2024 ASSESSMENT REPORT

MTM415117 MATHEMATICS METHODS

General Comments

The exam contained a large portion of questions that challenged students with their algebraic manipulation. Usually, students were able to identify what was required of them to progress in each question, but were hindered by their use of basic operations, laws and arithmetic. In Section B, students often lost marks in questions by using their graphics calculator in questions that specifically asked for algebraic working.

Section A

Function Study

Question 1

a. This question was well done.
b. This question was generally well done. However, common errors included not dividing by 4 as the first step which led to complications when converting to a logarithm, not dividing all terms by 2 as the final step, and not including correct inverse function notation for the final answer.
c. This question was well done.

Question 2

This question was generally well done, with common errors including poor arithmetic calculations, not applying the power to the full term, and not identifying the coefficient separately to the term. Students who tried to expand by other processes generally ran into trouble.

Question 3

This question was generally well done. The occasional incorrect application of low laws resulted in incorrect intercepts. Marks were also deducted when the asymptote was not labelled with an appropriate equation.

Question 4

a. Many students did not recognise that the coefficient required for the x term was 1, resulting in an answer which was insufficiently simplified.
b. Perhaps due to the unusual wording of this question, many students did not realise that they needed to use their answer from the previous part. For students to obtain full marks, it was necessary for them to use the correct wording from the formula sheet when describing transformations, specifically “dilation <u>by factor</u> 6 <u>in the direction of</u> the y-axis.

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Trigonometry

Question 5

a. Markers were surprised at the volume of numerical errors encountered within answers. The negative sign was sometimes omitted.
b. A surprising number of students were unable to simplify their fraction.

Question 6

Both parts of this question were well done. Some students chose to convert from degrees to radians. The generous marking allocation here meant that markers required some working to award full marks.

Question 7

a. This question was handled well. Common mistakes were in missing the reflection of the function or translating in the wrong direction, and not graphing the function over the entire domain.
b. To receive full marks, students needed to answer in a form resembling a function. Stating the word cosine was insufficient.
c. Most students opted to simplify the angle within sine. Students who pursued a complementary angle approach appeared more likely to make an error.

Question 8

a. This question was generally well done. Marks were deducted for omitting $\pm\sqrt{}$ or making numerical errors such as $49 - 16 = 36$.
b. This question was universally done well, with any errors being carried forward to receive full marks.

Differential Calculus

Question 9

a. This question was well done. A common mistake was in missing a minus sign.
b. This question was well done. Some students didn't change $\frac{2}{x^3}$ into negative index form and some students said that $\frac{3}{2}x^{\frac{1}{2}}$ was $\frac{3}{2\sqrt{x}}$.
c. This question was generally well done. It required use of the product rule. A common mistake was in interpreting the derivative of $ln(2x-3)$ as $\frac{1}{2x-3}$ without the coefficient of 2.

Question 10

Most students used the quotient rule correctly but then failed to show sufficient working in getting to the required result. There was some incorrect algebraic manipulation.

Question 11

a. Most students were able to calculate the gradient of the tangent but did not go on to then indicate that the gradient of the normal was the negative reciprocal of this result.

b. Most students identified the need to substitute co-ordinates and the gradient into the point gradient or linear formula. Substituting $(0,0)$ yields $k^6 = 2$ which leads to the solution $k = \pm 2^{\frac{1}{6}}$. Many students missed the negative solution.

Integral Calculus

Question 12

a. This was reasonably well done as would be expected. The simplification $2 \times \frac{3}{2} = 3$ was required.
b. Students who simplified the integrand $\frac{e^{5x+1}}{e^{2x-3}} = e^{3x+4}$ were successful. The only credit given to solutions that (erroneously) tried $\int \frac{f(x)}{g(x)} dx = \frac{\int f(x)dx}{\int g(x)dx}$ was for evidence of $\int e^{kx}\, dx = \frac{1}{k} e^{kx}\ (+c)$.

Question 13

a. This was well done by students who used the chain or product rule.
b. Many students understood that the correct approach was to turn the differentiation statement in the previous part into an antidifferentiation statement, but some struggled to find the correct factor to multiply this by so as to obtain the requested integral.

Question 14

a. Aside from some arithmetic errors, this was well done.
b. Most students mentioned 'below the x axis' but a stronger response was to say more of the area is below the x axis than above. Some students cited the fact that the terminal -1 is negative which is irrelevant.

Question 15

Successful solutions used $f'(2) = 0$, anti-differentiated for $f(x)$, and used $f(2) = 5$ alongside $f(0) = -5$ to generate 3 simultaneous equations. The most common errors were a failure to use all information, omitting the constant of integration, or integrating the constant b (with respect to x) as $\frac{b^2}{2}$ rather than bx. Students who found two simplified equations for a and b prior to solving were more successful than those who attempted to substitute their first expression for a in terms of b earlier on.

Probability

Question 16

a. This question was universally well done.
b. This question was generally well done. Some students erroneously used the number of outcomes in place of the associated fraction.
c. This question was done well.
d. Most students displayed a good understanding of the process but received partial marks due to arithmetic errors.

Question 17

a. This question was universally well done.
b. A common error was in adding the required areas rather than subtracting them.
c. A considerable number of students added two (instead of three) standard deviations or misread the inequality and thus answered 4 instead of 16.

Question 18

a. Almost all students recognised this as a binomial problem. Although some mistakes were made in omitting or finding the incorrect coefficient and/or having the powers the wrong way around, it was generally well answered.
b. Students who used the complementary method were generally well rewarded if they understood the inequality used. Those who didn't use a complementary method ran into problematic arithmetic.

Section B

Function Study

Question 19

Most students were able to write down the correct answer by using their calculator, but to obtain full marks students needed to show their algebraic working. This involved correctly using the logarithmic power and addition laws before making the subject of the logs equal to each other.

Question 20

a. Most students were able to prepare the equation by substituting the correct value of t and $\frac{A_0}{2}$ but many did not progress beyond this point by cancelling out A_0. Further mistakes were made in giving the answer correct to 6 significant figures rather than 6 decimal places as asked for by the examiner.
b. Students who were successful in the previous part of the question were further rewarded here.

Question 21

a. This question was universally well done.
b. Students who chose to use their graphic calculator to solve the pair of simultaneous equations were able to avoid any pitfalls in applying index laws. Marks were deducted for answers which combined the coefficient and base using multiplication.

Question 22

a. Most students understood that their graph needed to involve a and b and that substitution of any values should be avoided. Markers were looking for graphs which tended towards the asymptotes and were correctly labelled.
b. While this question started off well, many students felt compelled to expand their answer which often caused problems.
c. Most students failed to realise that the asymptote at b should not be included in the domain. Further mistakes were made in including 0 in the given range.

d. Many students correctly equated their composite answer from the previous part with the expression given but struggled to use equivalence to determine a.

Trigonometry

Question 23

a. Some students stated the period and then converted it to n which was not penalised.
b. This question was either easily done or poorly done. Common errors involved using the period of a sin graph rather than tan, substituting the inflection point to solve for a, and having the incorrect sign in front of b.

Question 24

a. The rearrangement of the equation and subsequent discovery of the basic angle was universally well done. However, many students struggled to identify the correct quadrants or use the domain to find all three solutions. Students who chose to use the general trigonometric equation were almost always unsuccessful.
b. Most students were able to use the period to determine 3 more solutions, including when they had answered the previous part incorrectly. In allocating marks, the markers looked for a statement of the period and/or 3 solutions which were within the given domain.

Question 25

a. Markers were surprised at the number of students who struggled to find all three variables, given the clear presentation of this question. A common mistake was interchanging the values of a and c.
b. Solutions were considered in terms of 'error carried forward'. Students knew the steps to take to find a solution for b but made algebraic and numerical errors along the way. The important word in this question is 'falling' and this needed to be considered when choosing the correct solution for b to three decimal places.
c. This question was often not attempted. For full marks to be awarded, students needed to give time in terms of both hours and minutes and round correctly.

Differential Calculus

Question 26

Markers were surprised to note the number of students who did not attempt this question. Those who did tended to do well. There were some inaccuracies aligning stationary point with x intercepts.

Question 27

This question was generally well done. Most students used the Product Rule correctly, though some didn't show sufficient algebraic working when finding the x-coordinate of the stationary point. Some students neglected to then calculate the y-coordinate, while some neglected to identify the type of stationary point found.

Question 28

Few students achieved a correct answer for this question. The fact that the turning point is midway between 0 and 20 leading to $b = 10$ was often overlooked. The most common approach was to instead substitute in the coordinates given on the diagram in hope of solving simultaneous to calculate the values of a, b and c. Without knowing the value of b, this approach wasn't helpful. Some students who established the correct values of a, b and c didn't go on to state that the minimum height above the ground was 75m.

Question 29

a. Pythagoras Theorem was not used well by a proportion of students. The most common error was to think that the square of 2b is 2b^2. With students required to show the given result, jumping ahead to the answer too soon (with insufficient working) was seen from many students.
b. Several students misinterpreted this question as an instruction to 'show that' the shaded area was 3bh/2. Quite a few students performed the substitution correctly but neglected to simplify their answer.
c. Few students were able to access this as it required an answer in the prior part of the question. Those who did relied on their calculator despite the question specifying the use of algebraic working.

Integral Calculus

Question 30

Many different approaches were seen including reversed terminals, or $\left|\int_\alpha^\beta f(x)dx\right|$, or $-1 \times \int_\alpha^\beta f(x)dx$ to accommodate the negatively signed area. While this was reasonably well done, more students than markers would expect failed to process the 'negatively signed' area.

Question 31

Most students included the solution of $\frac{1}{x^2} = \frac{1}{x^3}$ as $x =$1 for the left-hand terminal and stated the equation $\int_1^k \frac{1}{x^2} - \frac{1}{x^3} dx = \frac{1}{8}$. Thereafter, progress was variable. The explicit rejection of the 'second' solution $k = \frac{2}{3}$ was required to award full marks.

Question 32

a. This was well done. It was evident some students had their calculators in gradians or degrees mode. For this course radian mode should be the default for the calculus criteria and only deviated from with very good reason.
b. This was well done.
c. Most students integrated correctly but many ignored the constant of integration $+c$ or erroneously assumed $c = 0$.
d. This was well done by students who attempted it.

Question 33

a. Students who attempted this question were able to correctly determine the x values of intersection.
b. This question saw mixed responses. The $\int_{\text{left intersection}}^{\text{right intersection}} \text{upper function} - \text{lower function}\, dx$ approach appeared most reliable and straight forward in obtaining the correct result.

Students who relied on symmetry were required to explicitly state this without verifying it to be true.

Probability

Question 34

a. Both parts of this question were generally well done; however, a number of students set the lower boundary as 5 instead of 6.
b. This question was well done. Some students gave the standard deviation rather than the variance.

Question 35

A sizeable group of students found the z-score, but didn't go on to state the level of confidence.

Question 36

a. This was well done but easily missed by some students. For full marks to be awarded, the areas needed to be labelled and scaled appropriately.
b. A significant number of students did not start this question. Those who did were generally successful. Common mistakes were in not rounding to the correct number of decimal places.
c. Students who answered the previous part of the question were further rewarded here. Some students incorrectly stated that both the variance and mean stayed the same.

Question 37

a. This was very well done despite an error in the text.
b. This was very well done with most students stating that the increase was not reasonable as it appeared excessive. Partial marks were deducted when the invalid sample proportion was not rejected.

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External Assessment 2024

Solutions

Attach your candidate label here

MATHEMATICS METHODS

MTM415117

Section A

Pages: 20

Questions: 18

Information Sheet: 1

Preparation time for this exam: 15 minutes

Suggested working time: 80 minutes

Instructions:

Calculators are not allowed to be used in this section.

Section A will be collected after 80 minutes.

- There are **five (5) parts** to this section.
- Answer **all** questions and **all** items within each question.
- Write your answers in the spaces provided in this exam paper.
  - Spare diagrams have been provided at the end of Parts 1 and 2. Indicate in the box provided if you have used the spare diagrams.
- The exam is **three (3) hours** in length. It is suggested that you spend **approximately 80 minutes** in total answering the questions in this exam booklet.
- During the first 80 minutes you may move onto Section B, but you **cannot** use your calculator until told by your supervisor(s).
- The Mathematics Methods Information Sheet can be used throughout the exam.
- All answers must be written in **English**.
- You **must** make sure your answers address the listed criteria.

Marker use	
C4	/ 16
C5	/ 16
C6	/ 16
C7	/ 16
C8	/ 16

Part 1

Marker use

- Answer **all** questions in this part.
- This part assesses **Criterion 4**.

Question 1

Given that $f(x) = 4e^{2x-3}$

a) State the domain and range of $f(x)$. /2

$x \in \mathbb{R}$

$y \in (0, \infty)$

b) Find the inverse $f^{-1}(x)$. /3

$y \leftrightarrow x$: $x = 4e^{2y-3}$

$\frac{x}{4} = e^{2y-3}$

$\therefore 2y - 3 = \ln\left(\frac{x}{4}\right)$

$y = \frac{1}{2}\ln\left(\frac{x}{4}\right) + \frac{3}{2}$

$\therefore f^{-1}(x) = \frac{1}{2}\ln\left(\frac{x}{4}\right) + \frac{3}{2}$

c) State the domain of $f^{-1}(x)$. /1

$x \in (0, \infty)$

Question 2

Determine the coefficient of x^3 in $(2x-3)^5$.

The third term $= {}^5C_3 (2x)^3 (-3)^2$

$= \frac{5!}{3!\,2!}\, 8x^3 \times 9$

$= \frac{5\times4}{2} \times 72x^3$

$= 720x^3$ $\therefore$ The co-eff is 720

Marker use

/3

Question 3

Using Figure 1, sketch the graph of $f(x) = \log_2(x+2) - 2$.

Include asymptotes and intercepts in your sketch.

/3

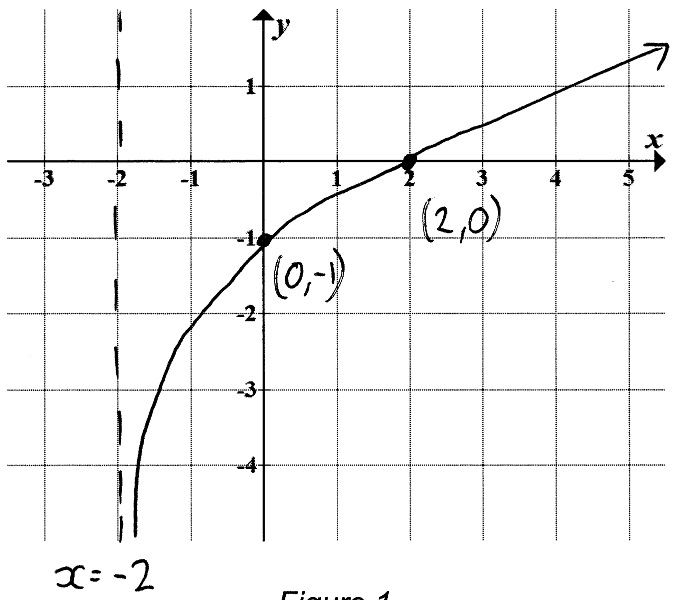


Figure 1

Spare diagram used (X) ☐

At $x=0$, $f(0) = \log_2(2) - 2$

$= 1 - 2$

$= -1$

x int: $0 = \log_2(x+2) - 2$

$2 = \log_2(x+2)$

$\therefore x+2 = 2^2$

$x = 4 - 2$

$\therefore x = 2$

Question 4

Marker use

For the function $f(x) = \sqrt{9(4x-8)} + 2$

a) Using algebra express $f(x)$ in the form $y = a\sqrt{x-h} + k$.

$f(x) = \sqrt{9}\,\sqrt{4x-8} + 2$

$= 3\sqrt{4(x-2)} + 2$

$= 3 \times \sqrt{4} \times \sqrt{x-2} + 2$

$= 6\sqrt{x-2} + 2$

/2

b) State the transformations required (in order) from $y = \sqrt{x}$ to transform $f(x)$ into the form $y = a\sqrt{x-h} + k$.

· Dilate by factor 6 in the y-axis direction

· Translate 2 units right and up 2 units

/2

Total P1 /16

Part 2

Marker use

- Answer **all** questions in this part.
- This part assesses **Criterion 5**.

Question 5

a) Convert $-135°$ into radians.

$= -3\pi/4$ /1

b) Convert $\frac{11\pi}{6}$ into degrees.

$= 330°$ /1

Question 6

Give exact values of the following:

a) $\cos(-150°)$ /2

$= -\cos(30°)$

$= -\sqrt{3}/2$

b) $\tan\left(\frac{3\pi}{4}\right)$ $= -\tan\left(\frac{\pi}{4}\right)$ /2

$= -1$

Question 7

Marker use

Given the graph of $y = \sin(x)$ below, for $x \in [-2\pi, 2\pi]$

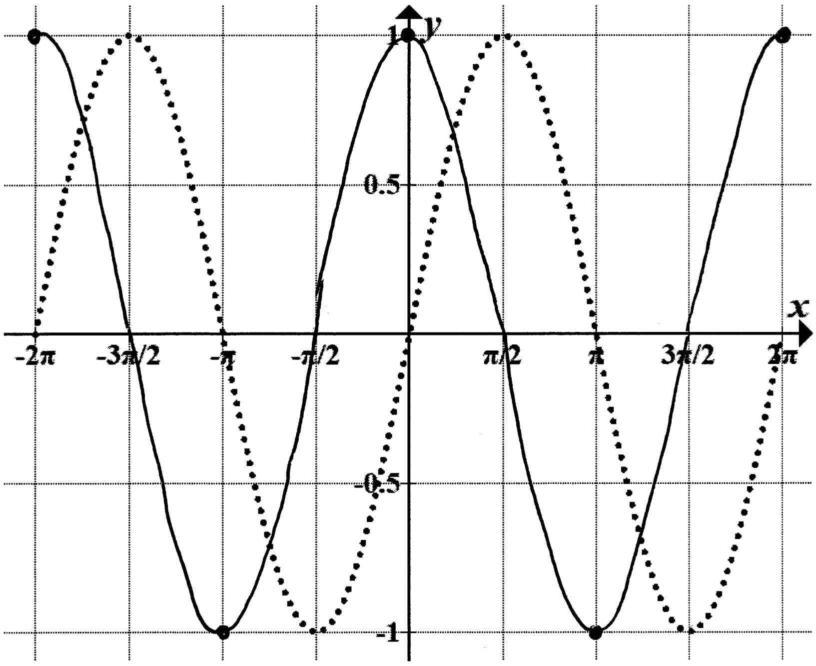


Figure 2

Spare diagram used (X) ☐

a) Using Figure 2, sketch on the same axes the graph of $y = -\sin\left(x - \frac{\pi}{2}\right)$. /3

b) Which trigonometric function has the same graph as the one you drew in a)? /1

$y = \cos(x)$

c) Calculate $-\sin\left(\frac{\pi}{3} - \frac{\pi}{2}\right)$. /2

$= -\sin\left(-\frac{\pi}{6}\right)$ OR $= \cos\left(\frac{\pi}{3}\right)$ (Complimentary Angles)

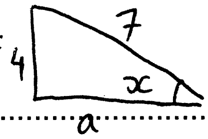
$= \sin\left(\frac{\pi}{6}\right)$ $= \frac{1}{2}$

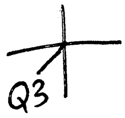
$= \frac{1}{2}$

Question 8

Given that $\sin x = -\frac{4}{7}$ and $-\pi \leq x \leq -\frac{\pi}{2}$, calculate:

a) $\cos x$





$a^2 + 4^2 = 7^2$ OR $\left(-\frac{4}{7}\right)^2 + \cos^2 x = 1$

$a^2 = 49 - 16$ $\therefore \cos^2 x = 1 - \frac{16}{49}$

$a = \pm\sqrt{33}$ $\cos x = \pm\sqrt{\frac{33}{49}}$

$\therefore \cos x = -\frac{\sqrt{33}}{7}$ $\therefore \cos x = -\frac{\sqrt{33}}{7}$ as Q3

Marker use

/3

b) $\tan x$

$= \frac{4}{\sqrt{33}}$

/1

Total P2

/16

Part 3

Marker use

- Answer **all** questions in this part.
- This part assesses **Criterion 6**.

Question 9

Differentiate the following

a) $y = 3\cos(\pi x) + 6\tan(2x)$

$dy/dx = -3\pi \sin(\pi x) + 12\sec^2(2x)$

/2

b) $y = 6x^3 - x + \pi - \frac{2}{x^3} + x^{\frac{3}{2}}$

$dy/dx = 18x^2 - 1 + \frac{6}{x^4} + \frac{3}{2}x^{\frac{1}{2}}$

/2

c) $y = e^{4x+1}\log_e(2x-3)$

$u = e^{4x+1}$ $\quad$ $v = \ln(2x-3)$

$u' = 4e^{4x+1}$ $\quad$ $v' = \frac{2}{2x-3}$

$dy/dx = 4e^{4x+1}\ln(2x-3) + \frac{2e^{4x+1}}{2x-3}$

/3

Question 10

Show that the derivative of $f(x) = \frac{\ln(x)}{e^x}$ is equal to $f'(x) = \frac{1-x\ln(x)}{xe^x}$.

$u = \ln(x)$ $\quad$ $v = e^x$

$u' = \frac{1}{x}$ $\quad$ $v' = e^x$

$\therefore f'(x) = \frac{\frac{1}{x}e^x - \ln(x)e^x}{(e^x)^2}$

$= \frac{\frac{1}{x} - \ln(x)}{e^x}$

$= \frac{1 - x\ln(x)}{xe^x}$

/3

Question 11

Marker use

A normal to the curve $f(x) = \frac{1}{x^2}$ is drawn at $x = k$.

One example of a solution is shown in Figure 3.

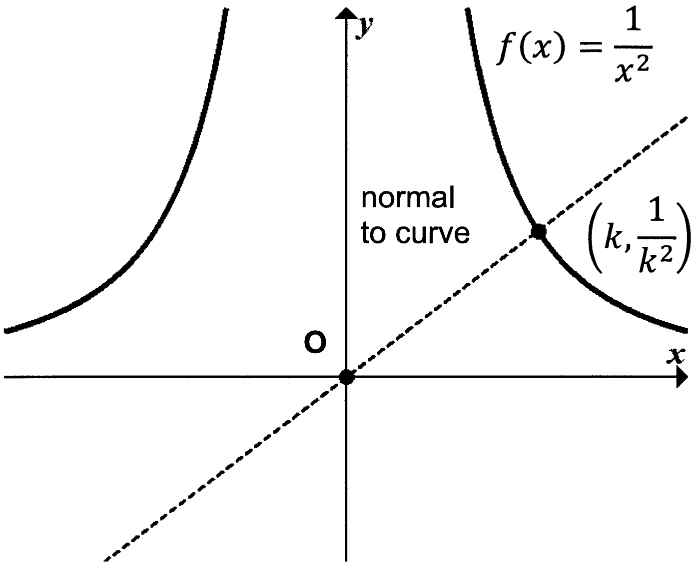


Figure 3

a) Using Figure 3, show that the gradient of the normal at $x = k$ is equal to $\frac{k^3}{2}$.

$f'(x) = -\frac{2}{x^3}$

$f'(k) = -\frac{2}{k^3}$

$\therefore m_N = \frac{-1}{\left(-\frac{2}{k^3}\right)} = \frac{k^3}{2}$ as required.

/3

b) Find the exact values of k such that the normal passes through the origin, O.

$y - \frac{1}{k^2} = \frac{k^3}{2}(x - k)$

Sub $(0,0)$:

$-\frac{1}{k^2} = -\frac{k^4}{2}$

$2 = k^6$

$\therefore k = \pm 2^{1/6}$ or $\pm \sqrt[6]{2}$

/3

Total P3

/16

Part 4

Marker use

- Answer **all** questions in this part.
- This part assesses **Criterion 7.**

Question 12

a) Evaluate

$\int \sqrt{2x-3}dx \quad = \int (2x-3)^{\frac{1}{2}} dx$

$= \dfrac{\frac{2}{3}(2x-3)^{3/2}}{2} + c$

$= \frac{1}{3}(2x-3)^{3/2} + c$

/2

b) Evaluate

$\int \frac{e^{5x+1}}{e^{2x-3}} dx \quad = \int e^{5x+1-(2x-3)} dx$

$= \int e^{3x+4} dx$

$= \frac{1}{3} e^{3x+4} + c$

/2

Question 13

Marker use

a) Show that the derivative of $y = (\ln x)^2$ is $\frac{dy}{dx} = \frac{2\ln x}{x}$. /2

$\frac{dy}{dx} = 2(\ln x) \times \frac{1}{x} = \frac{2\ln(x)}{x}$

b) Hence evaluate the integral: /3

$$\int_1^e \frac{\ln x}{2x}\,dx$$

As $\int \frac{2\ln(x)}{x}\,dx = (\ln x)^2 + c$

$\frac{1}{4}\int \frac{2\ln(x)}{x}\,dx = \frac{1}{4}(\ln x)^2 + c$

$\therefore \int \frac{\ln x}{2x}\,dx = \frac{1}{4}(\ln x)^2 + c$

$\therefore \int_1^e \frac{\ln x}{2x}\,dx = \left[\frac{1}{4}(\ln x)^2\right]_1^e$

$= \frac{1}{4}(\ln(e))^2 - \frac{1}{4}(\ln(1))^2$

$= \frac{1}{4}(1)^2 - \frac{1}{4}(0)^2$

$= \frac{1}{4}$

Question 14

a) Determine /2

$$\int_{-1}^{2} (6x - 6x^2)\,dx$$

$= \left[3x^2 - 2x^3\right]_{-1}^{2}$

$= (3(2)^2 - 2(2)^3) - (3(-1)^2 - 2(-1)^3)$

$= (12 - 16) - (3 + 2)$

$= -9$

b) Explain why the answer to a) is negative. /1

The answer is negative because of the interval chosen.

(Eg. Area: more below x-axis than above, Kinematics: object displaced backwards)

Question 15

Marker use

The graph of $f(x)$ has a stationary point at $(2, 5)$.

It also has a y-intercept at –5.

Its derivative is in the form $f'(x) = -3x^2 + ax + b$, for $a,\ b \in \mathbb{R}$.

Determine the function $f(x)$

/4

Subbing $f'(2) = 0$ gives $0 = -3(2)^2 + 2a + b$

$12 = 2a + b$ (1)

$f(x) = -\frac{3x^3}{3} + \frac{ax^2}{2} + bx + c$

y int gives $c = -5$

Subbing $f(2) = 5$ gives $5 = -(2)^3 + \frac{a(2)^2}{2} + 2b - 5$

$18 = 2a + 2b$ (2)

Solving (2) - (1) gives $b = 6$

$\therefore 18 = 2a + 12$

$2a = 6$

$a = 3$

$\therefore f(x) = -x^3 + \frac{3}{2}x^2 + 6x - 5$

Total P4

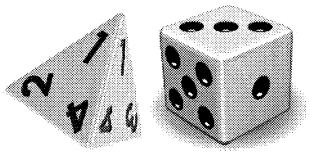
/16

Part 5

Marker use

- Answer **all** questions in this part.
- This part assesses **Criterion 8.**

Question 16



A new casino game is being featured where the player rolls a four and a six-sided die simultaneously. The game costs \$1 to play.

If the 4-sided die rolls higher than the 6-sided die, the player wins \$3 (including the \$1 to play).

If the 4-sided die rolls the same as the 6-sided die, the player receives their \$1 back.

If the 4-sided die rolls lower than the 6-sided die, the player loses their \$1.

a) Complete the Table 1 for the earnings for the player: /1

Earnings for Player		**Six-Sided Die Roll**					
		1	2	3	4	5	6
Four-Sided Die Roll	1	0	−1	−1	−1	−1	−1
	2	2	0	−1	−1	−1	−1
	3	2	2	0	−1	−1	−1
	4	2	2	2	0	−1	−1

Table 1

b) A random variable x is defined as the earnings for the player. /2

Complete the probability distribution table (Table 2) for the random variable x.

x	−1	0	2
$P(X = x)$	$14/24$	$4/24$	$6/24$

Table 2

c) Calculate $E(X)$. Is this a fair game? /2

$E(x) = -1 \times \frac{7}{12} + 0 \times \frac{1}{6} + 2 \times \frac{1}{4}$

$= -\frac{7}{12} + \frac{6}{12}$

$= -\frac{1}{12}$

$\therefore$ Not fair as $E(x) \neq 0$

Question 16 continues

Question 16 continued

Marker use

d) Calculate $var(X)$.

$Var(x) = (-1)^2 \times \frac{7}{12} + 0^2 \times \left(\frac{1}{6}\right) + 2^2 \times \frac{1}{4} - \left(-\frac{1}{12}\right)^2$ /2

$= \frac{7}{12} + 1 - \frac{1}{144}$

$= \frac{84 + 144 - 1}{144}$

$= \frac{227}{144}$

Question 17

A random variable X is normally distributed with a mean of 10 and a standard deviation of 2.

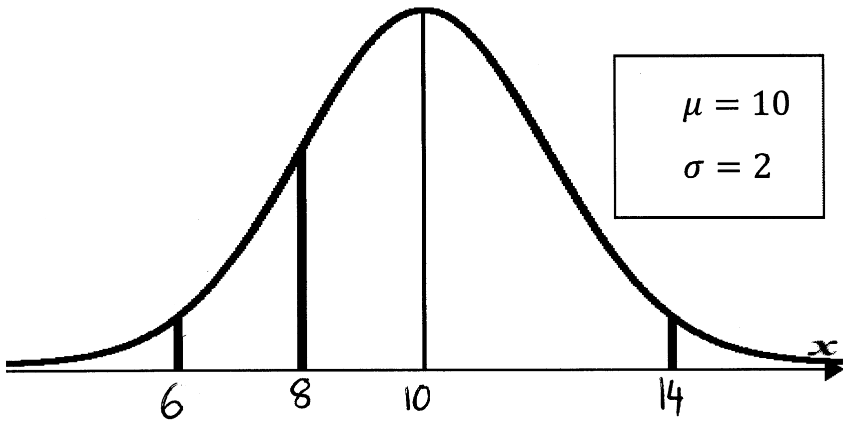


Figure 4

Using the approximate values for areas of the normal distribution, find values for the following:

a) $P(6 \le X \le 14)$

$= Pr(\mu - 2\sigma \le x \le \mu + 2\sigma)$ /1

$= 95\%$ or 0.95

Question 17 continues

Question 17 continued

b) $P(6 \leq X \leq 8)$ $= \frac{95-68}{2}$

$= 13.5\%$ or 0.135

Marker use

/2

c) Find the value of x so that $P(X \geq x) = 0.0015$.

$Pr(X \geq \mu + 3\sigma) = 0.0015$

$\therefore x = 10 + 3(2)$

$\therefore x = 16$

/2

Question 18

The probability of a machine in a factory producing a defective item is $\frac{1}{10}$.

X is a random variable given by the number of defective items produced when 4 items are made.

$X \sim Bi(4, \frac{1}{10})$

Calculate exact values for the following probabilities:

a) $P(X = 3)$

$= {}^4C_3 \left(\frac{1}{10}\right)^3 \left(\frac{9}{10}\right)^1$

$= 4 \times \frac{1}{1000} \times \frac{9}{10}$

$= \frac{36}{10000}$ OR $\frac{9}{2500}$

/2

b) $P(X < 3)$

$= 1 - Pr(X=3) - Pr(X=4)$

$= \frac{10000}{10000} - \frac{36}{10000} - \frac{1}{10000}$

$= \frac{9963}{10000}$

/2

End of Section A

Total P5

/16

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MATHEMATICS METHODS

MTM415117

Section B

Pages: 32

Questions: 19

Information Sheet: 1

Suggested working time: 100 minutes

Instructions:

Calculators are allowed to be used in this section.

- There are **five (5) parts** to this section.
- Answer **all** questions and **all** items within each question.
- Write your answers in the spaces provided in this exam paper.
  - Spare diagrams have been provided at the end of Parts 1, 2, 3 and 5. Indicate in the box provided if you have used the spare diagrams.
- The exam is **three (3) hours** in length. It is suggested that you spend **approximately 100 minutes** in total answering the questions in this exam booklet.
- During the first 80 minutes you may move onto Section B, but you **cannot** use your calculator until told by your supervisor(s).
- The Mathematics Methods Information Sheet can be used throughout the exam.
- All answers must be written in **English**.
- You **must** make sure your answers address the listed criteria.

Marker use	
C4	/ 20
C5	/ 20
C6	/ 20
C7	/ 20
C8	/ 20

Part 1

Marker use

- Answer **all** questions in this part.
- This part assesses **Criterion 4**.

Question 19

Solve

$$2\log_3 x - \log_3(x-1) = \log_3(x+3)$$

Show all algebraic working in your answer.

/3

$\log_3(x^2) - \log_3(x-1) = \log_3(x+3)$

$\log_3\left(\frac{x^2}{x-1}\right) = \log_3(x+3)$

$\therefore \frac{x^2}{x-1} = x+3$

$x^2 = (x+3)(x-1)$

$\therefore 0 = x^2 + 3x - x - 3 - x^2$

$0 = 2x - 3$

$\therefore x = 3/2$

Question 20

The number of Carbon-14 atoms in a sample is given by the equation

$$\boldsymbol{A = A_0 e^{-kt}}$$

where A_0 is the initial number of atoms, k is a constant, and A is the number of atoms remaining t years later.

a) If A is equal to half the value of A_0 after 5730 years, find the value of k.

Give your answers to 6 decimal places.

/2

Solve $A = A_0/2$ when $t = 5730$

$\therefore \frac{A_0}{2} = A_0 e^{-5730k}$

OR $\frac{1}{2} = e^{-5730k}$ (cancel A_0)

$\therefore k = 0.000121$

Question 20 continues

Question 20 continued

Marker use

b) Hence find the age of a sample of Carbon-14 atoms if there is instead 70% of the initial number remaining.

Solving $0.7A_0 = A_0 e^{-0.000121t}$

gives $t = 2948$ years

/2

Question 21

Marker use

The graph in Figure 5 is in the form:

$$f(x) = a \times b^x + k, \quad x \in \mathbb{R}$$

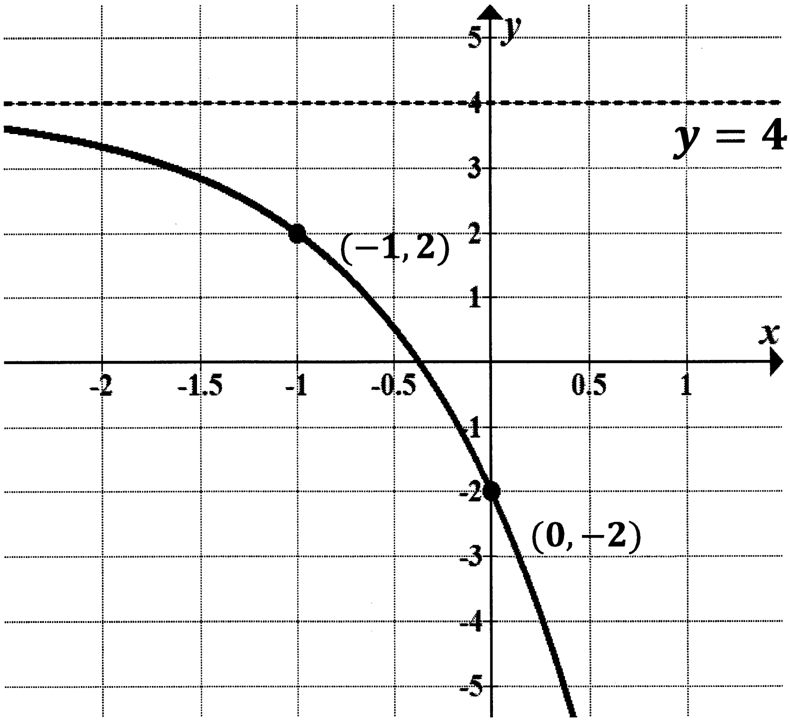


Figure 5

a) Using Figure 5, State the value of k.

Asy gives $k = 4$

/1

b) Determine the equation of $f(x)$.

Subbing $(0,-2)$ gives $-2 = a \times b^0 + 4$

$\therefore a = -6$

Subbing $(-1, 2)$ gives $2 = -6 \times b^{-1} + 4$

$\frac{1}{3} = \frac{1}{b}$

$\therefore b = 3$

$\therefore f(x) = -6 \times 3^x + 4$

/3

Question 22

Marker use

a) Using Figure 6, sketch the graph of the hyperbola:

$$f(x) = \frac{a}{x-b}$$

for $a > 0$ and $b > 0$ where $x \in \mathbb{R}\backslash\{b\}$.

Include asymptotes and exact values for intercepts in your sketch.

/3

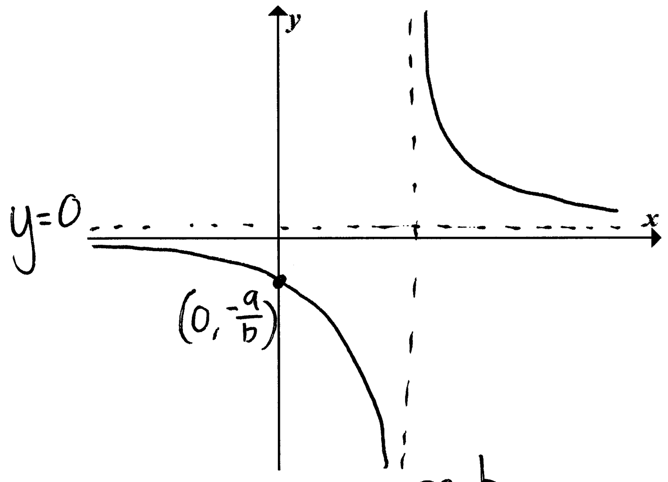


Figure 6

Spare diagram used (X) ☐

At $x=0$, $f(0) = \frac{a}{-b}$

b) Consider $g(x) = x^{-2}$ and $f(x) = \frac{a}{x-b}$.

/2

Determine the function $g(f(x))$ $= \left(\frac{a}{x-b}\right)^{-2}$ OR $\frac{(x-b)^2}{a^2}$

Question 22 continues

Question 22 continued

Marker use
/2
/2

c) State the domain and range for $g(f(x))$.

$x \in \mathbb{R} \setminus \{b\}$

$y \in (0, \infty)$

As $\mathbb{R} \setminus \{0\} \subseteq \mathbb{R} \setminus \{0\}$, it exists (range $f(x) \subseteq$ Domain $g(x)$) and the domain is the domain of the inner function, $f(x)$.

d) Find the function $f(x)$ such that:

$$g(f(x)) = 3(x-4)^2$$

Letting $\frac{1}{a^2}(x-b)^2 = 3(x-4)^2$ gives $a = \pm\frac{1}{\sqrt{3}}$

$= \frac{1}{\sqrt{3}}$ as $a > 0$

and $b = 4$

$\therefore f(x) = \frac{1/\sqrt{3}}{x-4}$ OR $\frac{1}{\sqrt{3}(x-4)}$

Total P1

/20

Part 2

Marker use

- Answer **all** questions in this part.
- This part assesses **Criterion 5**.

Question 23

The graph below is in the form

$$y = a\tan n(x+b)$$

where x is given in radians.

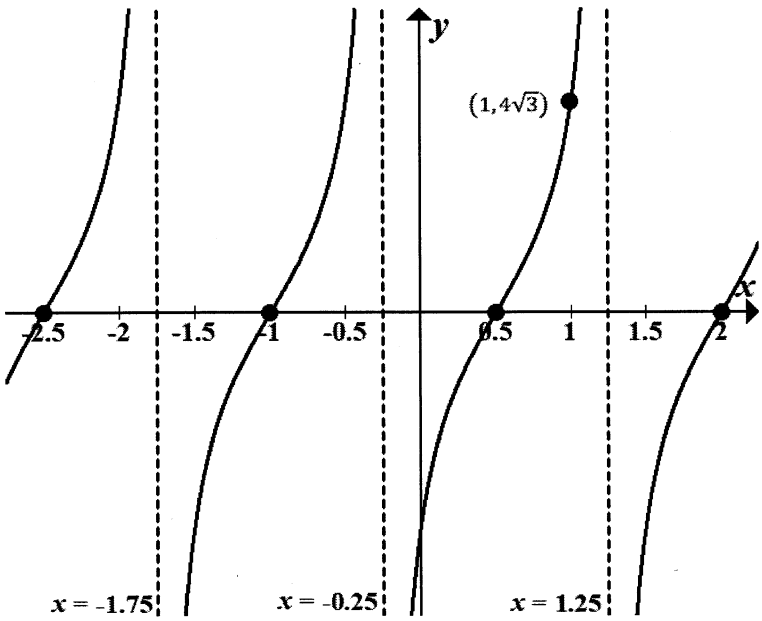


Figure 7

Spare diagram used (X) ☐

a) What is the period for this graph?

1.5^c

/1

b) Find the equation for the graph in Figure 7.

Solving $1.5 = \pi/n$ gives $n = \frac{2\pi}{3}$

Inflection gives $b = 1$

Subbing $(1, 4\sqrt{3})$ gives $a = 4$

$\therefore y = 4\tan\left(\frac{2\pi}{3}(x+1)\right)$ OR $y = 4\tan\left(\frac{2\pi}{3}(x-\frac{1}{2})\right)$

/3

Question 24

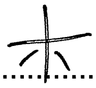
Marker use

a) Solve algebraically the equation

$$\sin\left(3x - \frac{\pi}{2}\right) + 1 = \frac{1}{2}$$

for $0 \leq x \leq \pi$. /4

$\sin\left(3x - \pi/2\right) = -1/2$



$\therefore 3x - \pi/2 = \pi + \pi/6,\ 2\pi - \pi/6$

BA: $\pi/6$

$3x - \frac{3\pi}{6} = 7\pi/6,\ \frac{11\pi}{6}$

$3x = \frac{10\pi}{6},\ \frac{14\pi}{6}$

$x = \frac{5\pi}{9},\ \frac{7\pi}{9}$

As period $= \frac{2\pi}{3} = \frac{6\pi}{9}$ and $x \in [0, \frac{9\pi}{9}]$

$x = \pi/9,\ \frac{5\pi}{9},\ \frac{7\pi}{9}$

b) By considering the period of the equation $y = \sin\left(3x - \frac{\pi}{2}\right)$, find three more solutions in the interval $[\pi, 2\pi]$. /3

If $x \in [\frac{9\pi}{9}, \frac{18\pi}{9}]$ then $x = \frac{11\pi}{9},\ \frac{13\pi}{9},\ \frac{17\pi}{9}$

(Assuming original part a) equation was intended)

Question 25

Marker use

The height of the water level at the entrance of a harbour follows a sinusoidal pattern.

Low tide has a height of 2 m and 6 hours later the following high tide is 8 m.

The height of the tide can be modelled by the equation:

$\boldsymbol{H(t) = a\sin(n(t+b)) + c}$, where $\boldsymbol{t}$ is hours after midnight.

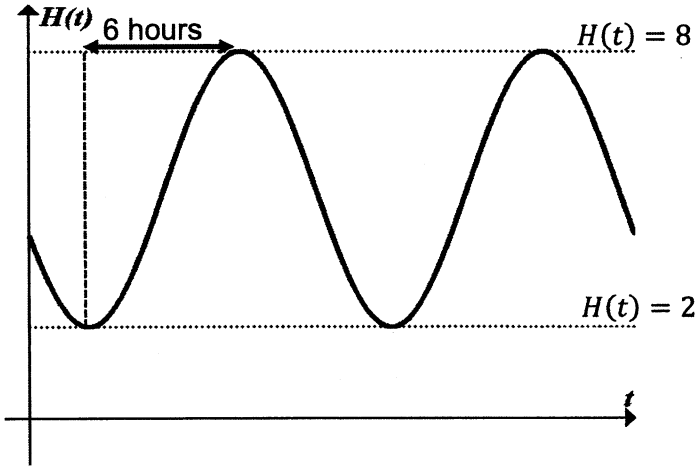


Figure 8

Spare diagram used (X) ☐

a) Using Figure 8, calculate **first** the values of a, n and c. /3

$c = \frac{8+2}{2} = 5$, $|a| = \frac{8-2}{2} = 3$ Period $= 12 = \frac{2\pi}{n}$

$\therefore n = \pi/6$

$\therefore a = \pm 3$

b) One night, the height of the water at midnight is 4 m and falling.

Find an appropriate value for b to three decimal places. /3

Midnight is when $t = 0$

Solving $4 = 3\sin(\pi/6(0+b)) + 5$

gives $b = 6.649$

(Alternatives include $b = -0.649$, 0.649, -5.351, ...)

exclude as rising   if $a=-3$ $\pm$ a period

Question 25 continues

Question 25 continued

Marker use

c) A boat is only able to pass through the harbour when the depth of the water is 7 m or greater. During which times (to the nearest minute) of the day can the boat pass through?

/3

Solving $7 = 3\sin\left(\frac{\pi}{6}(x + 6.649)\right) + 5$

gives $t = 6.74,\ 9.96,\ 18.74,\ 21.96$

$\therefore$ 6:45am to 9:57am and 6:45pm to 9:57pm

(round inwards as $H(t) \geq 7$)

Total P2

/20

Part 3

- Answer **all** questions in this part.
- This part assesses **Criterion 6**.

Marker use

Question 26

Sketch the derivative of the function below on the same axes.

Note that no working is required to be shown for this question.

/3

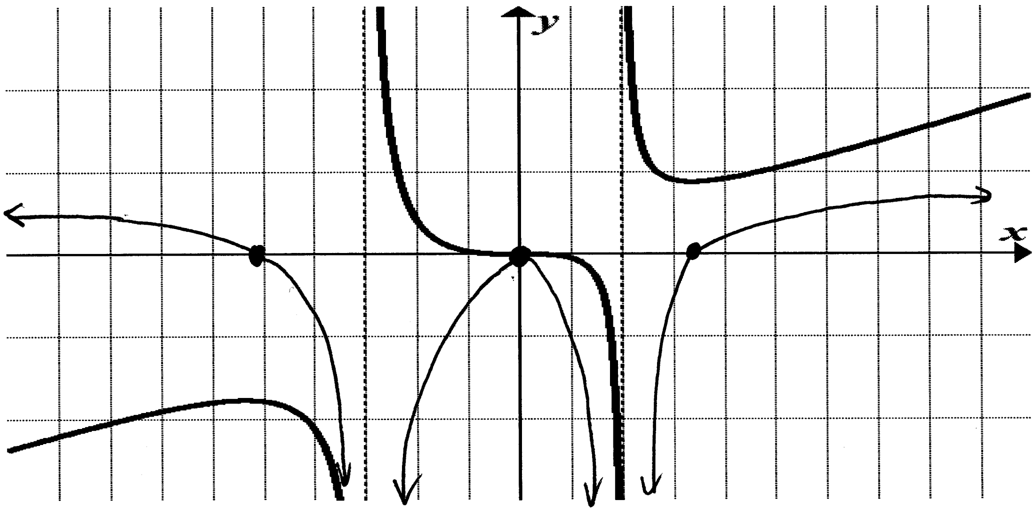


Figure 9

Spare diagram used (X) ☐

Question 27

Find and classify the stationary point(s) of the function

$$f(x) = x\,e^{2x}$$

Show all algebraic working.

/5

$u = x \qquad v = e^{2x}$

$u' = 1 \qquad v' = 2e^{2x}$

$f'(x) = e^{2x} + 2xe^{2x}$

$= e^{2x}(1 + 2x)$

Let $f'(x) = 0$ to find SPs:

$0 = e^{2x}(1 + 2x)$

$\therefore \quad x = -\frac{1}{2}$

$f(-\frac{1}{2}) = -\frac{1}{2}e^{-\frac{1}{2}\times 2}$

$= -\frac{1}{2e}$

x	-1	$-\frac{1}{2}$	0
$f'(x)$	$-e^{-2}$	0	1
Shape	\	—	/

OR

$f''(x) = 2e^{2x} + 4xe^{2x} + 2e^{2x}$

$f''(-\frac{1}{2}) = \frac{2}{e} > 0$

$\therefore \left(-\frac{1}{2}, -\frac{1}{2e}\right)$ is a minimum

Question 28

Marker use

A chain is hung across a canyon from the points $(0,100)$ and $(20,100)$.

The height of the chain above ground level can be modelled by the equation

$$H(x) = a(x-b)^2 + c \quad \text{for } a \neq 0$$

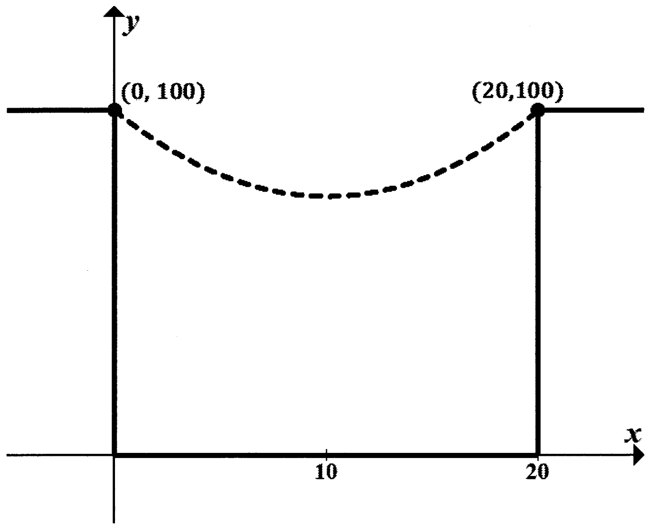


Figure 10

So that the chain is not too steep, the value of the rate of change at $x = 20$ is 5 m/m

(i.e. vertical metres per horizontal metres).

Find the minimum value of the height of the chain above ground level.

$H(x) = a(x-10)^2 + c$ (from TP)

$H'(x) = 2a(x-10)$

Subbing $H'(20) = 5$ gives $a = 1/4$

Subbing $H(0) = 100$ gives $c = 75$

$\therefore$ Chain is 75m above ground

/5

Question 29

Marker use

A camping equipment manufacturer wants to design a new tent.

A rear view is shown below.

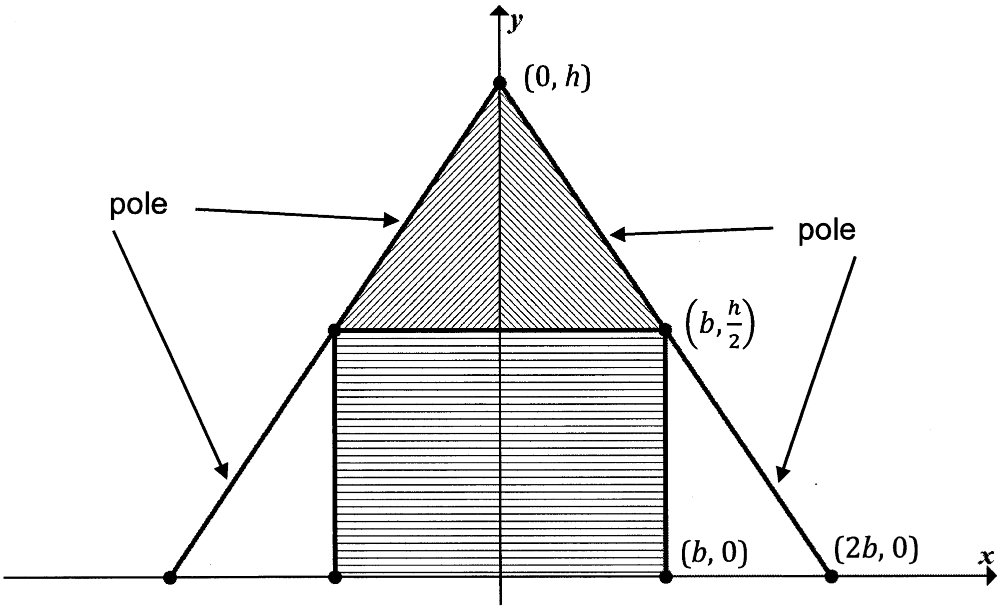


Figure 11

A pole that attaches from the ground $(2b, 0)$ to the top $(0, h)$ is 4 m long.

a) Show that $h = 2\sqrt{4 - b^2}$. /2

Pythag: $h^2 + (2b)^2 = 4^2$

$h^2 = 16 - 4b^2$

$h = \pm\sqrt{4(4-b^2)}$

$= 2\sqrt{4-b^2}$ $(h>0)$

b) Given that the shaded area of the tent is $\frac{3bh}{2}$, /1

find an expression for the area in terms of b.

Area $= \frac{3bh}{2}$

$= \frac{3b(2\sqrt{4-b^2})}{2}$ (sub h from part a))

$= 3b\sqrt{4-b^2}$

Question 29 continues

Question 29 continued

Marker use

c) Find algebraically the values of b and h that give the maximum area.

Note: You do **not** need to justify that it is a maximum.

/4

$u = 3b \qquad v = (4-b^2)^{1/2} = \sqrt{4-b^2}$

$u' = 3 \qquad v' = \frac{1}{2}(4-b^2)^{-1/2} \times (-2b) = \frac{-b}{\sqrt{4-b^2}}$

$\therefore \frac{dA}{db} = 3\sqrt{4-b^2} - \frac{3b^2}{\sqrt{4-b^2}}$

Let $dA/db = 0$:

$\frac{3b^2}{\sqrt{4-b^2}} = 3\sqrt{4-b^2}$

$3b^2 = 3(4-b^2)$

$b^2 = 4-b^2$

$2b^2 = 4$

$b^2 = 2$

$b = \pm\sqrt{2}$

$\therefore b = \sqrt{2}\,m \; (b>0)$ and $h = 2\sqrt{4-(\sqrt{2})^2}$

$= 2\sqrt{2}\,m$

Total P3

/20

Part 4

Marker use

- Answer **all** questions in this part.
- This part assesses **Criterion 7.**

Question 30

Write expressions for the shaded area enclosed by the functions in the following situations:

a)

/1

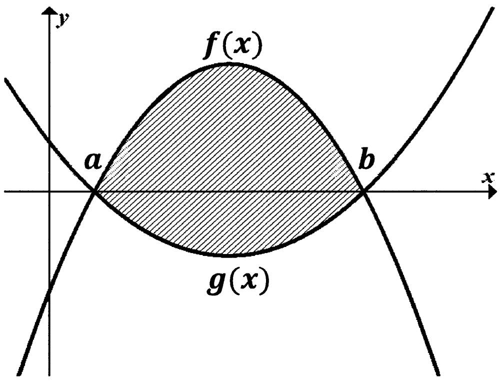


Figure 12

$\int_a^b f(x) - g(x)\,dx$

b)

/1

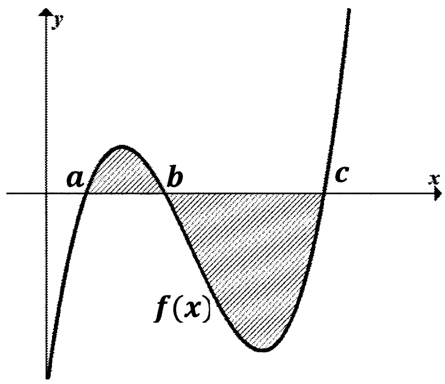


Figure 13

$\int_a^b f(x)\,dx + \left|\int_b^c f(x)\,dx\right|$

Question 30 continues

Marker use

c)

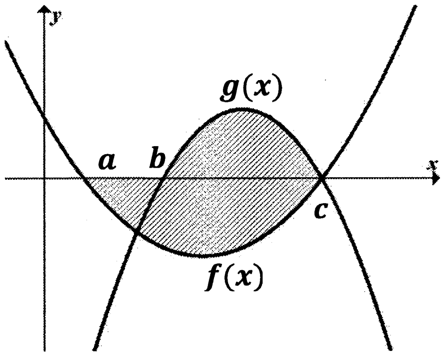


Figure 14

$\left|\int_a^c f(x)\,dx\right| + \int_b^c g(x)\,dx$ OR $\left|\int_a^b f(x)\,dx\right| + \int_b^c g(x) - f(x)\,dx$

/1

Question 31

Find the value of k, where $k > 1$, such that the area enclosed between the line $x = k$ and the curves $y = \frac{1}{x^3}$ and $y = \frac{1}{x^2}$ is equal to $\frac{1}{8}$.

Show all working in your answer.

/4

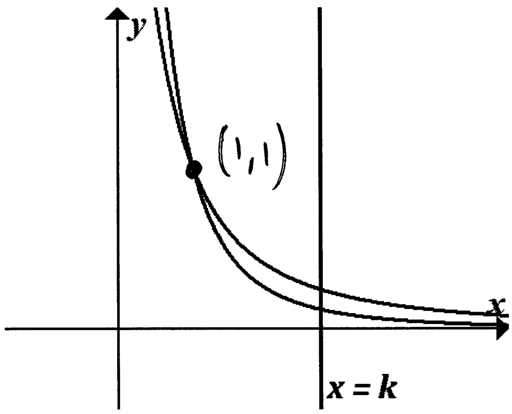


Figure 15

Intersection: $\frac{1}{x^3} = \frac{1}{x^2}$ gives $x=1$ $\therefore$ $(1,1)$

Solving $\frac{1}{8} = \int_1^k \frac{1}{x^2} - \frac{1}{x^3}\,dx$

gives $k \neq \frac{2}{3}$ OR $k = 2$

(Invalid as $k>1$)

Question 32

Marker use

A popcorn maker produces popcorn at a rate of

$$\frac{dP}{dt} = -15t^2 + 60t + 5\sin(2\pi t)$$

Where P is the number of pieces of popcorn produced in t minutes.

It begins producing popcorn at $t = 0$ and ends when $\frac{dP}{dt} = 0$ once more.

a) Find the value of $\frac{dP}{dt}$ at $t = 1$ minute. /1

At $t=1$, $dP/dt = 45$

b) Find the time at which popcorn production ends. /2

Solving $dP/dt = 0$

gives $P = 4$ minutes

c) Find the equation of the function $P(t)$. /3

$P(t) = \int -15t^2 + 60t + 5\sin(2\pi t)\, dt$

$= -5t^3 + 30t^2 - \frac{5}{2\pi}\cos(2\pi t) + c$

As $P(0) = 0$, $c = 5/2\pi$

$\therefore P(t) = -5t^3 + 30t^2 - 5/2\pi \cos(2\pi t) + 5/2\pi$

d) Hence, find the total amount of popcorn pieces produced. /2

$P(4) = 160$ pieces of popcorn

Question 33

Marker use

The three functions are sketched in Figure 16 in the domain $[0, \pi]$:

$f_1(x) = \sin(x)$

$f_2(x) = \sin\left(x - \frac{2\pi}{3}\right)$

$f_3(x) = \sin\left(x + \frac{2\pi}{3}\right)$

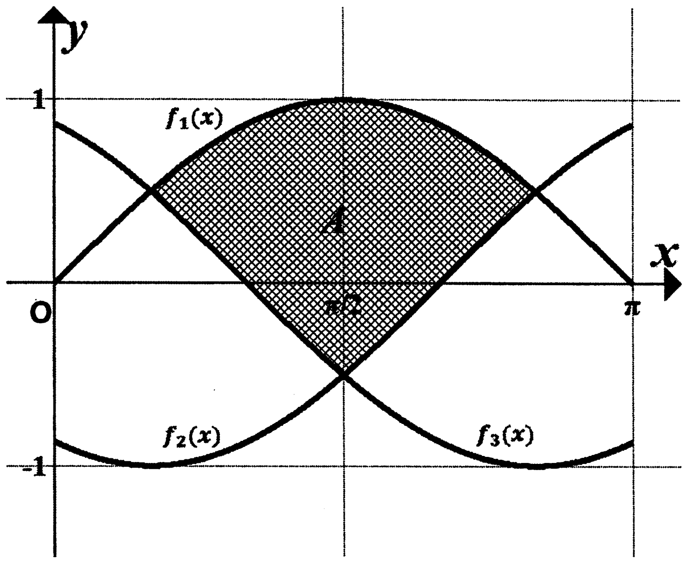


Figure 16

a) Identify all the boundaries of the area A above. /2

Solve $f_1(x) = f_2(x)$ gives $x = \frac{5\pi}{6}$ $\left(\frac{5\pi}{6}, \frac{1}{2}\right)$

Solve $f_1(x) = f_3(x)$ gives $x = \frac{\pi}{6}$ $\left(\frac{\pi}{6}, \frac{1}{2}\right)$

Solve $f_2(x) = f_3(x)$ gives $x = \frac{\pi}{2}$ $\left(\frac{\pi}{2}, -\frac{1}{2}\right)$

b) Determine the exact value for the area of A. /3

$$\text{Area} = \int_{\pi/6}^{\pi/2} f_1(x) - f_3(x)\, dx + \int_{\pi/2}^{\frac{5\pi}{6}} f_1(x) - f_2(x)\, dx$$

$= \sqrt{3}$ units of area

Total P4 /20

Part 5

Marker use

- Answer **all** questions in this part.
- This part assesses **Criterion 8.**

Question 34

A world-renowned chef randomly samples 10 vastly different ice-cream flavours.

Let X be the binomial random variable of how many of these she likes, each with a probability of $\frac{3}{10}$ of success. $X \sim Bi(10, \frac{3}{10})$

From these 10 flavours:

a) i. find the probability that she will like more than 5 flavours. /2

$Pr(X > 5) = 0.0473$

ii. find the probability that she will like exactly 5 flavours. /1

$Pr(X = 5) = 0.1029$

b) Calculate the mean and variance for this distribution. /2

$\mu = 10 \times 3/10$ and $\sigma^2 = 10(\frac{3}{10})(1 - 3/10)$

$= 3$ $= 21/10$

Question 35

200 random people are surveyed and 70 say that they prefer honey to jam on their toast. /4

A confidence interval for the true proportion of the population who prefer honey is $(0.2945,\ 0.4055)$.

Find the level of confidence that generates this interval.

Show full working in your answer.

$\hat{p} = \frac{70}{200}$ $\therefore$ $0.4055 = 0.35 + z\sqrt{\frac{0.35 \times 0.65}{200}}$

$= 0.35$ $\therefore$ $z = 1.6456$

$\therefore$ 90% level of confidence

(By solving $Pr(-1.6456 < Z < 1.6456$ on calc)

Question 36

Marker use

A normal distribution has 10% above 30 and 25% below 10.

a) Using Figure 17, sketch a diagram representing the information above. /1

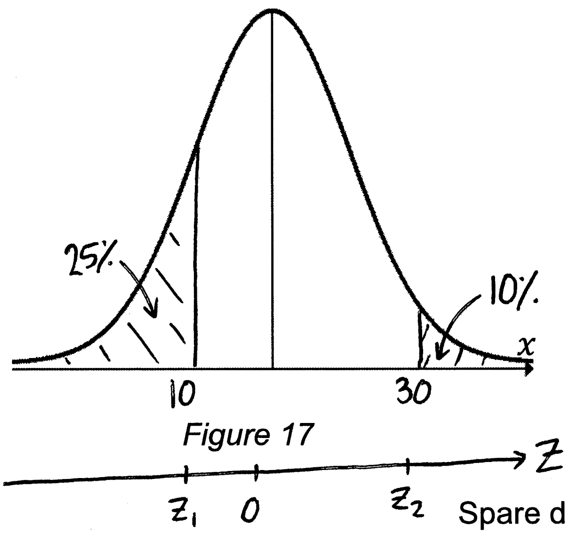


Figure 17

Spare diagram used (X) ☐

b) Find μ and σ to two decimal places. /4

$\Pr(Z < z_1) = 0.25$ and $\Pr(Z > z_2) = 0.10$

$\therefore z_1 = -0.6745$ $\therefore z_2 = 1.2816$

$\therefore -0.6745 = \dfrac{10-\mu}{\sigma}$ $\therefore 1.2816 = \dfrac{30-\mu}{\sigma}$

Solving simultaneously gives $\mu = 16.90$
and $\sigma = 10.22$

c) If instead, 25% of its area is above 30 and 10% of its area is below 10, state the new values of μ and σ. /1

Let $z_2 = 0.6745$ and $z_1 = -12816$ (swapped)

$\therefore \mu = 23.10$ and $\sigma = 10.22$

Question 37

Marker use

A recent survey of 1000 people undertaken by a particular puffer jacket company found that 50% of respondents own a puffer jacket made by that company.

a) Show that the 95% confidence interval for this survey finding is (46.80%, 53.10%), i.e. a margin of error of 3.10%, to 2 decimal places.

/2

$\hat{p} = \frac{500}{1000}$, $z = 1.96$, $n = 1000$

$= 0.5$

$\therefore$ 95% CI $= \left(0.5 - 1.96\sqrt{\frac{0.5 \times 0.5}{1000}} \, , \, 0.5 + 1.96\sqrt{\frac{0.5 \times 0.5}{1000}}\right)$

$= (0.4690 \, , \, 0.5310)$ as required

Company management want p to increase so that when another 1000 people are surveyed in a year's time, the margin of error is now 2%. The confidence interval for the new survey will still be 95%.

b) What value of $\hat{p}$ would be required for this to occur?

/3

Is this a reasonable increase in the proportion of puffer jacket ownership in 1 year?

Solving $0.02 = 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{1000}}$

gives $\hat{p} = 0.1180$ or $\hat{p} = 0.8819$

(invalid as < 0.5)

Increasing 50% to 88% seems like a lot, but could be doable by changing sales tactics, demographic, competition or climate.

Total P5

/20