OFFICE OF TASMANIAN ASSESSMENT, STANDARDS & CERTIFICATION

External Assessment 2021

Attach your candidate label here

MATHEMATICS METHODS

MTM415117

Part 1

Pages	24
Questions	18
Information Sheet	1

Reading time: 15 minutes – you may begin writing during this time

Suggested working time: 80 minutes

Instructions

Calculators are not allowed to be used.
Part 1 will be collected after 80 minutes.

- There are **five (5)** sections to this exam paper.
- Answer **all** questions and **all** parts within each question.
- Write your answers in the spaces provided in this exam paper.
  - Spare diagrams have been provided at the end of each section. Indicate in the box provided if you have used the spare diagram.
- During the first 80 minutes you may move onto Part 2, but you **cannot** use your calculator until told by your Supervisor(s).
- All answers must be written in **English**.
- You **must** make sure your answers address:
  - Criterion 4 understand polynomial, hyperbolic, exponential and logarithmic functions.
  - Criterion 5 understand circular functions.
  - Criterion 6 use differential calculus in the study of functions.
  - Criterion 7 use integral calculus in the study of functions.
  - Criterion 8 understand binomial and normal probability distributions and statistical inference.

Marker use	
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	16

Additional Exam Instructions

For questions worth **one (1)** mark, you are not required to show workings. Markers will look at the presentation of the answer(s) and at the argument(s) leading to the final answer(s).

For questions worth **two (2)** or more marks **you are required** to show relevant workings.

Marks will be allocated:

- according to the degree to which workings convey a logical line of reasoning
- for suitable justifications and explanations of methods and processes when requested.

Guide to Exam Structure

	Sections	Questions available	How many questions to answer	Suggested working time	Marks available
Part 1	Section A	4	4	16 minutes	16
	Section B	3	3	16 minutes	16
	Section C	4	4	16 minutes	16
	Section D	3	3	16 minutes	16
	Section E	4	4	16 minutes	16
Total		**18**	**18**	**80 minutes**	**80**
Part 2	Section A	4	4	20 minutes	20
	Section B	4	4	20 minutes	20
	Section C	5	5	20 minutes	20
	Section D	5	5	20 minutes	20
	Section E	4	4	20 minutes	20
Total		22	22	100 minutes	100
Total		**40**	**40**	**180 minutes (3 hours)**	**180**

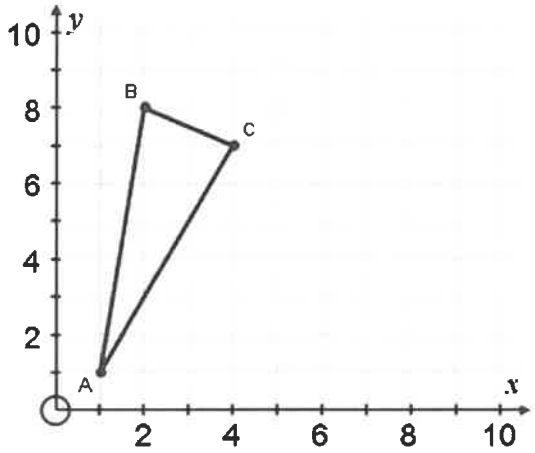
Section A

- Answer **all** questions in this section.
- This section assesses **Criterion 4**.

Question 1

Marker use

Three linear functions are sketched below over restricted domains to form the triangle ABC.



Spare diagram used (✓)

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On the same grid, draw the triangle $A'B'C'$ that results from the **inverse** of each point.

State the range of the side $\overline{A'B'}$.

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Section A continues

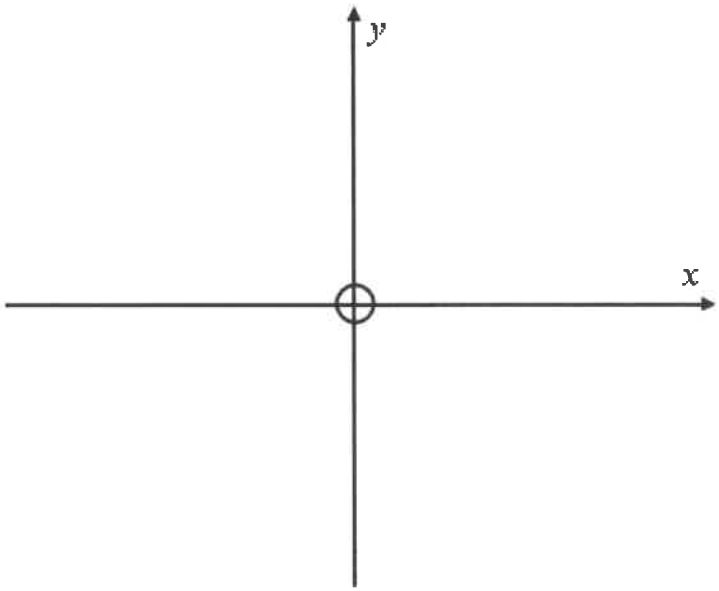
Question 2

Marker use

3

Sketch the graph of $y = \dfrac{-2}{(x-2)} - 3$ on the axes provided.

Label all axes intercepts and asymptotes.



Spare diagram used (✓)

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Section A continues

Marker use

Question 3

a) Determine the binomial expansion of $f(x)=(3x-2)^3$.

Fully simplify your solution.

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b) Hence, determine the graphical transformations from $f(x)$ to obtain $g(x)$ where:

$g(x)=-9x^3+18x^2-12x+\frac{8}{3}$.

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Section A continues

Question 4

Marker use

If $f(x)=3\ln(x-2)+5$ and $g(x)=e^{(x-3)}-1$ then the composite function $g[f(x)]$ exists for a restricted domain.

a) Show that $g[f(x)]=e^2(x-2)^3-1$.

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b) Determine the maximal domain of $g[f(x)]$.

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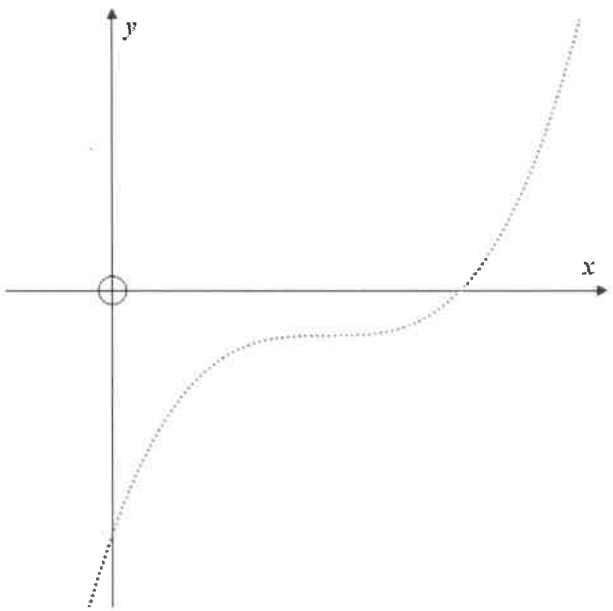
c) $y=e^2(x-2)^3-1$ is graphed below.

On the same axes sketch the composite function $g[f(x)]$.

Label any **relevant** intercepts and endpoints.

2



Spare diagram used (✓)

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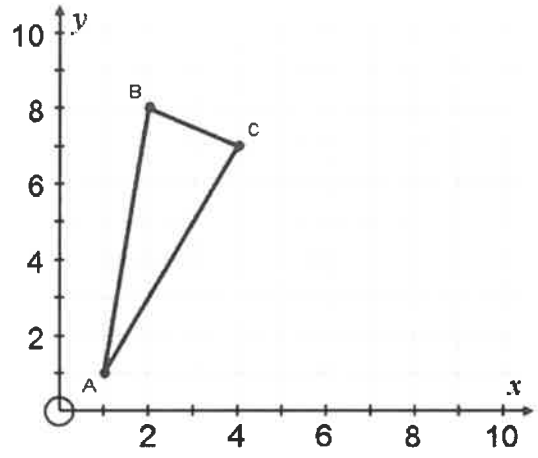
Total C4

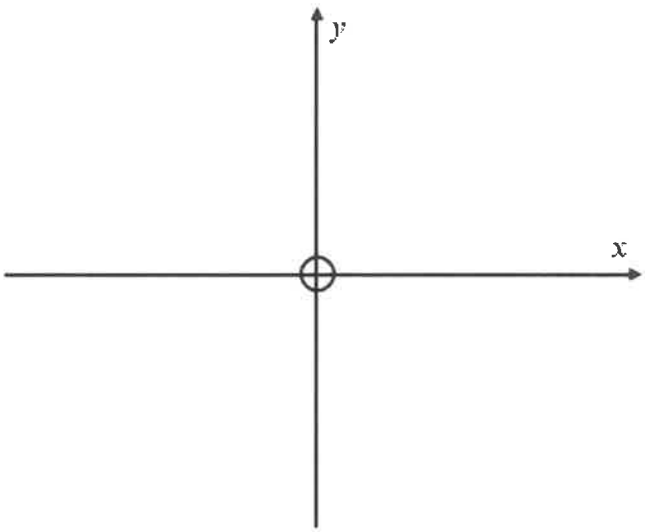
16

Spare Diagrams

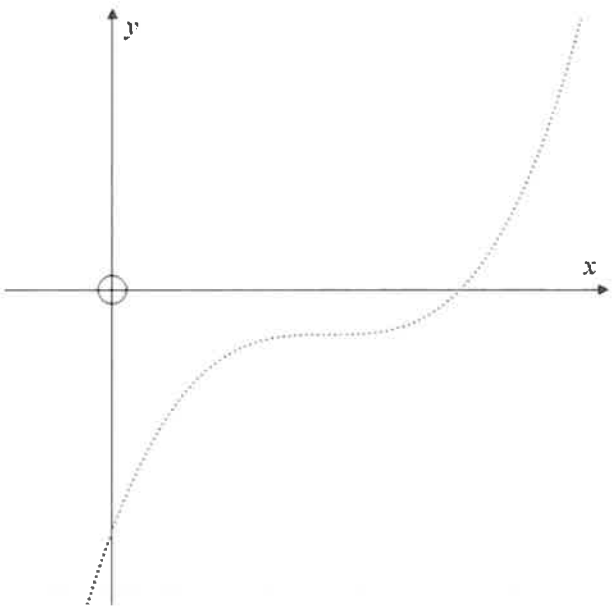
Question 1



Question 2



Question 4c



Section B

- Answer **all** questions in this section.
- This section assesses **Criterion 5**.

Question 5

Marker use

a) Find the vertical asymptotes closest to the origin when $f(x) = \tan(3x)$.

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b) Determine the range of $y = \sin(x),\ x \in \left[\frac{\pi}{4}, \frac{7\pi}{6}\right]$.

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Section B continues

Marker use

Question 6

Solve $\sin(\pi x+\pi)=\frac{\sqrt{2}}{2}$ for $x \in [-1,2]$.

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Section B continues

Question 7

Marker use

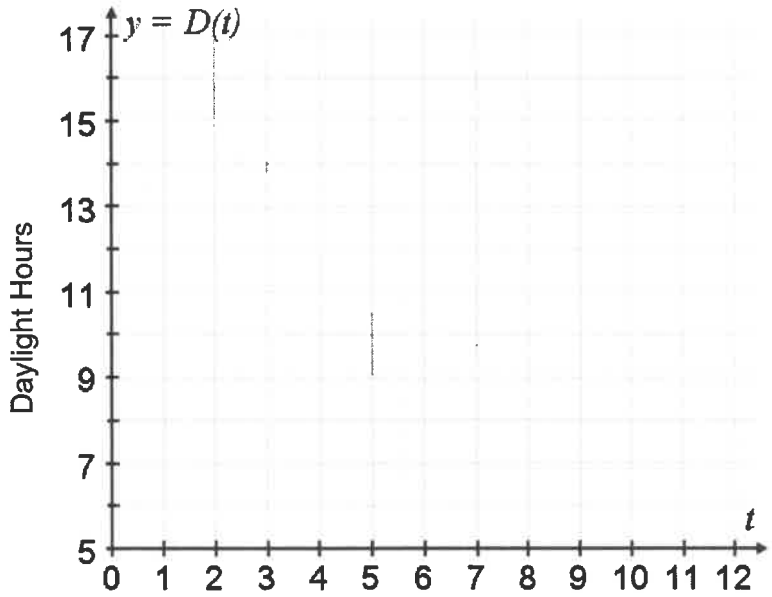
Daylight hours for Macquarie Island can be modelled by the function $D = -5\cos\left(\frac{\pi t}{6}\right) + 12$,

where t represents the months after the winter solstice which occurs at $t = 0$.

a) Complete a sketch for one period on the axes provided below.

Label the coordinates when $t = 0,\ 3,\ 6,\ 9$ and 12.

/4



Spare diagram used (✓) ☐

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b) Determine when Macquarie Island has more than $14\frac{1}{2}$ hours of daylight.

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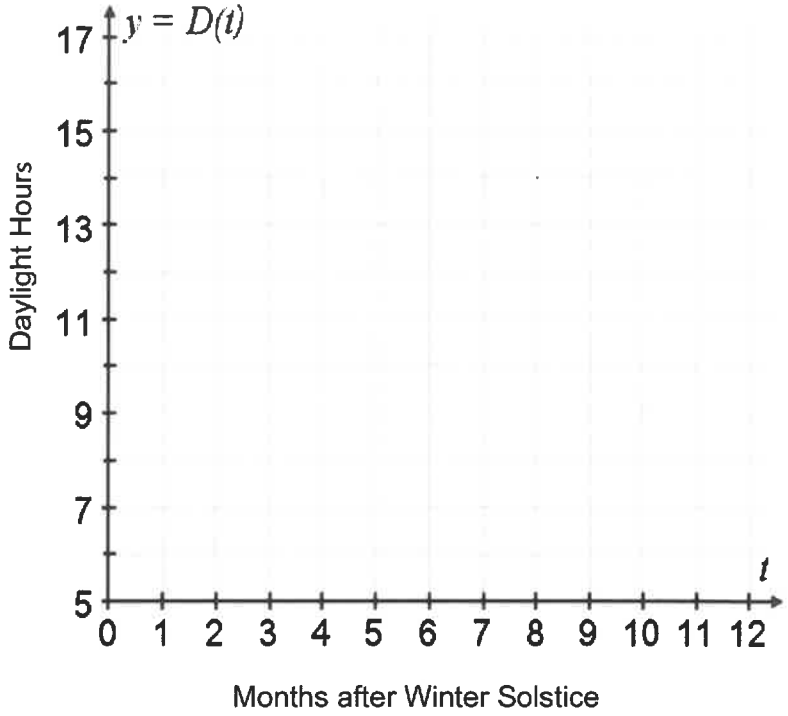
Total C5

/16

Spare Diagram

Question 7



Section C

- Answer **all** questions in this section.
- This section assesses **Criterion 6**.

Question 8

Marker use

a) Given $f(x)=\dfrac{x^2}{e^{2x}}$, use the quotient rule to determine a simplified factorised expression for $f'(x)$.

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b) Hence, determine the stationary point(s) for $f(x)$.

No need to justify the nature of stationary points.

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Section C continues

Marker use

Question 9

a) Determine $\lim_{h \to 0} \dfrac{(x+h)^2 - x^2}{h}$

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b) Explain what the limit from part a) represents.

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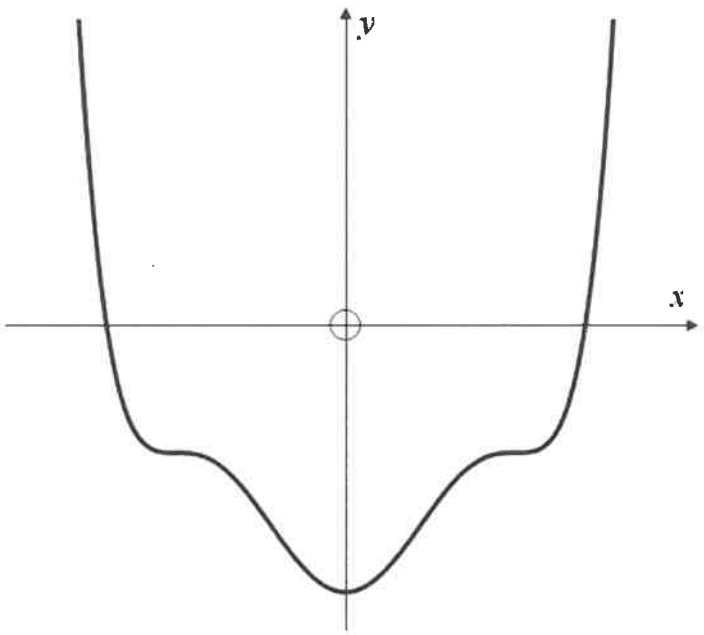
Question 10

The function below is symmetrical about the y axis, has two stationary points of inflection and a local minimum. Sketch a possible derivative function on the same axes.

3



Spare diagram used (✓)

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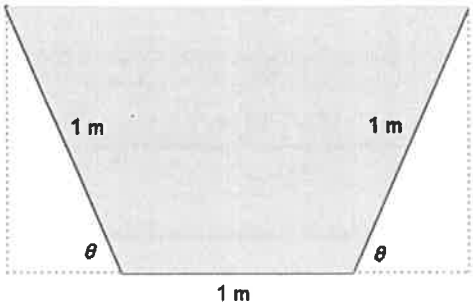
Section C continues

Marker use

Question 11

A metal wire three metres long is bent up at angles of θ so that 1 metre sections form the base and sides of a trapezoid.

The profile formed is shown below.



The **shaded** trapezoidal area is given by $A = \sin(\theta) + \cos(\theta)\sin(\theta)$.

Show, with justification, that $\theta = \frac{\pi}{3}$ maximises the area A.

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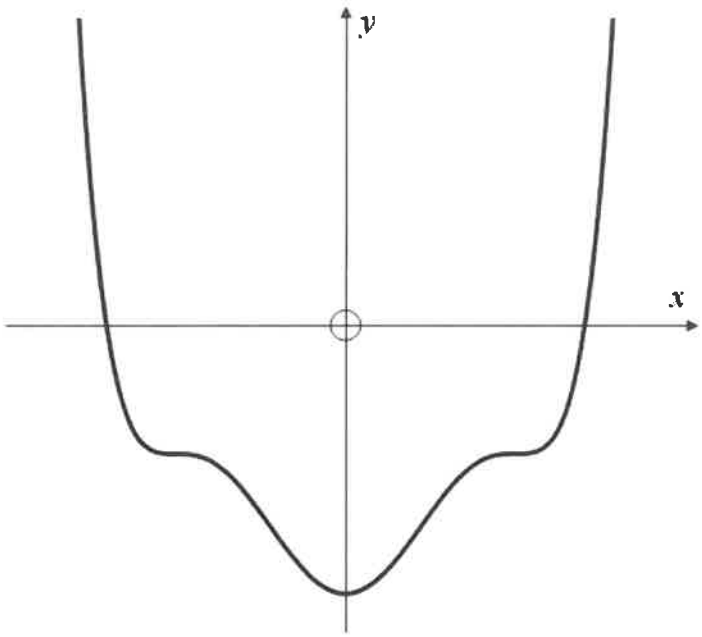
Total C6

16

Spare Diagram

Question 10



Section D

- Answer **all** questions in this section.
- This section assesses **Criterion 7**.

Question 12

Marker use

a) i. Determine $\int (2x-1)^4 dx$.

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ii. Hence, determine $\int \frac{(2x-1)^5}{(1-2x)} dx$.

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b) Given $\int_2^a \frac{1}{(x-1)} dx = 1$, determine an exact value for a, where $a > 2$.

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Section D continues

Marker use

Question 13

The temperature of a soup varies at a rate modelled by $\frac{dT}{dt} = 32e^{-0.4t}$ where T is the temperature (^{0}C) and t is the time in minutes after the temperature begins to change.

Determine an equation for T given at $t = 0$ the temperature was 20^{0}C.

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Marker use

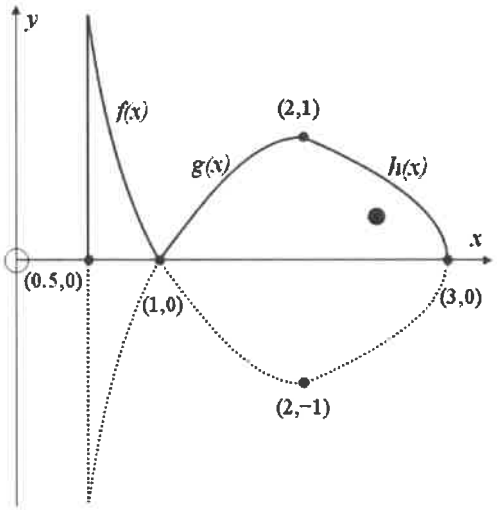
Question 14

The "fish" sketch below has an enclosed area between $x = 0.5$, $f(x)$, $g(x)$, $h(x)$ and **reflections** of these functions about the x axis where:

$f(x) = \frac{2}{x} - 2$ $\quad x \in [0.5, 1]$

$g(x) = -\cos\left(\frac{\pi x}{2}\right)$ $\quad x \in [1, 2]$

$h(x) = \sqrt{3 - x}$ $\quad x \in [2, 3]$



a) State a definite integral expression for the total area of the enclosed shape.

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b) Determine an exact value for this area.

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Total C7

16

Section E

- Answer **all** questions in this section.
- This section assesses **Criterion 8**.

Question 15

Marker use

Gracie tosses a coin until she either gets a head or completes five throws.

a) Determine the probability of the six possible events that make up the sample space. Show that the sum of these probabilities equals one.

Use H and T to denote heads and tails respectively.

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b) Determine the probability of getting a "head" in less than four throws.

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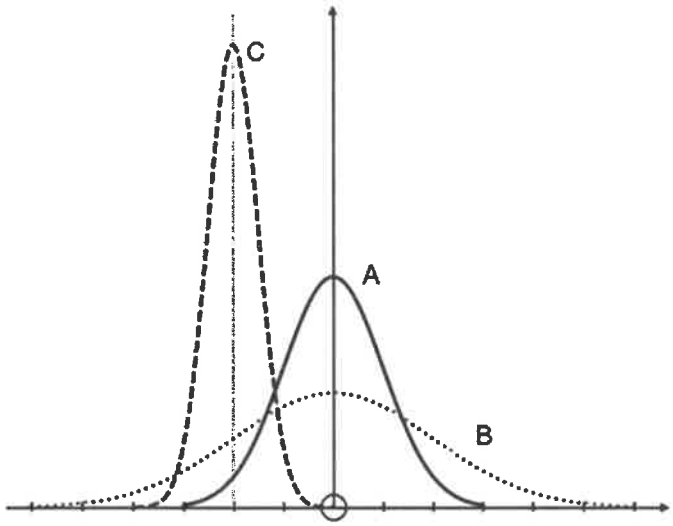
Section E continues

Marker use

Question 16

Three normal distributions, labelled A, B and C are shown in the diagram below.



Spare diagram used (✓) ☐

a) **Label** the horizontal scale with appropriate integer values, given A represents the standard normal distribution where the mean is zero and standard deviation is one.

1

b) Hence, state the means and standard deviations for the distributions B and C.

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Question 17

A distribution for $\hat{p}$ has a standard deviation of 0.02 and a mean of 0.80

Determine the sample size.

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Section E continues

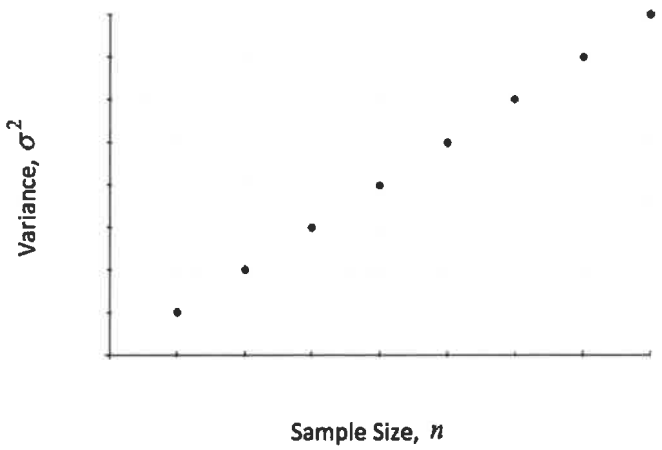
Marker use

Question 18

Variances for two binomial distributions are shown below.

a) Label the scales on both axes for each graph.

i. $X \sim Bi\left(n, \frac{1}{2}\right)$ for $n \in [1,8]$



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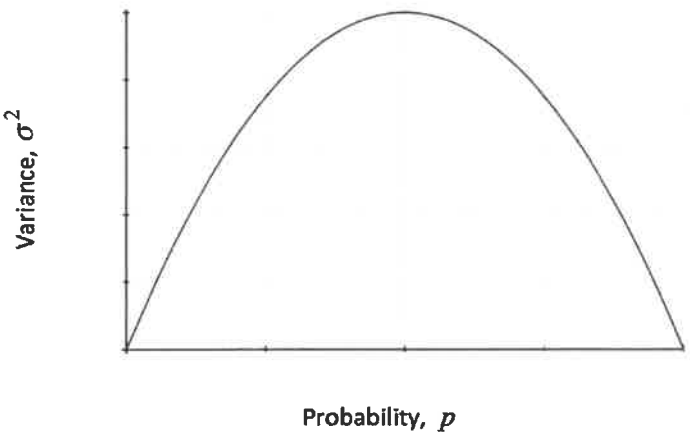
ii. $Y \sim Bi(20, p)$



2

Spare diagram used (✓) ☐

b) Explain why points are plotted for graph i. whilst graph ii. is a continuous curve.

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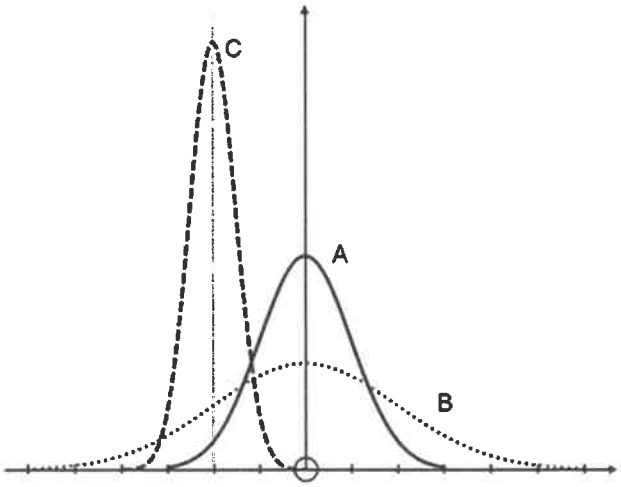
Total C8

16

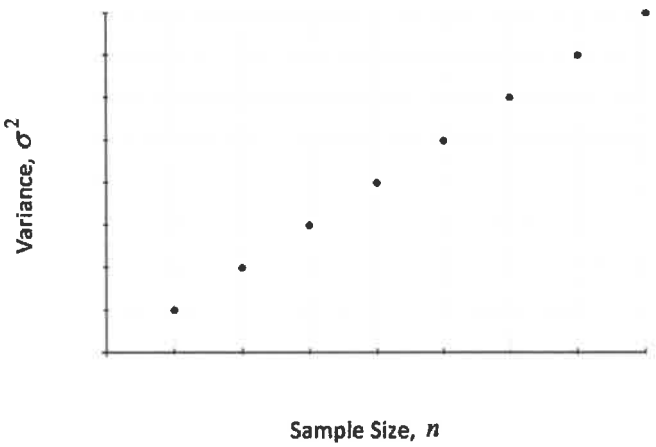
Spare Diagrams

Question 16

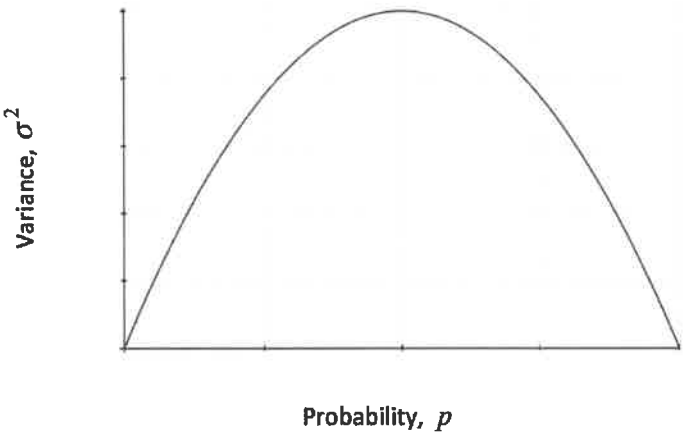


Question 18

i. $X \sim Bi\left(n, \frac{1}{2}\right)$ for $n \in [1,8]$



ii. $Y \sim Bi(20, p)$



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End of Part 1

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External Assessment 2021

MATHEMATICS METHODS

MTM415117

Part 2

Pages	28
Questions	22
Information Sheet	1

Suggested working time: 100 minutes

Instructions

Calculators are allowed to be used for Part 2.

- There are **five (5)** sections to this exam paper.
- Answer **all** questions and **all** parts within each question.
- Write your answers in the spaces provided in this exam paper.
  - Spare diagrams have been provided at the end of each section. Indicate in the box provided if you have used the spare diagram.
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Marker use	
	20
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	20
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	Section B	3	3	16 minutes	16
	Section C	4	4	16 minutes	16
	Section D	3	3	16 minutes	16
	Section E	4	4	16 minutes	16
Total		18	18	80 minutes	80
Part 2	Section A	4	4	20 minutes	20
	Section B	4	4	20 minutes	20
	Section C	5	5	20 minutes	20
	Section D	5	5	20 minutes	20
	Section E	4	4	20 minutes	20
Total		**22**	**22**	**100 minutes**	**100**
Total		**40**	**40**	**180 minutes (3 hours)**	**180**

Section A

- Answer **all** questions in this section.
- This section assesses **Criterion 4.**

Question 19

Marker use

a) Determine the equation of the inverse for $f(x) = 2\sqrt{x-1}$.

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b) Complete the table below for the domain and range of $f(x)$ and $f^{-1}(x)$.

2

	$f(x)$	$f^{-1}(x)$
Domain		
Range		

Section A continues

Question 20

Marker use

a) Apply log laws to transform $f(x)=\log_2(x-1)-\log_2(x-1)^3+\log_2 4$ into the form $f(x)=a\log_2(x-1)+k$.

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b) If $x=\log_a b$, $y=\log_b c$ and $z=\log_c a$, use the change of base theorem to show that $xyz=1$.

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Marker use

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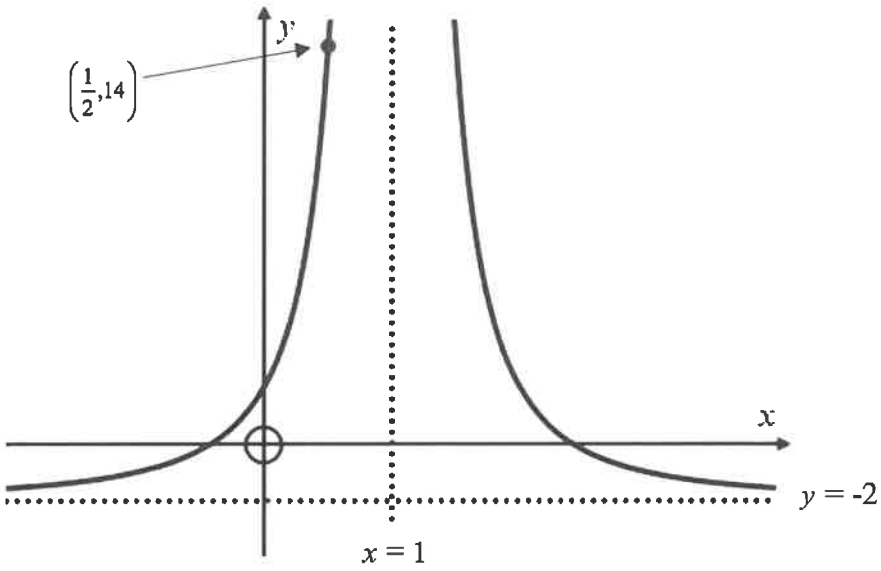
Question 21

Find and label, with exact values, the x and y intercepts of the truncus graphed below.



Spare diagram used (✓) ☐

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Section A continues

Marker use

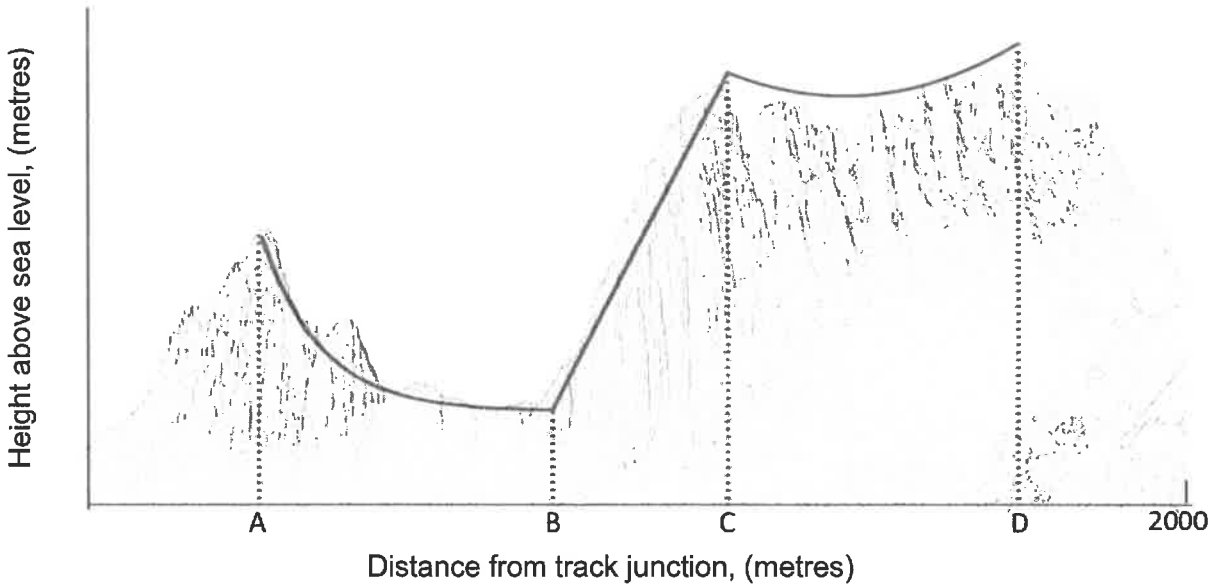
Question 22

A profile of Cradle Mountain is graphed below and modelled by the functions $f(x)$, $g(x)$ and $h(x)$ where:

$$f(x) = e^{\frac{-1}{100}(x-800)} + 1230 \quad \text{for} \quad x \in [\text{A}, \text{B}]$$

$$g(x) = \frac{289x}{300} + \frac{1381}{3} \quad \text{for} \quad x \in [\text{B}, \text{C}]$$

$$h(x) = \frac{x^2}{2000} - \frac{13x}{10} + 2345 \quad \text{for} \quad x \in [\text{C}, \text{D}]$$



The x coordinates are horizontal distances from a track junction and the y coordinates represent heights above sea level.

Units are in metres. The diagram above is not to scale.

Question 22 continues

Marker use

a) State the four graphical transformations from $y = e^x$ to $f(x) = e^{\frac{-1}{100}(x-800)} + 1230$.

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b) Smithies Peak occurs when $x = \mathrm{C}$. Cradle Peak is the highest at 1545m and occurs when. $x = \mathrm{D}$. Determine the **difference in distance** from the track junction and the **height variation** between the Smithies and Cradle Peaks.

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(State relevant equations and reasons for validating any solutions.)

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Total C4

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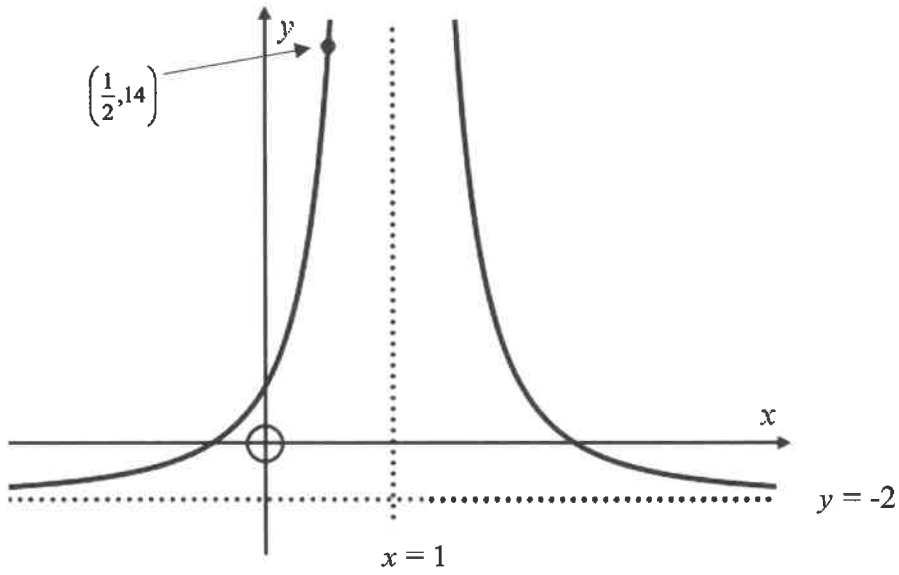
Spare Diagram

Question 21

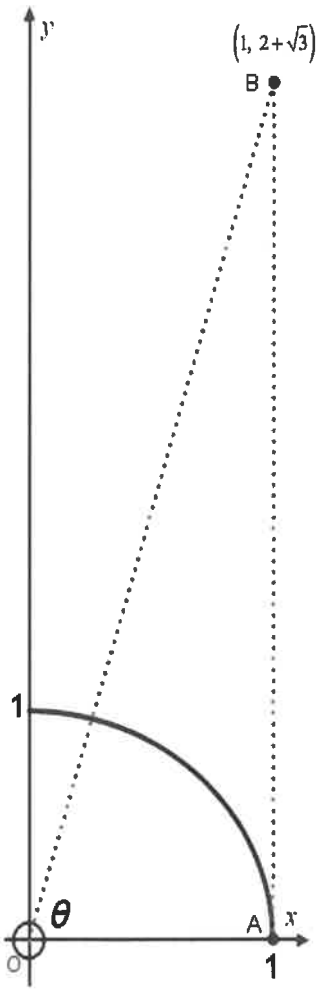


Section B

- Answer **all** questions in this section.
- This section assesses **Criterion 5**.

Question 23

Marker use



The first quadrant of a unit circle and the right-angled triangle AOB are shown to the left.

The triangle makes an angle θ at the origin and the coordinate of point B is $\left(1,\ 2+\sqrt{3}\right)$.

a) Label $\tan\theta$ on the diagram.

1

b) State and solve a trigonometric equation to determine an exact value for θ.

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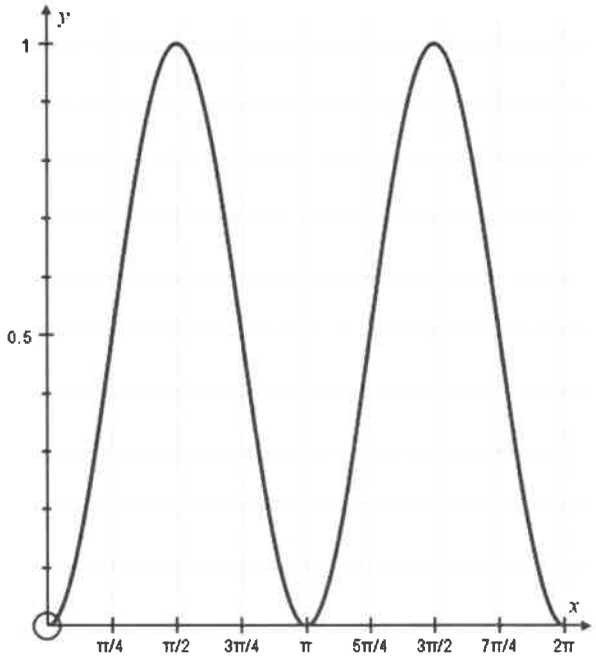
Spare diagram used (✓)

☐

Section B continues

Marker use

Question 24



For the graph shown above:

a) determine a cosine function of the form $f(x) = a\cos(bx) + c$.

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b) determine a sine function of the form $g(x) = d\sin\left[e\left(x - f\right)\right] + g$.

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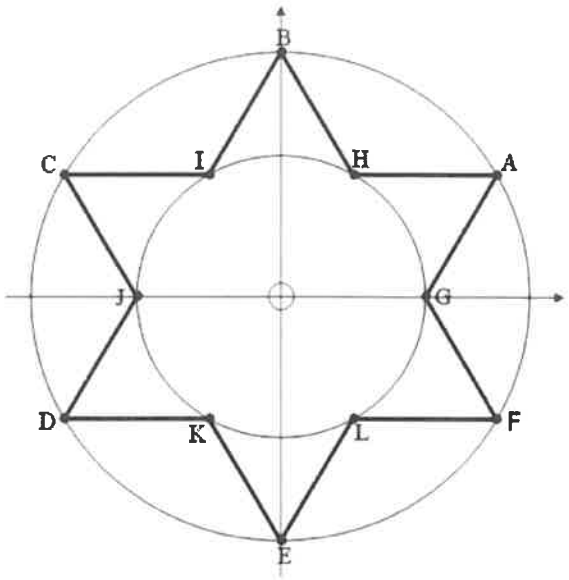
Section B continues

Marker use

Question 25

The star shown to the right has six outer points (A to F) equally spaced around a **unit circle**.



a) Find the co-ordinates of point A.

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Six inner points (G to L), evenly distanced from the closest outer points, lie on a circle with a smaller radius. The line $\overline{\text{AH}}$ is parallel to the x axis.

b) Show the inner circle radius is $\frac{\sqrt{3}}{3}$.

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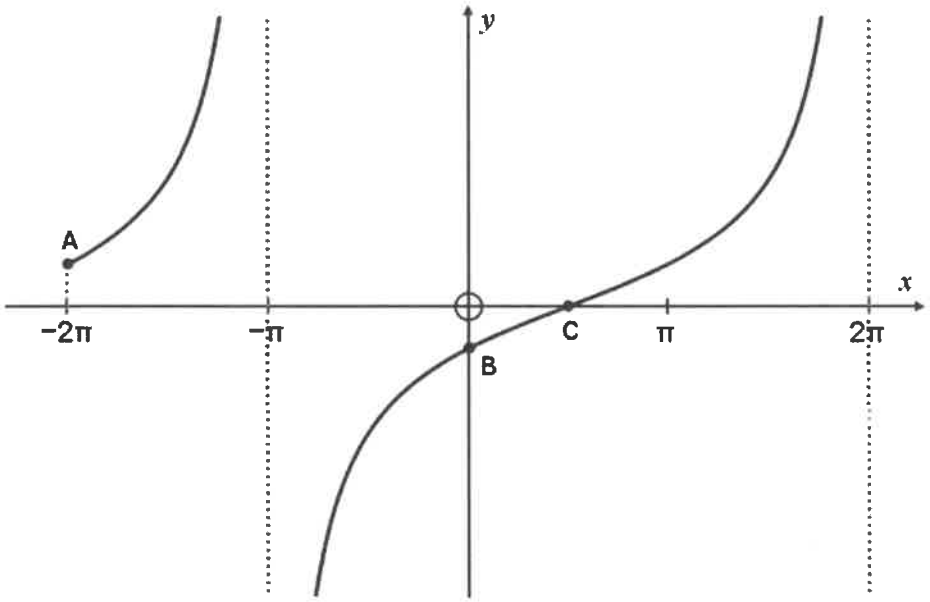
Section B continues

Marker use

Question 26

A function of the form $y = \tan n(x + b)$ is shown below with vertical asymptotes at $x = -\pi$ and 2π.



Determine the coordinates for the end point, A and the axes intercepts for B and C.

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Total C5

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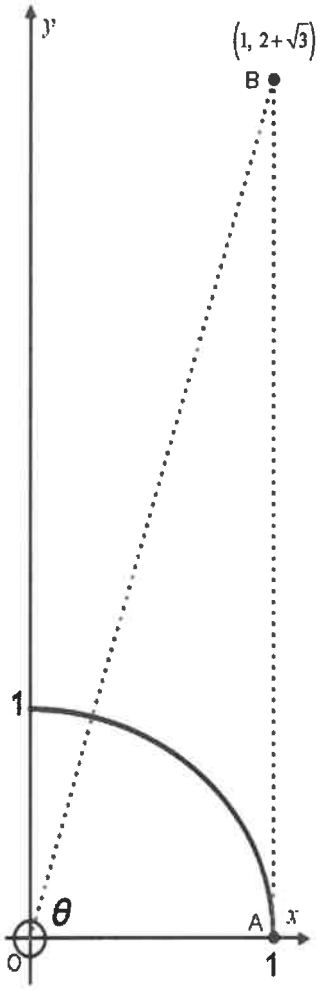
Spare Diagram

Question 23



Section C

- Answer **all** questions in this section.
- This section assesses **Criterion 6**.

Question 27

Marker use

$f(x) = 2ax^2 - bx$ where a and b are constants.

a) Determine an expression for the instantaneous rate of change at $x = 2$.

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b) Show that the average rate of change between $x = 1$ and $x = 3$ is the same as the instantaneous rate of change in part a).

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Section C continues

Marker use

Question 28

Find the x coordinate(s) where $f(x)=\ln(2x)$ and $g(x)=\dfrac{-9}{2x^2}$ have the same gradient.

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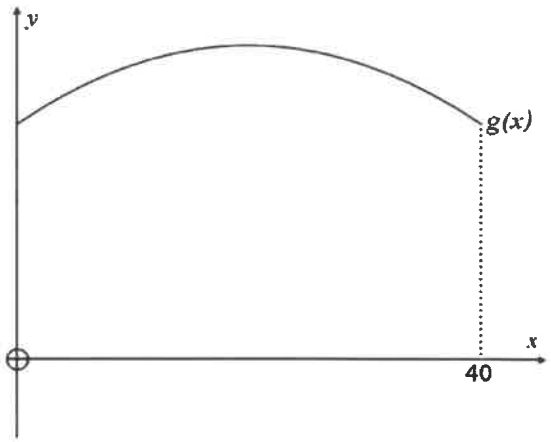
Question 29

$g(x)=\dfrac{-x^2}{400}+\dfrac{x}{10}+3$ models the walkway of a bridge that spans 40m.



Determine all values of x where the **magnitude** of the gradient is 0.06 or less.

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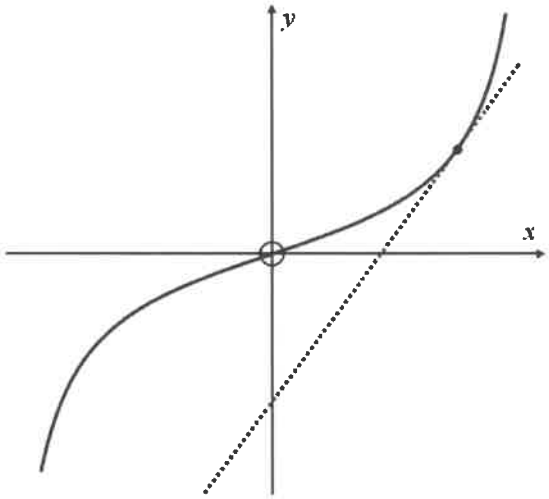
Section C continues

Marker use

Question 30

A graph of $y = \tan(2x)$ and its tangent at $x = \frac{\pi}{6}$ are shown below.



Determine the equation of the tangent.

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Marker use

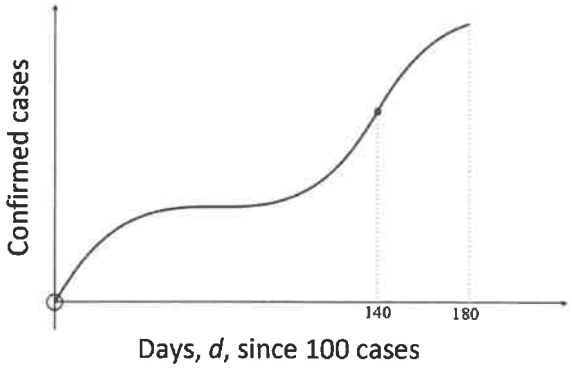
Question 31

The graph shown models the number of confirmed cases of a virus, $C(d)$, d days since the first 100 cases were recorded in a country.

The graph of $C(d)$ is a combination of the two functions $f(d)$ and $g(d)$ where:

$$f(d) = \frac{1}{50}(d-70)^3 + 6860, \quad d \in [0, 140]$$

$$g(d) = \frac{1}{50}(d-210)^3 + 20580, \quad d \in [140, 180]$$



a) Show that $C(d)$ is **differentiable** at $d = 140$

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b) Explain the meaning of $C'(140)$.

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Total C6

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Exam continues over the page

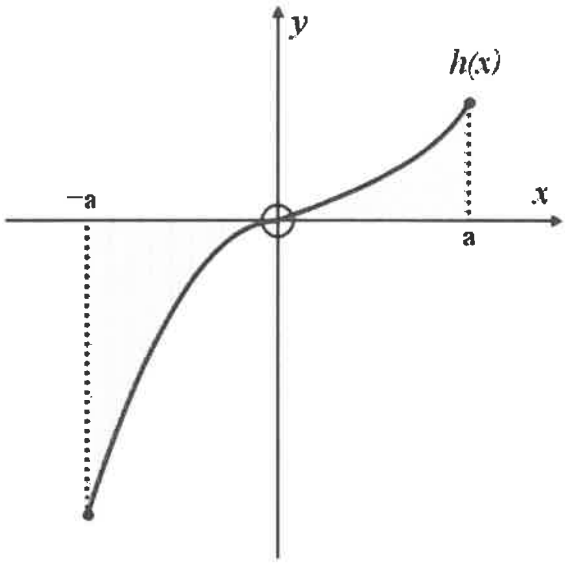
Section D

- Answer **all** questions in this section.
- This section assesses **Criterion 7**.

Question 32

Marker use

The function $h(x)$ is graphed to the right.



a) Write an expression to determine the shaded area.

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b) Explain the meaning of $\int_{-a}^{a} h(x)\,dx = -1$ in terms of the shaded area above.

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Section D continues

Marker use

Question 33

Evaluate $\int_a^b f(x)\,dx$ given:

$$\int_a^b f(x)\,dx - \int_1^4 2\,dx = \int_b^a f(x)\,dx.$$

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Question 34

An object moves from the start in a straight line with a velocity given by:

$$v = \cos\left(\frac{\pi t}{2}\right) + 4t$$

where v is in metres/second and t is in seconds.

Provide calculus reasoning to show the distance covered by the object in the first 4 seconds is 32 metres.

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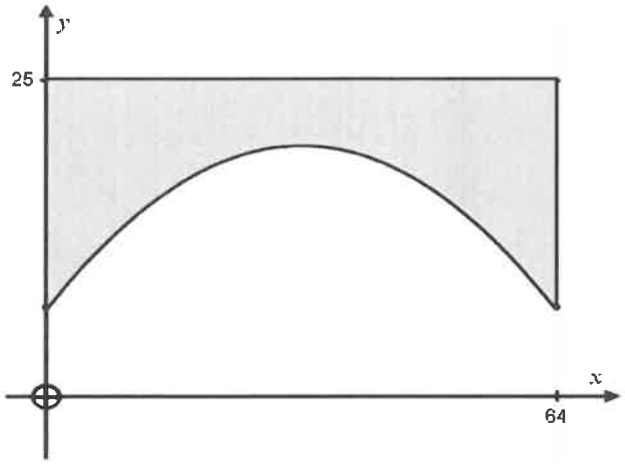
Marker use

Question 35

An arch and road surface 64 metres long are modelled by the functions:

$$f(x) = 170\sin\left(\frac{\pi}{256}(x+96)\right) - 150 \quad \text{and} \quad g(x) = 25$$

The graph below shows the cross-sectional area bounded by the arch and road.



a) Write an integral expression to find the cross-sectional area.
Determine this area accurate to one decimal place.

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b) The width of the road is 12 metres. Calculate the volume of rock required to fill the space between the arch and road surface.

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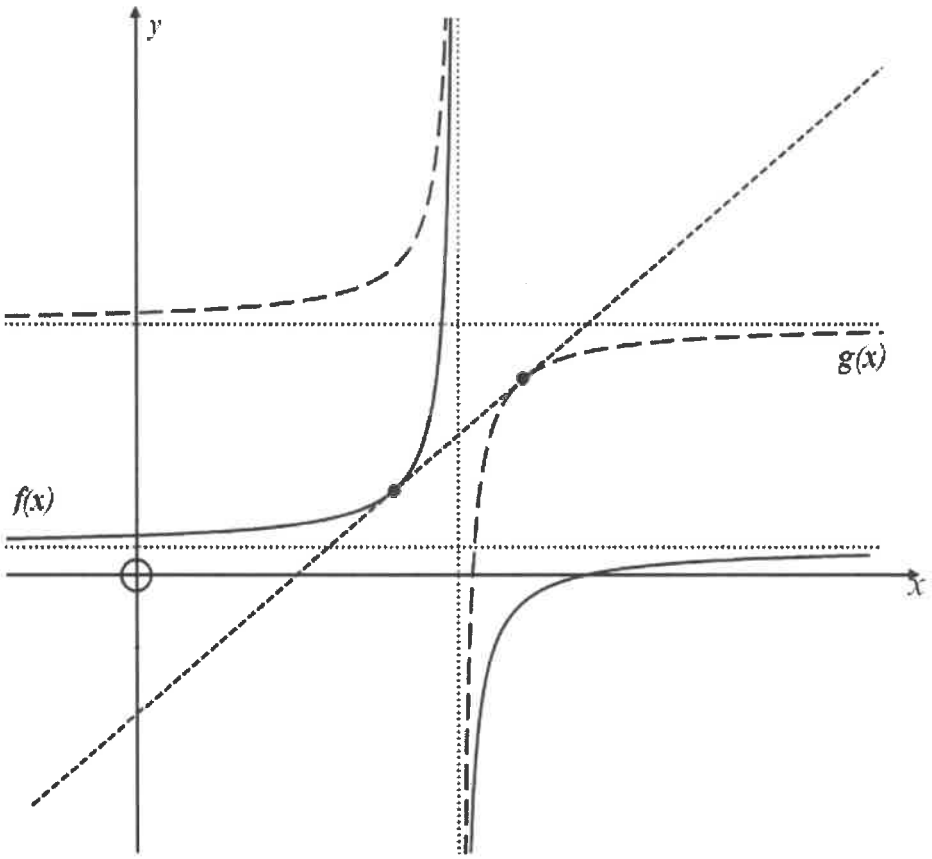
Section D continues

Marker use

Question 36

Two hyperbolas, $f(x)$ and $g(x)$ have the same gradient function equal to $\frac{4}{(2x-5)^2}$.

The line $4x-y-5=0$ is a tangent to both functions and intersects at two points as shown in the graph below.



Question 36 continues

Marker use

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Determine the equations for $f(x)$ and $g(x)$.

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Total C7

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Section E

- Answer **all** questions in this section.
- This section assesses **Criterion 8**.

Question 37

Marker use

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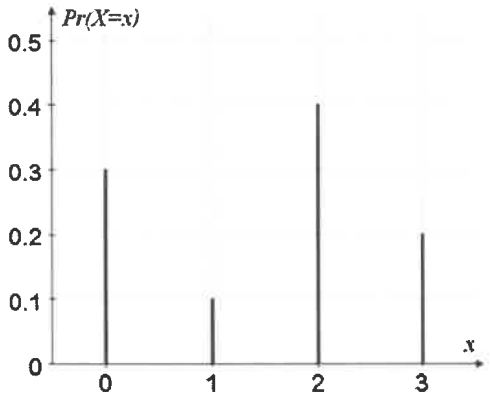
The probability distribution for the discrete random variable X is graphed below. Determine the expected value and standard deviation for this distribution.



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Question 38

Table tennis balls are required to meet a "40+" standard which sets allowable diameters between 40 and 40.2 mm.

It is thought that about 99% of a premium ball meets the "40+" requirement.

Determine the smallest sample needed to establish this to within a margin of error of 3% at a 95% confidence interval.

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Section E continues

Question 39

IQ test results are normally distributed with a mean of 100 and a standard deviation of 15.

Marker use

a) Marie Curie reportedly had an IQ of 180. Given Australia has a population of about 26 million, determine how many Australians are likely to have an IQ greater than Marie Curie.

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b) Determine the value of b, to the nearest integer, such that $\Pr(110 < x < b)$ represents approximately 15% of IQs.

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Section E continues

Question 40

Marker use

a) Given $X \sim Bi(15, p)$ and $Pr(X = 0) = 0.02$, show p is approximately equal to 0.23

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b)

i. A local restaurant seats 15 people and is fully booked for an extended period.
On average, 23% of customers order a "gluten free" option.
Determine the probability that three or more customers order "gluten free" on a given night.

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ii. The restaurant opens six nights a week. Determine the probability that three or more customers will order "gluten free" on 5 out of the 6 nights.

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Total C8

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End of Part 2

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