graph is π.

(1 mark)

(d) $n = \frac{2\pi}{\pi} = 2$

Phase shift is $\frac{\pi}{4}$ right, so $b = -\frac{\pi}{4}$

Amplitude $= \frac{8-0}{2} = 4$, so $a = 4$

Graph shifted upwards 4, so $c = 4$

$\therefore \; y = 4\sin\left(2\left(x - \frac{\pi}{4}\right)\right) + 4$

(3 marks)

Q26. (a) Point A will be at the top after 15 seconds, so when $t = 15$.

(1 mark)

(b) A will now be at $\left(\cos\left(\frac{\pi\times5}{30}\right), \sin\left(\frac{\pi\times5}{30}\right)\right)$

$= \left(\cos\left(\frac{\pi}{6}\right), \sin\left(\frac{\pi}{6}\right)\right)$

$= (0.866, 0.5)$ or $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

(2 marks)

(c) Point A will now be at position $\left(\cos\left(\frac{\pi t}{15}\right), \sin\left(\frac{\pi t}{15}\right)\right)$.

(1 mark)

(d) A will now be at $\left(\cos\left(\frac{\pi\times100}{15}\right), \sin\left(\frac{\pi\times100}{15}\right)\right)$

$= \left(\cos\left(\frac{10\pi}{15}\right), \sin\left(\frac{10\pi}{15}\right)\right)$

$= \left(\cos\left(\frac{2\pi}{3}\right), \sin\left(\frac{2\pi}{3}\right)\right)$

$= (-0.5, 0.866)$ or $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

(2 marks)

Q27. $y = -10\sin\left(\frac{\pi}{10}(x+5)\right) + 5$

(a) $0.5 = \sin\left(\frac{\pi}{10}(x+5)\right)$

$\frac{\pi}{10}(x+5) = \frac{\pi}{6}$ or $\frac{5\pi}{6}$

$6(x+5) = 10$ or $6(x+5) = 50$

$x + 5 = \frac{5}{3}$ or $x + 5 = \frac{25}{3}$

$x = -\frac{10}{3}$ or $x = \frac{10}{3}$

(3 marks)

(b) The waterline beam is the distance from $-\frac{10}{3}$ to $\frac{10}{3}$ which is $\frac{20}{3}\ m$.

(1 mark)

(c) Solving $-10\sin\left(\frac{\pi}{10}(x+5)\right) + k = \pm3$ on the calculator gives $k = 5.9$.

(3 marks)

B3 Differential Calculus

Q28. $xy = 16$, so $y = \frac{16}{x}$

$\therefore$ we need to minimise $x + \frac{16}{x}$

$\frac{d}{dx}\left(x + \frac{16}{x}\right) = 1 - 16x^{-2}$

Solving $1 - 16x^{-2} = 0$ to find the minimum

$\therefore \ 1 = 16x^{-2}$

$\therefore \ x^2 = 16$

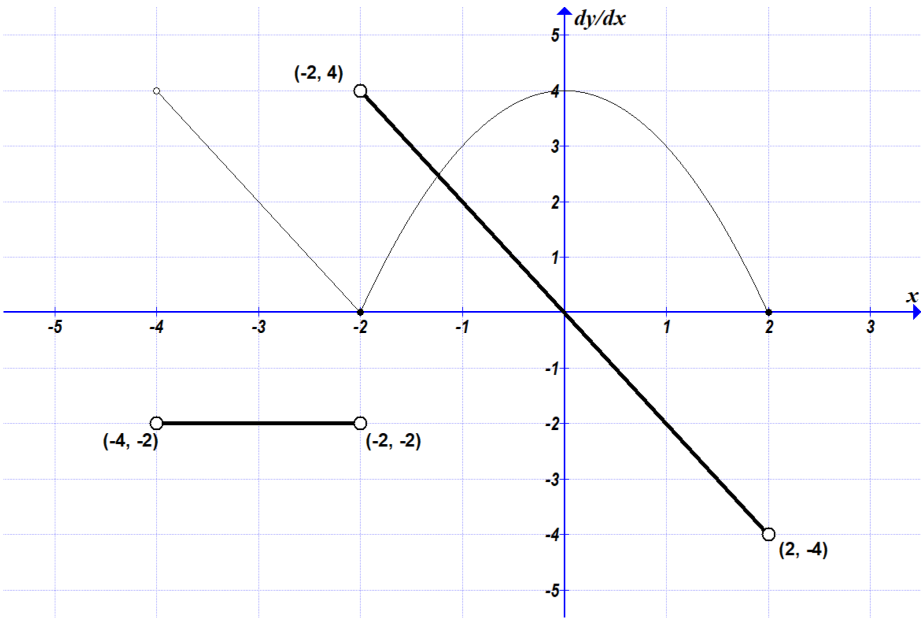
$\therefore \ x = \pm 4$

But x is positive so $x = 4$ and $y = \frac{16}{4} = 4$.

So the minimum value of $x + y$ is $4 + 4 = 8$.

(3 marks)

Q29.



Gradient of line is $\frac{0-4}{-2--4} = \frac{-4}{2} = -2$, so a constant value of -2 from $(-4, -2)$ to $(-2, -2)$.

Equation of parabola is $y = -x^2 + 4$, so $\frac{dy}{dx} = -2x$, the line passes from $(-2, 4)$ to $(2, -4)$.

Each endpoint is open because the derivative is not the same on both sides of any point.

(3 marks)

Q30. (a) $f(x) = \ln(2x-4)+3$ $g(x) = e^{2x-5}+2$

$f\left(\frac{5}{2}\right) = \ln\left(2\times\frac{5}{2}-4\right)+3$ $g\left(\frac{5}{2}\right) = e^{2\times\frac{5}{2}-5}+2$

$= ln1 + 3 = 3$ $= e^0 + 2 = 3$

Thus both $f(x)$ and $g(x)$ intersect at $\left(\frac{5}{2}, 3\right)$.

(2 marks)

(b) $f'(x) = \frac{1}{2x-4}\times 2$ $g'(x) = e^{2x-5}\times 2$

Sub. $x = \frac{5}{2}$ Sub. $x = \frac{5}{2}$

$f'\left(\frac{5}{2}\right) = \frac{2}{2\times\frac{5}{2}-4} = \frac{2}{1} = 2$ $g'\left(\frac{5}{2}\right) = e^{2\times\frac{5}{2}-5}\times 2 = 1\times 2 = 2$

Thus both $f(x)$ and $g(x)$ have the same gradient at $x = \frac{5}{2}$.

(3 marks)

(c) Since both points pass through $\left(\frac{5}{2}, 3\right)$ and the slope of their tangents at $x = \frac{5}{2}$ is 2, they are tangent to each other.

(1 mark)

Q31. (a) $T(-1) = -\frac{9}{160}(-1)^3 + \frac{27}{40}(-1)^2 = 0.73125°C$

(1 mark)

(b) Solve $\frac{dT}{dh} = 0$ to find turning points.

$$\frac{dT}{dh} = -\frac{27}{160}h^2 + \frac{27}{20}h$$

$= -\frac{27}{160}h^2 + \frac{27}{20}h$

$\therefore\ -h\left(\frac{27}{160}h - \frac{27}{20}\right) = 0$

$\therefore\ h = 0$ or $h = 8$

When $h = 0$, $T = 0$ When $h = 8$, $T = -\frac{9}{160}\times 8^3 + \frac{27}{40}\times 8^2 = 14.4$

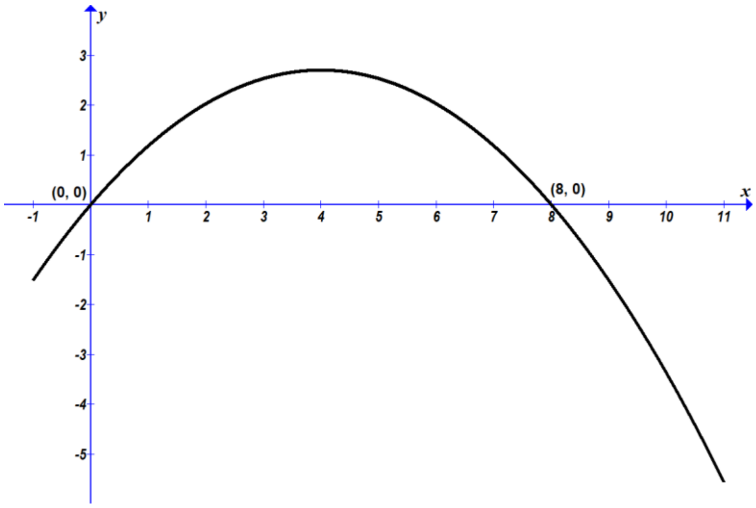
$\frac{d^2T}{dh^2} = -\frac{27h}{80} + \frac{27}{20}$

At $h = 0$, $\frac{d^2T}{dh^2} = \frac{27}{20} > 0$ When $h = 8$, $\frac{d^2T}{dh^2} = -\frac{27\times 8}{80} + \frac{27}{20} < 0$

$\therefore\ (0,0)$ is a minimum and $(8, 14.4)$ is a maximum.

(4 marks)

(c)



(2 marks)

(d) The temperature is increasing when $\frac{dT}{dh} > 0$, so $h \in (0, 8)$.

This is between 7am and 3pm.

(1 mark)

B4 – Integral Calculus

Q32. (a) $\frac{1}{3}\int \frac{1}{x}dx = \frac{1}{3}\ln|x| + C$

(2 marks)

(b) $\frac{\ln|3x|}{3} + C = \frac{ln3+\ln|x|}{3} + C = \frac{1}{3}\ln|x| + \frac{ln3}{3} + C$

The answer is the same except the integration constant C is different, since $\frac{ln3}{3}$ is a constant.

(1 mark)

Q33. (a) $\int_1^2 \frac{1}{x^2}dx = \int_1^2 x^{-2}dx$

$= \left[\frac{x^{-1}}{-1}\right]_1^2$

$= -2^{-1} - -1^{-1}$

$-\frac{1}{2} + 1 = \frac{1}{2}$

(2 marks)

(b) $\int_1^k \frac{1}{x^2} dx = 0.99$

$\therefore \left[\frac{x^{-1}}{-1}\right]_1^k = 0.99$

$\therefore -\frac{1}{k} + 1 = 0.99$

$\therefore -\frac{1}{k} = -0.01 = -\frac{1}{100}$

$\therefore k = 100.$

(2 marks)

(c) It appears that as $k \to \infty$, $\int_1^k \frac{1}{x^2} dx$ tends towards 1.

(1 mark)

Q34. Area left tunnel

h = $\int_0^{19} \left(-\frac{2}{5}x^2 + 8x\right) dx - \int_2^{18} \left(-\frac{1}{2}x^2 + 10x - 18\right) dx$

$= \left[-\frac{2}{15}x^3 + 4x^2\right]_0^{19} - \left[-\frac{1}{6}x^3 + 5x^2 - 18x\right]_2^{18}$

$= \left(-\frac{13718}{15} + 1444\right) - (0) - (-\frac{18^3}{6} + 5 \times 324 - 324 - (-\frac{1}{6} \times 8 + 5 \times 4 - 36))$

$= \frac{2822}{15}\ m^2$

Total Area = Left tunnel x 2

$= \frac{5644}{15}\ m^2$ or $752.53\ m^2$.

(7 marks)

Q35. (a) Using product rule,

$\frac{d}{dx}(xsinx) = 1 \times sinx + x \times cosx$

$= xcosx + sinx$

(2 marks)

(b) $\int_0^{\frac{\pi}{2}} xcosx\,dx = \int_0^{\frac{\pi}{2}}(xcosx + sinx)\,dx - \int_0^{\frac{\pi}{2}} sinx\,dx$

$$= [xsinx]_0^{\frac{\pi}{2}} + [cosx]_0^{\frac{\pi}{2}}$$

$= \frac{\pi}{2}\sin\frac{\pi}{2} - 0sin0 + \cos\frac{\pi}{2} - cos0$

$= \frac{\pi}{2} \times 1 - 0 + 0 - 1$

$= \frac{\pi}{2} - 1$

(3 marks)

B5 - Probability

Q36. (a) The probability is still $\frac{1}{6}$.

(1 mark)

(b) $P(4\ sixes) = \left(\frac{1}{6}\right)^4 = \frac{1}{1296}$.

(1 mark)

(c) $P(0\ sixes) = \left(\frac{5}{6}\right)^4 = \frac{625}{1296}$

(1 mark)

(d) $X \sim Bi\left(5, \frac{1}{6}\right)$

$P(X \geq 3) = 0.0355 = 3.55\%$

(2 marks)

Q37. (a) $X \sim N(40, 10^2)$

$P(X > 45) = 0.3085$

(2 marks)

(b) We need x so that $P(X \geq x) = 0.2$

From calculator, $x = 48.42$

The longest 20% of forearms are longer than 48.42 cm.

(2 marks)

(c) This is a Binomial Distribution with $n = 10$ and $p = 0.3085$

$Y \sim Bi(10, 0.3085)$

$P(Y \geq 2) = 0.8635$

(2 marks)

Q38. From the standard normal distribution, $Z \sim N(0,1^2)$

$P(Z \geq z) = 0.1$ gives $z = 1.2816$.

Using $z = \frac{x-\mu}{\sigma}$, $\sigma = \frac{x-\mu}{z}$

$\therefore \quad \sigma = \frac{150-135}{1.2816} = 11.70\ cm$

(3 marks)

Q39. (a) $M = z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 1.96\sqrt{\frac{0.53(1-0.53)}{500}} = 0.04375$

So the 95% confidence interval is $(0.53 - 0.04375, 0.53 + 0.04375)$

$= (48.63\%, 57.37\%)$

(2 marks)

(b) We need $0.53 - z\sqrt{\frac{0.53(1-0.53)}{500}} = 0.5$

Solving on the calculator gives $z = 1.3441$

$P(Z > 1.3441) = 0.08946$

The confidence interval is the middle, with 8.946% on each side,

So the maximum confidence interval that can be quoted is $100\% - 2 \times 8.946\%$

which is equal to 82.11%.

(4 marks)

TASMANIAN ASSESSMENT, STANDARDS & CERTIFICATION