2025 ASSESSMENT REPORT

MTM415117 MATHEMATICS METHODS

General Comments

The examination was generally straightforward, with many students completing it with time to spare and reporting limited opportunity to demonstrate the full extent of their knowledge. This is reflected in half the cohort achieving marks above 75% in each criterion. While overall performance was strong across both sections, gaps in basic numeracy skills were evident in several questions, particularly those relating to probability. It is important to note that this examination does not encompass the full breadth of the course content and includes several questions drawn from a prerequisite, less advanced course. Accordingly, it should not be considered an indication of the overall difficulty of the course or future assessments.

All marks are out of 36 except for probability, which was out of 34.

The exam will now be commented on question by question.

Section A

Criterion 4

Question 1

a. Done well. Requires x and y to be stated for full marks.
b. Done well. A common error was using incorrect coordinates or incorrectly substituting the x- and y-values of a point (often swapped).

Question 2

Very well done.

Question 3

Some students struggled to divide by -2 correctly. The use of $f^{-1}(x)$ notation in the final answer was required for full marks.

Question 4

Half a mark was deducted from some students for not identifying the coefficient of the term. A few students wasted time by expanding the whole expression.

Question 5

Very well done. A few students lost marks for not labelling their y-intercepts or asymptotes correctly. It is good practice to label an extra point, although in this case, marks were not deducted, as it was not specified. Students should be reminded to draw asymptotes using dotted or dashed lines.

Criterion 5

Question 6

Generally well answered.

Question 7

Generally well answered. Several students did not correctly identify the sign of cos and tan from the quadrant.

Question 8

Students who used the general solution for this question generally did not get the correct result. This is mainly due to algebraic errors and incorrect use of the equation. Other methods were far more successful.

Question 9

Generally well answered. Several students lost marks for not providing the equations of their asymptotes.

Question 10

Generally well answered.

Criterion 6

Question 11

All three parts were done well by many students. A common error in part c) was neglecting to multiply by 4 when differentiating $\ln(1 + 4x)$, incorrectly obtaining a result of $\frac{1}{(1+4x)}$ rather than $\frac{4}{(1+4x)}$.

Question 12

Done well by most students, though marks were sometimes lost for incorrect notation (for example, $lim\ h \to 0$ missing completely or incorrectly positioned within the working) or for errors made when expanding brackets.

Question 13

Many students neglected to multiply by ½ (the current power) or by $2x$ (the derivative of $x^2 + 5$), so had an incorrect expression for $g'(x)$. Several students gave the final answer incorrectly as $\pm\frac{2}{3}$, when it should have been just $\frac{2}{3}$.

Question 14

Many students were able to differentiate the equation of the curve and to correctly calculate that the gradient at $(3, 1)$ is 0, but few were then able to deduce that the normal line must be vertical and must therefore have the equation $x = 3$. Many students stated incorrectly that since the gradient of the normal is undefined, the normal itself is undefined/doesn't exist.

Criterion 7

Question 15

Well done. The errors that occurred (infrequently) were no constant of integration $(+c)$, a sign error with the $\sin(3x)$ term and multiplying $\sin(3x)$ by 3 rather than dividing.

Question 16

Well done. The only notable recurring error that occurred was trying to embellish an already simple answer $4e^8 - 4e^4$ into $4e^{\frac{8}{4}} = 4e^2$ or $4e^4$.

Question 17

Mostly well done though there were some errors in solving the quadratic. Around 15% of students decided to integrate the function (correctly) to $\frac{1}{4}(2x-7)^2$ which added algebraic and arithmetic overheads in solving $\frac{(2k-7)^2}{4} - \frac{7^2}{4} = -6$ these included overlooking the negative square root and dealing with fractions and directed numbers. Nonetheless a few of the students using this approach were successful. (Using the alternate correct integrand $x^2 - 7x$ gives integer arithmetic).

A few students put incorrect constraints on the possible values of k, such as stating that k must be positive.

Question 18

Answers of variable quality. Pleasingly most students did use the correct notation for an integral $\int_a^b I(x)\boldsymbol{dx}$. Most common mistakes were to forget to write units squared and to either not know the value of e^0 or assume an antiderivative evaluated at 0 is 0 (as is the case for polynomials).

Question 19

Many good answers but also many with faulty reasoning; that is, it is important to carefully distinguish between the function and the derivative. Many put the given points $(2, -28)$ and $(0,0)$ into the expression for the derivative, noting that simplifying algebra is incorrect and an incorrect function is the result. On the positive side, most students realised the fact that there are two bits of information attached to $(2, -28)$ being a stationary point ($f(2) = -28$ and $f'(2) = 0$.

One unfortunate mistake was that many forgot about the arbitrary constant when integrating the given derivative. Because this constant was actually zero, students obtained the correct answer without realising they had omitted a step in the argument. A good number of students simplified $-28 = 8 + expression$ to $-20 = expression$ rather than the correct $-36 = expression$.

Finally, a considerable number just found the values of the constants a and b and stopped there without writing down the final form of the function as requested.

Criterion 8

Question 20

Generally well answered but a significant proportion forgot to square the standard deviation to find the variance.

Question 21

Some students multiplied 0 and 0.84 to get 0.84 resulting in an expected value of 1 or saying $(-1)^2 = -1$ leading to an expected value with $X^2 = 0$. Few students were able to find the square root of 0.16.

Question 22

Few students recognised that $\frac{0.6 \times 0.4}{24} = 0.01$ and even fewer students were able to take the square root of 0.01. Solutions based on 1.96 or 2 standard deviations either side of the mean were accepted. Many students left answers in unsimplified form and only received half marks.

Question 23

a. Two reasons were required to receive full marks, such as: the outcome being success or failure, the probability of success remaining constant, or the events are independent of each other.

b. Most students recognised that $Pr(X \geq 1) = 1 - Pr(X = 0)$. Other students used $Pr(X \geq 1) = Pr(X = 1) + Pr(X = 2)$.

Question 24

Just writing the answer without workings scored the full mark in the first two parts. Wrong answers with some logic to their working received 0.5 marks. Students who drew normal distribution curves and marked in areas and standard deviations for part c) generally achieved the correct answer. Common errors included $a = 130$ or 80.

Section B

Criterion 4

Question 25

a. Very well done. Students were not required to expand the answer.
b. Most students correctly used their calculator to find both solutions. Some students incorrectly substituted $x = 10$, or forget the +1 in $f(g(x))$.

Question 26

Transformations of the domain were also required to receive full marks. Students should be reminded that this is required when a domain is defined. Many students incorrectly applied the horizontal translation, writing $\ln(-x + 2)$ instead of $\ln\big(-(x + 2)\big)$. Students were also required to state $g(x)$ in their final answer as specified in the question.

Question 27

a. Very well done by students. In this case, the number of required decimal places was not stated, so $k = 0.06$ was marked as correct. Students are reminded not to round their answers too significantly.
b. Very well done. Units were required for full marks.
c. Also well done. Since the question was worth more than 1 mark, some form of working was required.

Question 28

a. Done well by most students. Students are encouraged not to use the number format -2E-4 when given by their calculator. Ensure the "number format" in CASIO Classpad is set to 'normal 2' to avoid this or understand how to write this in scientific notation.
b. All 3 solutions to $C = 0$ were expected to be included, with any impossible solutions then excluded. There were quite a few "error carried forward (ECF)" marks awarded in this question.
c. $[0,10]$ and $(0,10)$ were both considered correct.
d. Many students successfully found the two positive solutions to $C = 0.05$. However, many did not realise the drug was effective between these two times. Some students only stated one value. Some students also didn't realise that the question was asking for when the drug was effective, rather than ineffective. This is a friendly reminder to always carefully read the question.

Criterion 5

Question 29

Generally well done. The most common error was in finding the period and value for n. Students struggled to interpret that the two points provided spanned 1.5 periods.

Question 30

Largely well answered by most students. By far the most common error was incorrectly implementing the horizontal translation as $-2\cos\left(\frac{x}{4}-\frac{\pi}{4}\right)$ instead of $-2\cos\left(\frac{\left(x-\frac{\pi}{4}\right)}{4}\right)$.

Question 31

Generally well answered. Full marks were awarded for solving on the graphics calculator. Students who attempted the question by hand often made algebraic errors in their solution.

Question 32

a. Most students were successful in obtaining values for the amplitude and the vertical translation, numerical errors notwithstanding. Identifying the period proved to be more challenging. There were multiple versions of the correct functions possible. A half-mark was deducted if students produced a correct cosine function.
b. Generally well answered. Errors were carried forward from part a), and full marks awarded.
c. Most students identified what was needed to solve the question. Some produced answers with no indication of any working; a mark was deducted in these cases. Marks were awarded regardless of how the question was answered – i.e. in decimals, or in hours and minutes, or rounded to the nearest hour.
d. This question was the first on the exam paper with any real rigour, and most students did not obtain the correct function. Some students were successful in identifying the new period as being 12.2 hours, and some of these then correctly deduced the corresponding value of 'n'. Very few students realised a horizontal translation of 0.05 left (or other variants) was also required. Approximately 3% of students obtained full marks for this question.
e. If errors were correctly carried forward from part d), full marks were awarded here.

Criterion 6

Question 33

a. Generally done well or not attempted at all. It's a classic problem, so should be familiar to students.
b. Students were fairly equally split between (i) those commenting correctly on the fact that a cut of 7cm isn't possible since the cardboard isn't big enough to do that and negative lengths will result (this being the intended answer for this question); (ii) those who substituted $x = 7$ into the volume formula given in the previous part, and who incorrectly obtained an answer that wasn't 84, thus concluding that this is why the student is wrong in thinking that the volume is 84; (iii) a sizeable number of students who were confused by the word 'greater' and who calculated that $x = 7$ does lead to $V = 84$ but who then pointed out that 84 isn't greater than

84; (iv) a number who realised that 84 must be incorrect since it exceeds the maximum volume later found in the next part.

c. Largely done well, save for students neglecting to give all three dimensions for the box and also not following instructions to round off answers in the indicated fashion. Many students failed to explain why $x = 1.569$ was important whilst $x = 5.097$ should be discarded.

Question 34

A very well-done question. Some students failed to appreciate the need to multiply by -1 and obtained a derivative of $\frac{1}{(1-x)}$ rather than $\frac{-1}{(1-x)}$.

Question 35

a. This was done well.

b. Errors were quite common when trying to identify whether the gradient was positive or negative to either side of the stationary points, which led many students to incorrectly classify the two stationary points, often erroneously labelling one of them as a maximum.

Question 36

This was done well by many, though some errors were made in solving for k once the correct equation had been established.

Question 37

This was a poorly done question, which few students obtained full marks. Common errors were made with all three parts of the graph: (i) The first part was sometimes drawn sloping downwards and/or not passing through -4 on the x-axis; (ii) The second part was not drawn at all by quite a number of students, who perhaps thought that a gradient value of zero requires no derivative graph to be drawn; (iii) The third part was frequently drawn at the wrong height, though the gradient of the third part of the graph of $f(x)$ could fairly easily be calculated as being -3. It was very common for the open circles to be missing completely, or for answers to contain a mixture of open and closed circles. All six should have been open.

Criterion 7

Question 38

The question was done successfully by the majority of students. A common error was omitting the constant c in the final answer.

Question 39

To be awarded full marks for this question students needed to recast the derivative statement as an antiderivative statement and then use the properties of the definite integral. To get a correct solution some algebra and the antiderivative of e^{kx} needed to be employed.

Question 40

Another question targeting properties of the definite integral. Most errors that occurred were the result of algebraic short comings.

Question 41

Many students failed to parametrise the coordinates of B and C in terms of a. Most students successfully calculated $\int_0^a \sqrt{x}\,dx$ but many struggled with demonstrating that this shaded area is $\frac{2}{3}$ of the area of the rectangle.

Question 42

Generally well answered by students. Many students played it doubly safe by using $\left|\int_a^0 f(x) - g(x)dx\right|$ when $\int_a^0 f(x) - g(x)dx$ suffices. The most common mistake arose from reading $e^x - 1$ as e^{x-1} (even though e^{x-1}does not pass through $(0,0)$ as shown on the graph).

Criterion 8

Question 43

a. Students who rounded their answer to 56% lost 0.5 marks. Many forgot to turn their answer into a percentage.
b. Answers of 9502 eggs or 9503 eggs were accepted. Other values consistent with rounded probabilities gained partial marks Probabilities need to be stated to 4 decimal places after rounding.

Question 44

a. This was answered well by the vast majority of students. To gain the mark, students were required to show some working. Many students left their answer for the expected value as a fraction and got the full mark.
b. Common errors in finding the expected value of X^2 included squaring the probabilities instead of the scores and not knowing their multiplication tables. Some students forgot to subtract the square of the expected value in calculating the variance.

Question 45

Most students were able to find $\vec{p}$ and were able to calculate $z = \pm 1.998$. Many students incorrectly assumed that as this was close to 2 that it represented the 95% confidence interval.

Question 46

To score full marks required the use of the graph supplied to mark in the appropriate areas. Using their calculators, students needed to find z scores to 4 decimal places. Units were needed for the mean and standard deviation.

Question 47

a. To get full marks, students need to show the correct representation of the binomial distribution. $X \sim Bi(4, 0.75)$ Most students forgot to do this but found the correct answer $\Pr(X \geq 2) = 0.9492$.

b. This question was not included in the assessment as part of the marking process, following review of its suitability for consistent and equitable assessment.

TASMANIAN ASSESSMENT, STANDARDS & CERTIFICATION

External Assessment 2025

Solutions

Attach your candidate label here

MATHEMATICS METHODS

MTM415117

Section A

Pages: 20
Questions: 24
Information Sheets: 1

Preparation time for this exam: 15 minutes

Suggested working time: 80 minutes

Instructions:

Calculators are not allowed to be used in this section.

Section A will be collected after 80 minutes.

- There are **five (5) parts** to this section.
- Answer **all** questions and **all** items within each question.
- Write your answers in the spaces provided in this exam paper.
  - A spare diagram has been provided at the end of Part 1. Indicate in the box provided if you have used the spare diagram.
- The exam is **three (3) hours** in length. The suggested working time for this section is **approximately 80 minutes**.
- During the first 80 minutes of the exam you may move onto Section B, but you **cannot** use your calculator until told by your supervisor(s).
- The Mathematics Methods Information Sheet can be used throughout the exam.
- All answers must be written in **English**.
- You **must** make sure your answers address the listed criteria.

Marker use	
C4	/ 16
C5	/ 16
C6	/ 16
C7	/ 16
C8	/ 16

Part 1

Marker use

- Answer **all** questions in this part.
- This part assesses **Criterion 4**.

Question 1

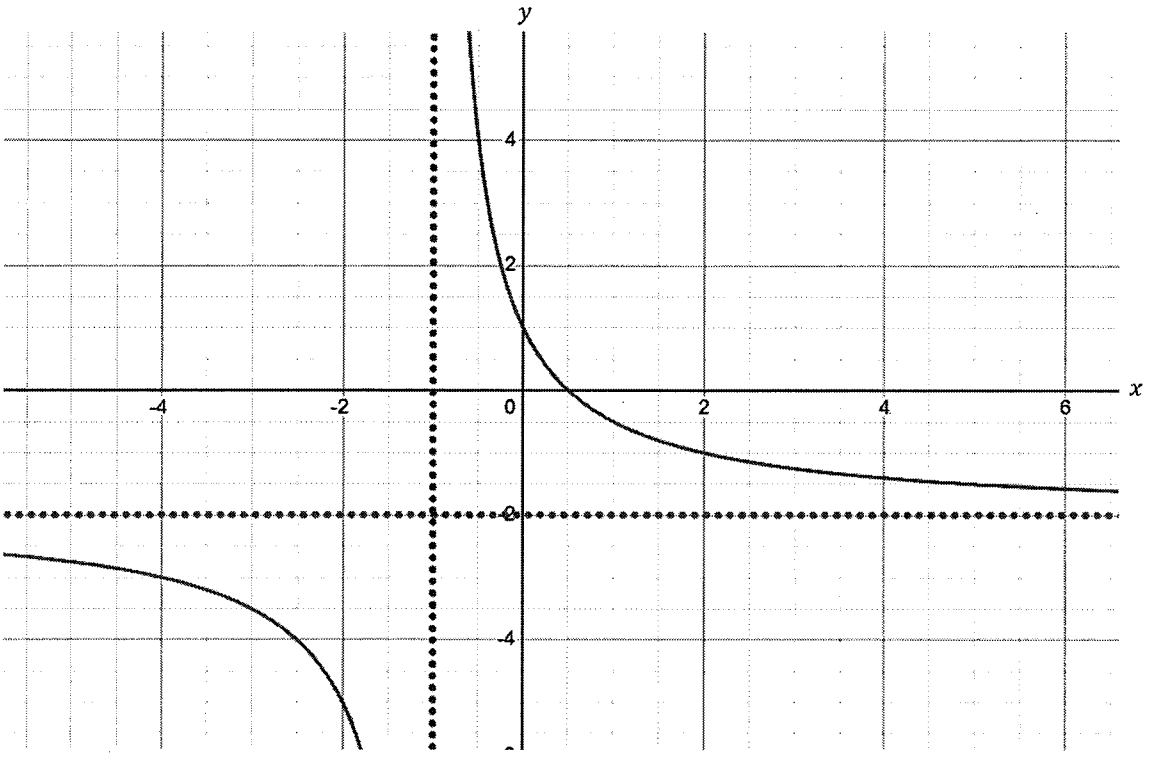


Figure 1

a) State the domain and range for the graph of the following function:

Domain: $x \in \mathbb{R} \setminus \{-1\}$ /2

Range: $y \in \mathbb{R} \setminus \{-2\}$

b) Determine the equation of the graph shown above. /3

$y = \frac{a}{x+1} - 2$

Sub $(0,1)$ to find a:

$1 = \frac{a}{1} - 2$

$\therefore a = 3$ $\therefore y = \frac{3}{x+1} - 2$

Question 2

Marker use

Solve $\log_3(3x-2)=2$. /2

$3^2 = 3x - 2$

$9 = 3x - 2$

$11 = 3x$

$\therefore x = 11/3$

Question 3

Given that $f(x) = -2e^{x-4} + 1$ find the inverse $f^{-1}(x)$. /3

$x \leftrightarrow y$: $x = -2e^{y-4} + 1$

$x - 1 = -2e^{y-4}$

$\frac{-x+1}{2} = e^{y-4}$

$\therefore y - 4 = \ln\left(\frac{-x+1}{2}\right)$

$\therefore f^{-1}(x) = \ln\left(\frac{-x+1}{2}\right) + 4$

Question 4

Pascals: 1 4 6 4 1

Determine the coefficient of x^2 in the expansion of $(5-2x)^4$. /3

The third term gives x^2: ${}^4C_2(5)^2(-2x)^2$

$= \frac{4!}{2!\,2!}(25)(4x^2)$

$= 6 \times 100x^2$

$= 600x^2$

$\therefore$ The coefficient is 600

Question 5

Marker use

Sketch the graph of

$$f(x) = \frac{4}{(x-2)^2} + 1, x \neq 2$$

/3

on Figure 2 below.

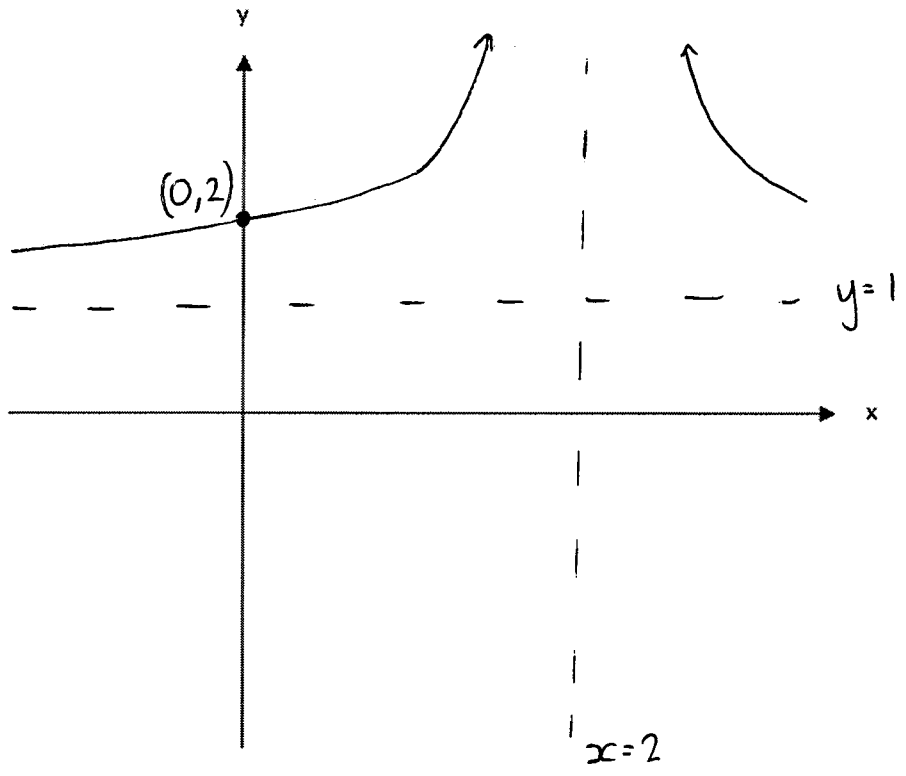


Figure 2

Spare diagram used (x) ☐

Total P1

/16

Part 2

Marker use

- Answer **all** questions in this section.
- This section assesses **Criterion 5**.

Question 6

Find the exact value of $\cos\frac{7\pi}{6} + \tan\frac{\pi}{4}$.

/3

$= \cos\left(\pi + \pi/6\right) + \tan\left(\pi/4\right)$

$= -\cos\left(\pi/6\right) + \tan\left(\pi/4\right)$

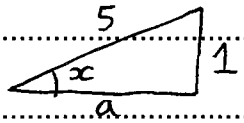
$= -\sqrt{3}/2 + 1$ OR $\frac{2-\sqrt{3}}{2}$

Question 7

If $\sin x = \frac{1}{5}$ and $\frac{\pi}{2} < x < \pi$ find exact values for $\cos x$ and $\tan x$.

/4

$= \frac{opp}{hyp}$



$a^2 + 1^2 = 5^2$

$a = \pm\sqrt{25-1}$

$\therefore a = \pm 2\sqrt{6}$

✓ $\therefore \cos x = -\frac{2\sqrt{6}}{5}$ and $\tan x = -\frac{1}{2\sqrt{6}}$

$= -\frac{\sqrt{24}}{5}$ $= -\frac{\sqrt{6}}{12}$

$= -\frac{1}{\sqrt{24}}$

Question 8

Marker use

Find all solutions of the equation $\cos(2x - \frac{\pi}{4}) = -\frac{\sqrt{2}}{2}$ in the domain $0 \leq x \leq 2\pi$.

/4

✓+ ✓ BA = $\pi/4$

$\therefore \quad 2x - \pi/4 = \pi - \pi/4, \ \pi + \pi/4$

$2x - \pi/4 = 3\pi/4, \ 5\pi/4$

$2x = 4\pi/4, \ 6\pi/4$

$x = \pi/2, \ 3\pi/4$

As period is $\frac{2\pi}{2} = \pi$ and domain is $x \in [0, 2\pi]$

$\therefore x = \pi/2, \ 3\pi/4, \ 3\pi/2, \ 7\pi/4$

Question 9

Sketch the graph of $y = -4\tan\left(x - \frac{\pi}{2}\right)$ for $-\frac{\pi}{2} \leq x < 2\pi$.

/3

Period = π

Translated $\pi/2$ units right

Reflected in the x-axis

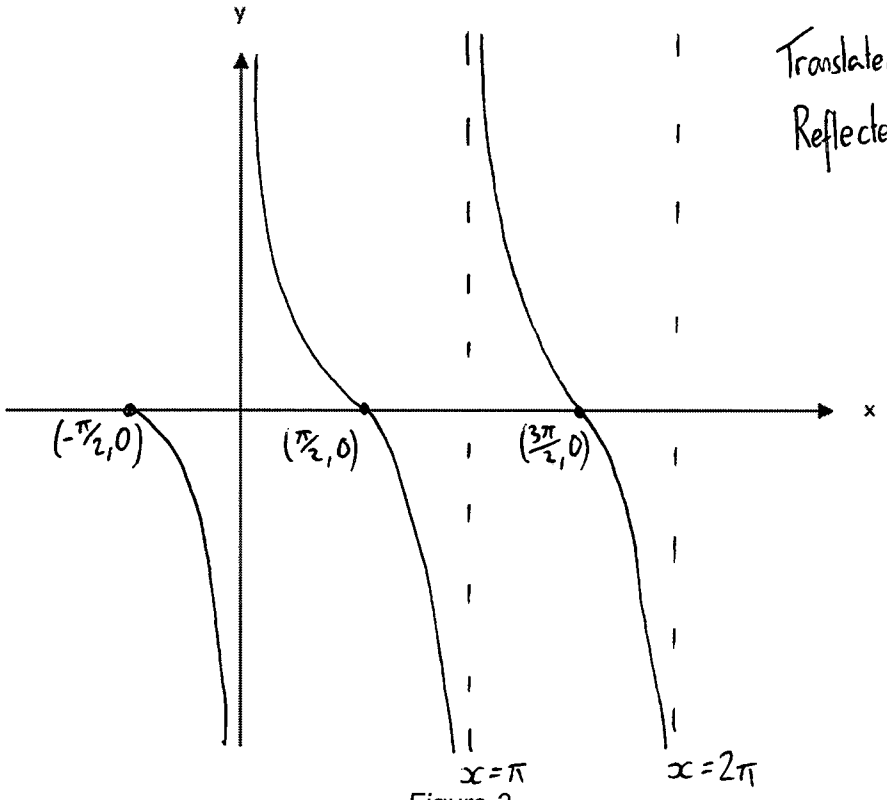


Figure 3

Spare diagram used (x) ☐

Question 10

Marker use

/2

Determine the value of $\cos^2 135° + \sin 150° + \sin^2 135°$.

$= \cos^2(135°) + \sin^2(135°) + \sin(180° - 30°)$

$= 1 + \sin(30°)$ (pythag)

$= 1 + \frac{1}{2}$

$= \frac{3}{2}$

OR $\left(\cos\left(\frac{3\pi}{4}\right)\right)^2 + \sin\left(\frac{5\pi}{6}\right) + \left(\sin\left(\frac{3\pi}{4}\right)\right)^2$

$= \left(\frac{\sqrt{2}}{2}\right)^2 + \frac{1}{2} + \left(\frac{\sqrt{2}}{2}\right)^2$

$= \frac{3}{2}$

Spare Diagram

Question 9

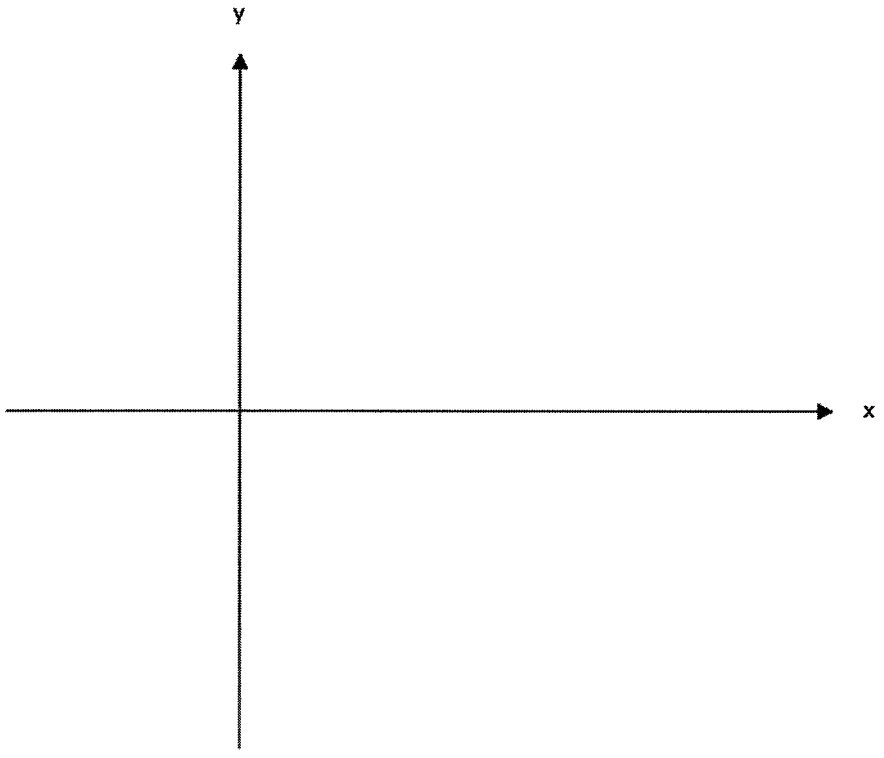


Figure 3

Total P2

/16

Part 3

Marker use

- Answer **all** questions in this section.
- This section assesses **Criterion 6.**

Question 11

Differentiate the following:

a) $f(x) = -5x^2 - \cos 2x - lnx + 4 - \frac{x}{3}$. /2

$f'(x) = -10x + 2\sin(2x) - \frac{1}{x} - \frac{1}{3}$

b) $g(x) = \frac{\tan x}{x^2}$. /2

$u = \tan x \quad v = x^2$

$u' = \sec^2 x \quad v' = 2x$

$\therefore g'(x) = \frac{x^2\sec^2(x) - 2x\tan x}{x^4}$

$= \frac{x\sec(x) - 2\sin x}{x^3\cos(x)}$

c) $h(x) = e^{1+2x}\,ln(1+4x)$. /2

$u = e^{1+2x} \quad v = \ln(1+4x)$

$u' = 2e^{1+2x} \quad v' = \frac{1}{1+4x} \times 4$

$\therefore h'(x) = 2e^{(1+2x)}\ln(1+4x) + e^{(1+2x)} \cdot \frac{4}{1+4x}$

Question 12

Marker use

Differentiate $y = 3x^2 - 4x + 6$ from first principles.

/4

$$dy/dx = \lim_{h \to 0} \frac{3(x+h)^2 - 4(x+h) + 6 - (3x^2 - 4x + 6)}{h}$$

$$= \lim_{h \to 0} \frac{3x^2 + 6xh + 3h^2 - 4x - 4h + 6 - 3x^2 + 4x - 6}{h}$$

$$= \lim_{h \to 0} 6x + 3h - 4$$

$$= 6x - 4$$

Question 13

Find the exact value of $g'(2)$ when $g(x) = \sqrt{x^2 + 5}$. $= (x^2 + 5)^{1/2}$

/3

$$g'(x) = \tfrac{1}{2}(x^2 + 5)^{-1/2} \cdot (2x)$$

$$= \frac{x}{\sqrt{x^2 + 5}}$$

$$\therefore g'(2) = \frac{2}{\sqrt{2^2 + 5}}$$

$$= 2/3$$

Question 14

Marker use

/3

Find the equation of the normal to the curve

$$y = \frac{x^3}{3} - x^2 - 3x + 10$$

at the point $(3, 1)$.

$dy/dx = x^2 - 2x - 3$

Sub $x = 3$:

$= (3)^2 - 2(3) - 3$

$= 9 - 6 - 3$

$= 0$

$\therefore$ Tangent is horizontal so the normal is vertical.

$\therefore$ Normal is $x = 3$.

Total P3

/16

Part 4

Marker use

- Answer **all** questions in this section.
- This section assesses **Criterion 7**.

Question 15

Determine the indefinite integral of $5 - \cos 3x$.

$\int 5 - \cos(3x)\, dx$

$= 5x - \frac{1}{3}\sin(3x) + c \quad , \quad c \in \mathbb{R}$

/2

Question 16

Find the exact value of $\int_2^4 8e^{2x} dx$

$= \left[4e^{2x}\right]_2^4$

$= 4e^8 - 4e^4$

/2

Question 17

Solve $\int_0^k (2x - 7)dx = -6$ for the constant k.

$\left[x^2 - 7x\right]_0^k = -6$

$k^2 - 7k - 0 = -6$

$k^2 - 7k + 6 = 0$

$(k-6)(k-1) = 0$

$\therefore \quad k = 1 \text{ or } k = 6$

/4

Question 18

Marker use

Find the exact area between the curve, $y = e^x + 5$, the x-axis and the lines $x = 0$ and $x = 2$.

/3

Area $= \int_0^2 e^x + 5\,dx$

$= [e^x + 5x]_0^2$

$= (e^2 + 10) - (e^0 + 0)$

$= e^2 + 10 - 1$

$= e^2 + 9$ units of area

Question 19

The function $f(x)$ has a stationary point at $(2, -28)$ and a y-intercept at 0.

The derivative of the function $f(x)$ is $f'(x) = 3x^2 + ax + b$ where a, b are real.

/5

Determine the function $f(x)$.

SP gives $f'(2) = 0$: $0 = 3(2)^2 + a(2) + b$

$-12 = 2a + b$ (1)

$f(x) = \int 3x^2 + ax + b\,dx$

$= x^3 + \frac{a}{2}x^2 + bx + c$

Sub $f(0) = 0$: $0 = 0 + 0 + 0 + c$

$\therefore c = 0$

Sub $f(2) = -28$: $-28 = 8 + 2a + 2b$

$-36 = 2a + 2b$ (2)

(1) - (2) gives $24 = -b$ $\therefore b = -24$ and $a = 6$

$\therefore f(x) = 3x^2 + 6x - 24$

Total P4

/16

Part 5

Marker use

- Answer **all** questions in this section.
- This section assesses **Criterion 8.**

Question 20

A binomial distribution has a mean of 5 and a standard deviation of 2. Determine the number of trials and the probability of success.

/3

$np = 5$ (1)

$np(1-p) = (2)^2$ (2)

Subbing (1) into (2) gives $5(1-p) = 4$

$\therefore p = \frac{1}{5}$

Subbing into (1) gives $n = 25$

Question 21

The discrete random variable, X, has a probability distribution as given in the table below:

X	-1	0	1
$P(X = x)$	0.08	0.84	0.08

Table 1

Determine the expected value, variance and standard deviation of the distribution.

/3

$E(X) = -1(0.08) + 0(0.84) + 1(0.08)$

$= 0$ OR by Symmetry

$Var(X) = (-1)^2(0.08) + 0^2(0.84) + 1^2(0.08) - (0)^2$

$= 0.16$

$SD(X) = \sqrt{0.16} = 0.4$

Question 22

Marker use

Find the approximate 95% Confidence Interval of P for a random sample (n = 24), and a sample proportion, $\hat{p}$ of 0.6.

/3

95% CI: $\left(0.6 - 2\sqrt{\frac{0.6 \times 0.4}{24}},\ 0.6 + 2\sqrt{\frac{0.6 \times 0.4}{24}}\right)$

$= \left(0.6 - 2 \times \sqrt{\frac{1}{100}},\ 0.6 + 2 \times \sqrt{\frac{1}{100}}\right)$

$= (0.4, 0.8)$

Question 23

Gemima is a talented basketball player who is successful on 80% of her 'free throws' shooting attempts. A 'free throw' is a shot from a marked line on the basketball court. She is confident that on 98% of the times that she has 2 'free throws' she will make at least 1 out of 2 'free throws'.

a) Explain why a binomial distribution is appropriate to use in this scenario.

/1

Only 2 possibilities (Success/Fail) and probability doesn't change.
The throws are independant of each other.

b) Show that her calculations are not correct.

/2

$X \sim Bi(2, 0.8)$

$Pr(X \geq 1) = 1 - Pr(X = 0)$

$= 1 - {}^{2}C_{0}\,(0.8)^{0}\,(0.2)^{2}$

$= 1 - 0.04$

$= 0.96$ OR 96% (which is not 98%)

Question 24

Marker use

A random variable X is normally distributed with a mean of 100 and a standard deviation of 10. Using approximate values for areas of the normal distribution, find values for:

a) $P(110 \leq X \leq 120)$.

$= \frac{95 - 68}{2}$

$= 13.5\%$

/1

b) $P(90 \leq X \leq 120)$.

$= 68 + \frac{95}{2}$

$= 81.5\%$

/1

c) a such that $P(X \leq a) = 0.975$.

As $97.5 = 50 + \frac{95}{2}$

$a = 120$

/2

Total P5

/16

Part 1

Marker use

- Answer **all** questions in this section.
- This section assesses **Criterion 4.**

Question 25

Given $f(x) = 2x + 5$ and $g(x) = \frac{3}{x^2} + 1$

a) Determine the function $g\{f(x)\}$.

$= \frac{3}{(2x+5)^2} + 1$ /2

b) Solve $g\{f(x)\} = 10$ for x. Give solution(s) correct to **two (2)** decimal places.

Solve $10 = \frac{3}{(2x+5)^2} + 1$ /2

$\therefore x = -2.79$ or $x = -2.21$

Question 26

The graph of the function $f: [2, 5] \to \mathbb{R}, f(x) = \ln x$ is reflected in the y-axis, translated 2 units to the left and then translated 3 units up to create a new function $g(x)$. /4

Determine the new function $g(x)$.

- $\ln(-x)$ · $[-5, -2]$
- $\ln(-(x+2))$ · $[-7, -4]$
- $\ln(-(x+2)) + 3$ · $[-7, -4]$

$g: [-7, -4] \to \mathbb{R}, g(x) = \ln(-x-2) + 3$

Question 27

Marker use

Radium is a radioactive element which decays according to the formula $M = Ae^{-kt}$ where M is the mass of radium after t days and A and k are constants.

10 g of radium was present at the start of an observation.

After 5 days there was 7.4 g of radium remaining.

a) Determine the values of A and k.

Solve $10 = Ae^{-k(0)}$ and $7.4 = Ae^{-k(5)}$ /2

$\therefore A = 10$ and $k = \frac{-\ln(0.74)}{5}$

$= 0.06022$

b) What will be the mass of radium after 25 days?

2.22g /1

c) After how many days will there be 2 g remaining?

Solve $2 = 10e^{-0.06022t}$ /2

$\therefore t = 26.73$ days (27 to the nearest day).

Question 28

Marker use

A person contracting a particular disease requires treatment with a certain drug.

The concentration C of that drug in the bloodstream t hours after taking a dose of the drug is given by $C = 0.02t + at^3$, where a is a constant.

Five (5) hours after taking the first dose, the concentration is equal to 0.075.

a) Find the value of a.

Solve $0.075 = 0.02(5) + a(5)^3$ /1

$a = -0.0002$

b) For how many hours is the drug still in the bloodstream?

Solve $0 = 0.02(t) - 0.0002t^3$ /2

$t = -10, 0, 10$

$\therefore$ 10 hours

c) Write down the relevant domain of C.

$t \in [0, 10]$ /1

Question 28 continues

Question 28 continued

Marker use

The drug is known to be ineffective when $C < 0.05$.

d) For how many hours is the first dose effective, correct to **one (1)** decimal place?

/3

Solve $0.05 = 0.02t - 0.0002t^3$

$t = 2.70, 8.38$

$\therefore$ Time effective $= 8.38 - 2.70$

$= 5.68$

$= 5.7$ hours

Total P1

/20

Part 2

Marker use

- Answer **all** questions in this section.
- This section assesses **Criterion 5**.

Question 29

The graph below is of the form $y = a\cos(n(x-b)) + c$.

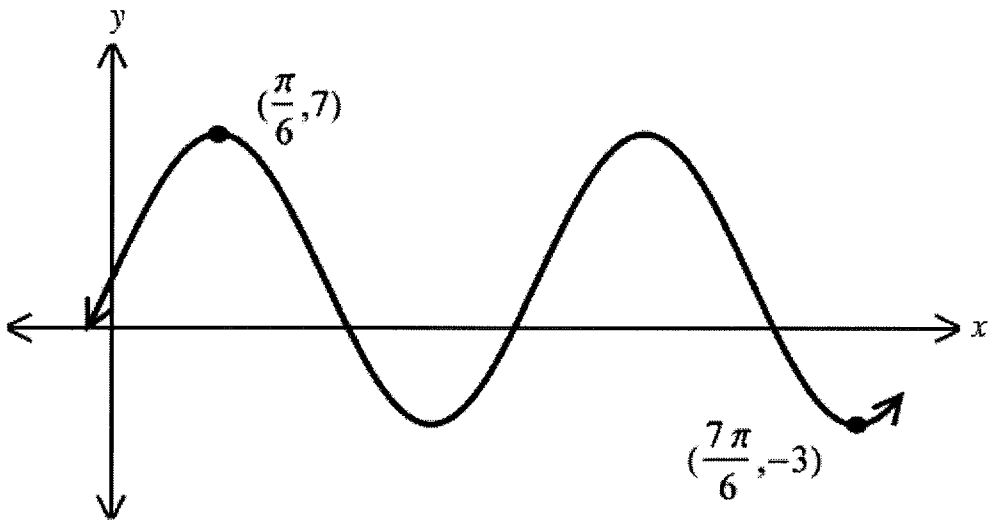


Figure 4

Determine possible values for a, b, c and n.

$|a| = \frac{7-(-3)}{2} = 5$ Translation gives $b = \frac{\pi}{6}$

$c = \frac{7+(-3)}{2} = 2$ Period $= \frac{2}{3} \times \left(\frac{7\pi}{6} - \frac{\pi}{6}\right) = \frac{2\pi}{3}$

$\therefore \frac{2\pi}{3} = \frac{2\pi}{|n|}$

$\therefore |n| = 3$

$\therefore a = 5,\ b = \frac{\pi}{6},\ c = 2$ and $n = 3$

/4

Question 30

Marker use

/4

The graph of $f(x) = \cos x$ undergoes the following transformations (in the order given):

- dilation by a factor of 2 in the y-direction $2\cos(x)$
- dilation by a factor of 4 in the x-direction $2\cos(x/4)$
- reflection in the x-axis $-2\cos(x/4)$
- translation of $\frac{\pi}{4}$ units to the right and 3 units down. $-2\cos\left(\frac{x-\pi/4}{4}\right) - 3$

Write down the equation of the transformed function $g(x)$.

$\therefore g(x) = -2\cos\left(\frac{x}{4} - \frac{\pi}{16}\right) - 3$

Question 31

/2

Determine the exact solutions of

$3\sin\left(2x + \frac{\pi}{2}\right) = \sqrt{3}\cos\left(2x + \frac{\pi}{2}\right)$ over the interval $[0, \pi]$.

$3\cos(2x + \pi/2)$ $3\cos(2x + \pi/2)$

Solve $\tan(2x + \frac{\pi}{2}) = \frac{\sqrt{3}}{3}$

$\therefore x = \frac{\pi}{3}, \frac{5\pi}{6}$

Question 32

Marker use

The Derwent River in Tasmania experiences semidiurnal tides (two (2) high and two (2) low tides each day). At a particular monitoring station in Hobart, the water depth varies between 2.4 metres at low tide and 4.8 metres at high tide.

On a particular day, high tide occurs at 3 am and 3 pm, while low tide occurs at 9 am and 9 pm. Mid tide at midnight → no horizontal translation.

a) Create a sine function $d(t)$ that models the water depth (in metres) at the monitoring station, where t represents the number of hours since midnight. /3

$a = \frac{4.8-2.4}{2} = 1.2$ Period = 12 hours

$\therefore \frac{2\pi}{n} = 12$

$c = \frac{4.8+2.4}{2} = 3.6$ $\therefore n = \pi/6$

$\therefore d(t) = 1.2\sin(\pi/6\, t) + 3.6$

OR $-1.2\sin(\frac{\pi}{6}(t-6)) + 3.6$, $-1.2\sin(\frac{\pi}{6}(t+18)) + 3.6$, $1.2\sin(\frac{\pi}{6}(t+12)) + 3.6$

b) What is the water depth at 6 am on this day? /1

$d(6) = 3.6$m

c) A small boat requires at least 3.5 metres of water depth to safely navigate this section of the river. During what hours between 10 am and 8 pm is it safe for the boat to pass through? /3

Solve $d(t) = 3.5$ for $t \in (10, 20)$

$\therefore t = 11.84, 18.16$

$\therefore$ Between 11:50am and 6:10pm (to the nearest minute).

Question 32 continues

Question 32 continued

Marker use

d) It was suggested that more accurate tide times needed to be used to ensure that the boat was safe. The new tide times were: high tides at 3 am and 3:12 pm, while low tides occur at 9:06 am and 9:18 pm. 15.2 hours /2

Determine the new sine function that models the water depth $d(t)$.

The period is now 12.2 hours $\therefore 12.2 = \frac{2\pi}{n}$

$\therefore n = \frac{10\pi}{61}$

Sub (3, 4.8) to check for translation → $b = -\frac{1}{20}$

$\therefore d(t) = 1.2\sin\left(\frac{10\pi}{61}\left(t + \frac{1}{20}\right)\right) + 3.6$

e) During what hours between 10 am and 8 pm on this particular day, is it now safe for the boat to pass through with more accurate tide times? /1

Solve $d(t) = 3.5$ for $t \in (10, 20)$

$\therefore t = 11.98, 18.41$

$\therefore$ Between 11:59am and 6:24pm

Total P2 /20

Part 3

Marker use

- Answer **all** questions in this section.
- This section assesses **Criterion 6**.

Question 33

A 12 cm by 8 cm rectangular sheet of cardboard is to be made into a box by cutting equal sized squares from each corner and folding up the four edges. Let x be the height of the box as shown in Figure 5.

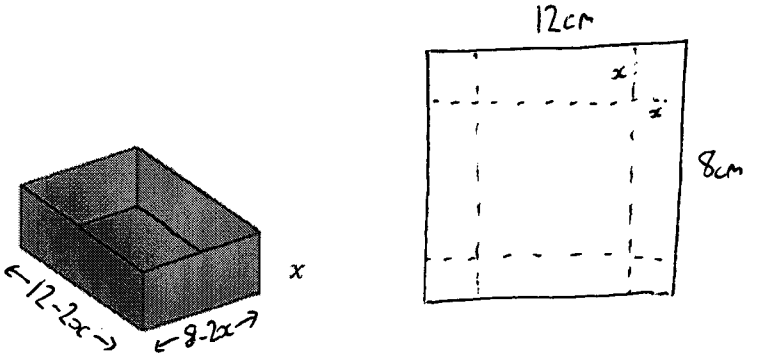


Figure 5

a) Show that the volume, $V(x)$ can be represented by $V(x) = 4x^3 - 40x^2 + 96x$.

/1

$V(x) = (12-2x)(8-2x)(x)$

$= 4x^3 - 40x^2 + 96x$

b) Using the volume formula for a cut of 7 cm, a student suggests this would result in a greater volume of 84 cm^3. Explain why this is incorrect.

/1

If $x = 7$, $12-2(7) = -2$ and $8-2(7) = -6$ which are invalid.

This is because length and width cannot be negative.

Question 33 continues

Question 33 continued

Marker use

c) Using calculus techniques determine the dimensions of the box (to **two (2)** decimal places) with the largest volume.

/4

Solve $\frac{dV}{dx} = 0$

$\therefore x = 1.569, \quad x = 5.097$ (invalid)

x	0	1.569	2	5.097	6
$\frac{dV}{dx}$	96	0	-16	0	48
Shape	/	−	\	−	/

OR $V''(x) = 24x - 80$

$V''(1.569) < 0$

$\therefore$ Max at $x = 1.57$

$\therefore$ Dimensions are $8.86 \times 4.86 \times 1.57$ cm

d) Determine the maximum volume of the box (to the nearest cm^3).

/1

$V(1.57) = 67.6$

$= 68 cm^3$

Question 34

Marker use

If $f(x) = ln(1-x)$, find the gradient at the point where $x = \frac{1}{2}$. /2

$f'(x) = -\frac{1}{1-x}$

$f'(\frac{1}{2}) = -2$

Question 35

$u = x^3 \quad v = e^{4x}$

$u' = 3x^2 \quad v' = 4e^{4x}$

a) Find the derivative of $y = x^3e^{4x}$. /1

$dy/dx = 4x^3e^{4x} + 3x^2e^{4x}$

b) Hence, find the co-ordinates of and classify the nature of the stationary points on the graph $y = x^3e^{4x}$. /4

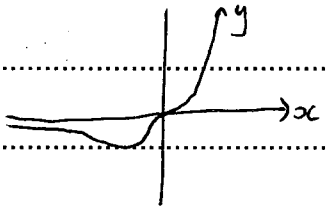
Solve $dy/dx = 0$

$\therefore x = 0$ or $x = -3/4$

$y = 0 \qquad y = \frac{-27}{64e^3}$

x	-1	$-\frac{3}{4}$	$-\frac{1}{2}$	0	1
$\frac{dy}{dx}$	$-\frac{1}{e^4}$	0	$\frac{1}{4e^2}$	0	$7e^4$
Shape	/	–	/	–	/

OR



$\therefore \left(-\frac{3}{4}, \frac{-27}{64e^3}\right)$ is a local (absolute) minimum

$(0, 0)$ is a stationary point of inflection

Question 36

Marker use

/3

Find the value of k if the function:

$f(x) = (2x-1)^2(kx+1)$ has a gradient of zero when $x = -\frac{5}{2}$.

$f'(x) = 12x^2k - 8xk + 8x + k - 4$

Solve $f'(-\frac{5}{2}) = 0$

$\therefore k = \frac{1}{4}$

Question 37

The sketch below is of the function $y = f(x)$.

Sketch the graph of the derivative function $y = f'(x)$ on the same set of axes.

/3

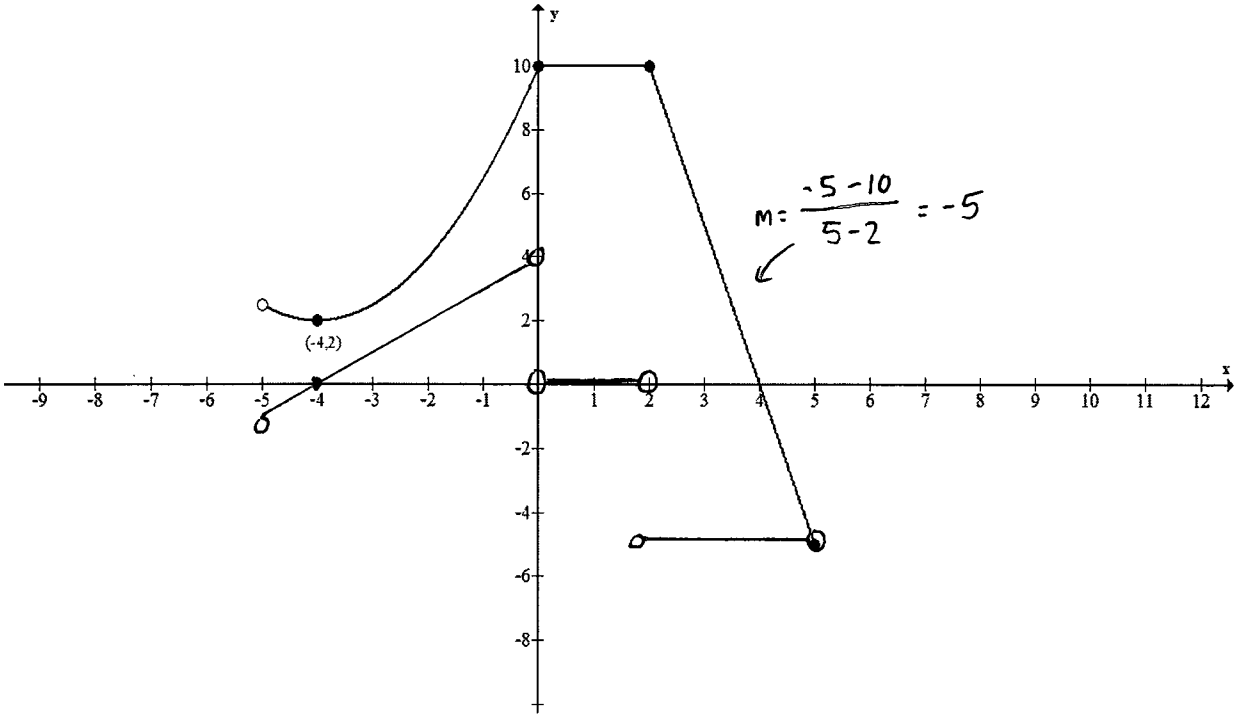


Figure 6

Spare diagram used (x) ☐

Total P3

/20

Part 4

Marker use

- Answer **all** questions in this section.
- This section assesses **Criterion 7**.

Question 38

/3

Determine $\int \left(\sin\left(\frac{x}{2}\right) - \frac{5}{x^2} + 2e^{3x}\right) dx$.

$= \int \sin\left(\frac{1}{2}x\right) - 5x^{-2} + 2e^{3x}\, dx$

$= -2\cos\left(\frac{x}{2}\right) + \frac{5}{x} + \frac{2}{3}e^{3x} + C$, $C \in \mathbb{R}$

Question 39

/3

Given that the derivative of xe^{kx} is $(kx+1)e^{kx}$, determine $\int xe^{kx}\, dx$.

$\therefore \int (kx+1)e^{kx}\, dx = xe^{kx} + C$

$\int kxe^{kx} + e^{kx}\, dx = xe^{kx} + C$

$\therefore \int kxe^{kx}\, dx = xe^{kx} + C - \int e^{kx}\, dx$

$= xe^{kx} - \frac{1}{k}e^{kx} + C$

$\therefore \int xe^{kx}\, dx = \frac{1}{k}xe^{kx} - \frac{1}{k^2}e^{kx} + C$

Question 40

/2

If $\int_3^5 (5 - 3f(x))\, dx = 15$ determine the exact value of $\int_3^5 f(x)\, dx$.

$\therefore \int_3^5 5\, dx - 3\int_3^5 f(x)\, dx = 15$

$[5x]_3^5 - 3\int_3^5 f(x)\, dx = 15$

$10 - 3\int_3^5 f(x)\, dx = 15$

$-3\int_3^5 f(x)\, dx = 5$

$\therefore \int_3^5 f(x)\, dx = -\frac{5}{3}$

Question 41

Marker use

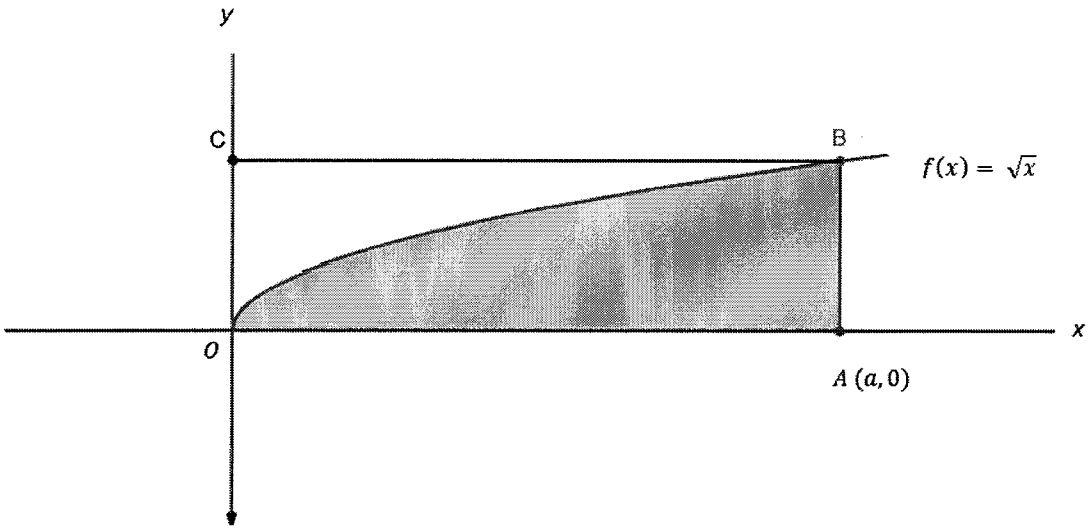


Figure 7

a) If point A has co-ordinates $(a, 0)$ state the co-ordinates of B and C in terms of a.

B = $(a, \sqrt{a})$ and C = $(0, \sqrt{a})$ /1

b) Show that the area of the shaded region is $\frac{2}{3}$ the area of the rectangle OABC. /4

$\text{Area}_{OABC} = a \times \sqrt{a} = a^{3/2}$

$\text{Area}_{SHADED} = \int_0^a \sqrt{x}\,dx$

$= \left[\frac{2}{3} x^{3/2}\right]_0^a$

$= \frac{2}{3} a^{3/2} - 0$

$= \frac{2}{3} a^{3/2}$

$= \frac{2}{3} \text{Area}_{OABC}$

Question 42

Marker use

In the domain $[-4, 4]$, the curves $f(x) = e^x - 1$ and $g(x) = 2\sin x$ intersect where $x = a$, $x = 0$ and $x = b$ as shown.

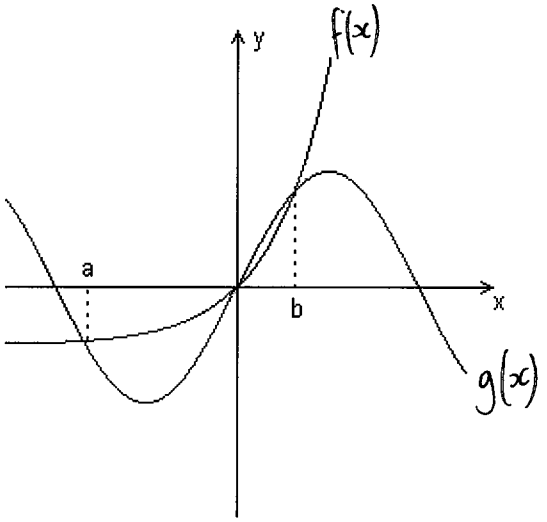


Figure 8

a) Label $f(x)$ and $g(x)$ on the graph above. /1

b) Write an expression using integrals for the area bound between the curves on the interval $[a, b]$. /2

$\int_a^0 f(x) - g(x)\,dx + \int_0^b g(x) - f(x)\,dx$

c) Determine the values of a and b correct to **three (3)** decimal places, and hence evaluate the area enclosed between the curves on the interval $[a, b]$ correct to **three (3)** decimal places. /4

Solving $f(x) = g(x)$ gives $a = -2.658$ and $b = 0.978$

Subbing values into b) and solving gives

Area = 2.244 units of area.

Total P4 /20

Part 5

Marker use

- Answer **all** questions in this section.
- This section assesses **Criterion 8.**

Question 43

The weights of free-range eggs from a particular farm in Tasmania are normally distributed with a mean of 67.5 g and a standard deviation of 2.8 g.

a) Determine the percentage of eggs that will have weights between 63.5 g and 68.5 g. /1

$X \sim N(67.5, 2.8^2)$

$Pr(63.5 < X < 68.5) = 56.29\ \%$

b) If a typical day has a total production of 9600 eggs, how many eggs are likely to have a weight greater than 61 g? /2

$Pr(X > 61) = 0.989868$

$9600 \times 0.989868 = 9502.73$

≈ 9503 eggs (or 9502)

Question 44

Marker use

The number of swimmers, x, in a particular lane at a local swimming pool at 7 am has a probability distribution given by:

x	0	1	2	3	4
$P(X = x)$	$\frac{1}{k}$	$\frac{3}{k}$	$\frac{5}{k}$	$\frac{7}{k}$	$\frac{9}{k}$

Table 2

a) Show that the value of k is 25.

$\frac{1}{k} + \frac{3}{k} + \frac{5}{k} + \frac{7}{k} + \frac{9}{k} = 1$

$\therefore \quad k = 25$

/1

b) Determine the expected number of swimmers in the lane each day.

$E(x) = 0\left(\frac{1}{25}\right) + 1\left(\frac{3}{25}\right) + 2\left(\frac{5}{25}\right) + 3\left(\frac{7}{25}\right) + 4\left(\frac{9}{25}\right)$

$= 2.8$

/1

c) Determine the variance in the number of swimmers in the lane each day.

$Var(x) = 0^2\left(\frac{1}{25}\right) + 1^2\left(\frac{3}{25}\right) + 2^2\left(\frac{5}{25}\right) + 3^2\left(\frac{7}{25}\right) + 4^2\left(\frac{9}{25}\right) - (2.8)^2$

$= 1.36 \quad \text{OR} \quad \frac{34}{25}$

/2

Question 45

Marker use

/4

400 people are randomly sampled regarding a new proposal. 220 of the people sampled were in favour of the proposal and a confidence interval of $(0.5003, 0.5997)$ was given.

Find the level of confidence that would generate this interval.

$\hat{p} = \frac{220}{400} = \frac{11}{20} = 0.55$

$\sigma_{\hat{p}} = \sqrt{\frac{0.55 \times 0.45}{400}}$

$X \sim N\left(0.55, \frac{0.55 \times 0.45}{400}\right)$

$Pr(0.5003 < X < 0.5997) = 0.9543$

$\approx 95.4\%$ Confidence Level

Alternatively, $z = 1.998$

$Pr(-1.998 < z < 1.998) = 0.9543.$

Question 46

Marker use

/5

The heights of students in a school are normally distributed.

20% of these students have heights less than 150 cm and 11% have heights greater than 180 cm.

Determine the mean and standard deviation of the heights of the students in this school.

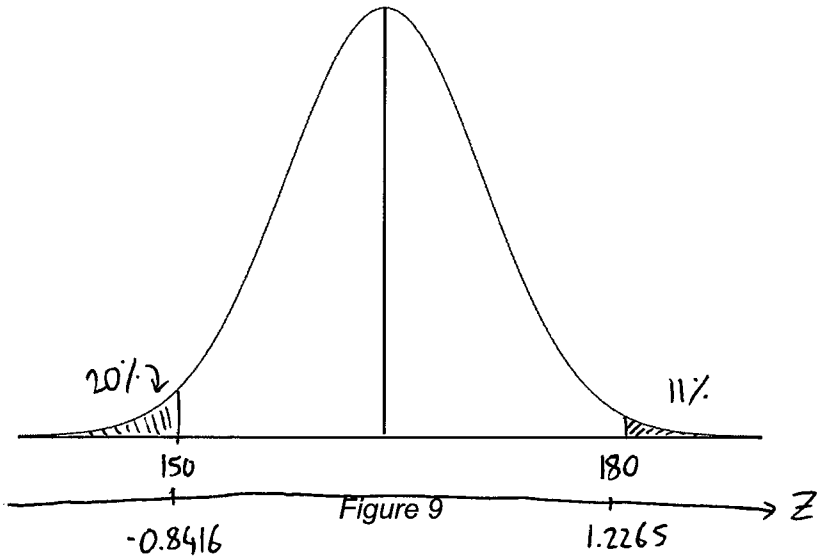


Figure 9

Spare diagram used (x) ☐

$\therefore \frac{150-\mu}{\sigma} = -0.8416$ (1)

$\frac{180-\mu}{\sigma} = 1.2265$ (2)

Solving (1) and (2) simultaneously gives:

$\mu = 162.2$ cm

$\sigma = 14.5$ cm

Question 47

Marker use

In an endurance event, the probability that a competitor will complete the course is 0.75. Teams consist of four (4) competitors. A team scores points if at least half its members complete the course.

a) Find the probability that a randomly chosen team scores points. /2

X = # of competitors in team that complete the course.

$X \sim Bi(4, 0.75)$

$Pr(X \geq 2) = 0.9492$

b) Find the probability that a randomly chosen team scores points, given that at least **one (1)** of its members fails to complete the course. /2

Total P5 /20