Mathematics Methods (MTM415117)

External Assessment Information Sheet

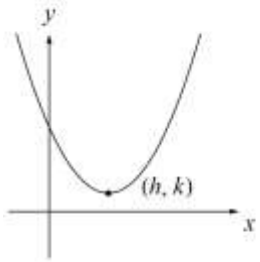
FUNCTION STUDY

Quadratic Formula: If $ax^2+bx+c=0$, then $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

Graph Shapes:

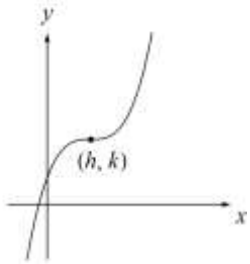
Quadratic

$y=a(x-h)^2+k$



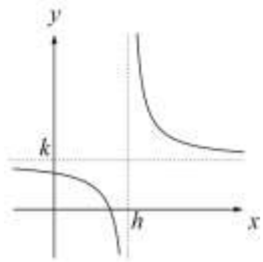
Cubic

$y=a(x-h)^3+k$



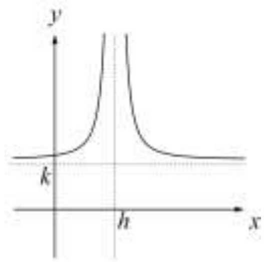
Hyperbola

$y=\dfrac{a}{x-h}+k$



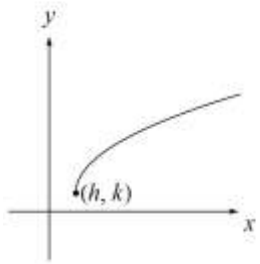
Truncus

$y=\dfrac{a}{(x-h)^2}+k$



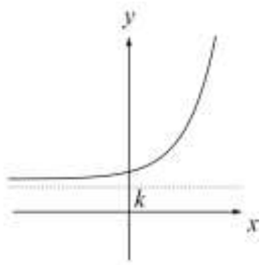
Square Root

$y=a\sqrt{x-h}+k$



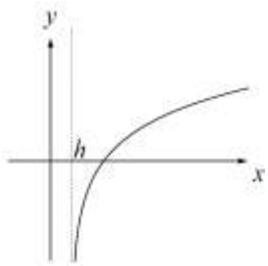
Exponential

$y=a\times b^{x-h}+k$



Logarithmic

$y=a\log_n(x-h)+k$



Graphical Transformations:

The graph of:

$y=-f(x)$ is a reflection of the graph of $y=f(x)$ in the *x* axis

$y=f(-x)$ is a reflection of the graph of $y=f(x)$ in the *y* axis

$y=af(x)$ is a dilation of the graph of $y=f(x)$ by factor *a* in the direction of the *y* axis

$y=f(ax)$ is a dilation of the graph of $y=f(x)$ by factor $\frac{1}{a}$ in the direction of the *x* axis

$y=f(x-h)$ is a translation of the graph of $y=f(x)$ by *h* units to the right

$y=f(x)+k$ is a translation of the graph of $y=f(x)$ by *k* units upwards

TASMANIAN ASSESSMENT, STANDARDS & CERTIFICATION

Index Laws

$a^x \times a^y = a^{x+y}$

$a^x \div a^y = a^{x-y}$

$\left(a^x\right)^y = a^{x \times y}$

$(a)^{\frac{1}{y}} = \sqrt[y]{a}$

$(a)^{\frac{x}{y}} = \sqrt[y]{a^x}$

Log Laws

$\log_a xy = \log_a x + \log_a y$

$\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$

$\log_a x^n = n \log_a x$

$\log_a x = \frac{\log_b x}{\log_b a}$

Useful log results

Definition: If $y = a^x$ then

$\log_a y = x$

$\log_a 1 = 0$

$\ln 1 = 0$

$\log_a a = 1$

$\ln e = 1$

Inverse Functions

$f\left\{f^{-1}(x)\right\} = f^{-1}\left\{f(x)\right\} = x$

Binomial Expansion

$(x+y)^n = {}^nC_0\, x^n + {}^nC_1\, x^{n-1} y + {}^nC_2\, x^{n-2} y^2 + \ldots + {}^nC_{n-1}\, x\, y^{n-1} + {}^nC_n\, y^n$

CIRCULAR FUNCTIONS

Conversion:

To convert from radians to degrees multiply by $\frac{180}{\pi}$

To convert from degrees to radians multiply by $\frac{\pi}{180}$

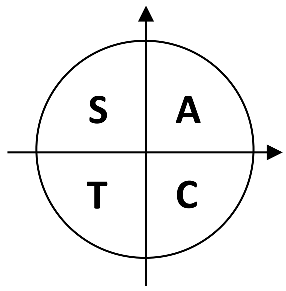
Basic Identities:

$\sin^2 x + \cos^2 x = 1$ $\quad$ $\tan x = \frac{\sin x}{\cos x}$ $\quad$ $\cot x = \frac{1}{\tan x}$ $\quad$ $\sec x = \frac{1}{\cos x}$ $\quad$ $\operatorname{cosec} x = \frac{1}{\sin x}$

Exact Values:

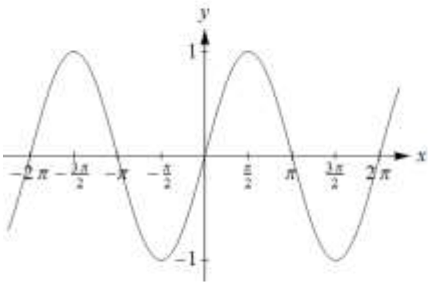
x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\sin x$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
$\cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0	1
$\tan x$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	undefined	0	undefined	0

CAST Diagram:

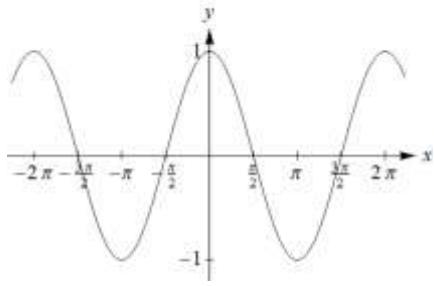


Trigonometric Graphs:

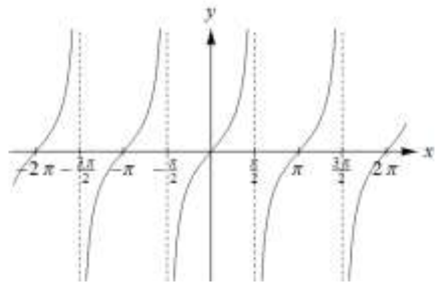
$y = \sin x$



$y = \cos x$



$y = \tan x$



Graphical Transformation:

The graph of
$y = a \sin n(x+b)+c$ or $y = a \cos n(x+b)+c$ has:

amplitude: $|a|$

period: $\frac{2\pi}{|n|}$

phase shift: *b* (shift of *b* units to the left)

vertical shift: *c* units upwards

The graph of
$y = a \tan n(x+b)+c$ has:

dilation:by factor *a* in the direction of the *y* axis

period: $\frac{\pi}{|n|}$

phase shift: *b* (shift of *b* units to the left)

vertical shift: *c* units upwards

Trigonometric Equations:

If $\sin x = a$ then $x = n\pi + (-1)^n \arcsin a$, $n \in Z$

If $\cos x = a$ then $x = 2n\pi \pm \arccos a$, $n \in Z$

If $\tan x = a$ then $x = n\pi + \arctan a$, $n \in Z$

CALCULUS

Definition of Derivative: $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

Differentiation and Integration

Differentiation Formulae	
Function	**Derivative**
x^n	nx^{n-1}
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$ or $\frac{1}{\cos^2 x}$
e^x	e^x
$\log_e x$ or $\ln x$	$\frac{1}{x}$

Integration Formulae	
Function	**Integral**
a	$ax + c$
x^n	$\frac{x^{n+1}}{n+1} + c$
$(ax+b)^n$	$\frac{(ax+b)^{n+1}}{a(n+1)} + c$
e^x	$e^x + c$
$\frac{1}{x}$	$\ln\lvert x\rvert + c$
$\sin x$	$-\cos x + c$
$\cos x$	$\sin x + c$

Differentiation Rules

	Function	Rule	Function	Rule
Product Rule	$f(x).g(x)$	$f(x).g'(x) + f'(x).g(x)$	$u.v$	$v.\frac{du}{dx} + u.\frac{dv}{dx}$
Quotient Rule	$\frac{f(x)}{g(x)}$	$\frac{g(x).f'(x) - f(x).g'(x)}{\{g(x)\}^2}$	$\frac{u}{v}$	$\frac{v.\frac{du}{dx} - u.\frac{dv}{dx}}{v^2}$
Chain Rule	$g\{f(x)\}$	$g'\{f(x)\}.f'(x)$	$y = f(u)$ and $u = g(x)$	$\frac{dy}{du}.\frac{du}{dx}$

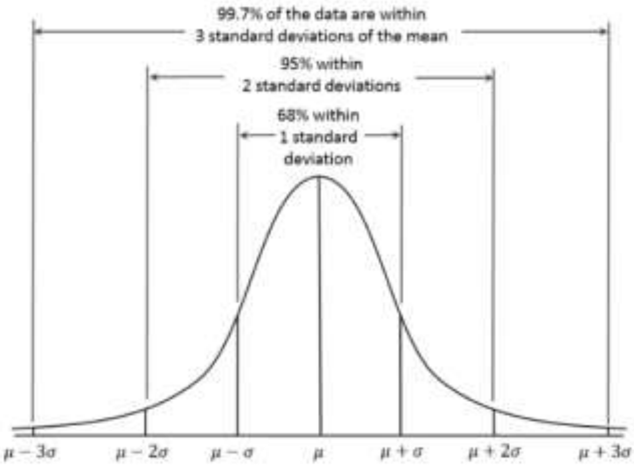
PROBABILITY DISTRIBUTIONS

Combinations: ${}^{n}C_{r} = \frac{n!}{r!(n-r)!}$ $\qquad n! = n(n-1)(n-2)\times\cdots\times 3\times 2\times 1$

	Discrete Random Distribution	Binomial Distribution
$\Pr(X=x)$	as table	$\Pr(X=x) = {}^{n}C_{x}\, p^{x}(1-p)^{n-x}$
Expected Value	$E(X) = \sum (x.\Pr(X=x))$	$\mu = np$
Variance	$\sigma^2 = E(X^2) - [E(X)]^2$	$\sigma^2 = np(1-p)$

Standard Normal: $z = \frac{x-\mu}{\sigma}$

The 68 – 95 – 99.7 approximations to the normal distribution



Sample Proportion

$\hat{p} = \frac{X}{n}$ $\qquad$ mean: $E(\hat{p}) = p$ $\qquad$ standard deviation: $\sigma_{\hat{p}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

Confidence intervals

$$\left(\hat{p} - z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}, \hat{p} + z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right)$$ for a 95% confidence interval, $z = 1.96$

Where the Margin of error (M) is:

$$M = z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$