OFFICE OF TASMANIAN ASSESSMENT, STANDARDS & CERTIFICATION

External Assessment 2022

Attach your candidate label here

MATHEMATICS METHODS

MTM415117

Section A

Pages	24
Questions	16
Information Sheet	1

Preparation time for this exam: 15 minutes

Suggested working time: 80 minutes

Instructions:

Calculators are not allowed to be used in this section. Section A will be collected after 80 minutes.

- There are **five (5)** parts to this section.
- Answer **all** questions and **all** items within each question.
- Write your answers in the spaces provided in this exam paper.
  - Spare diagrams have been provided at the end of each part. Indicate in the box provided if you have used the spare diagrams.
- The exam is **three (3)** hours in length. It is suggested that you spend **approximately 80 minutes** in total answering the questions in this section.
- During the first 80 minutes you may move onto Section B, but you cannot use your calculator until told by your supervisor(s).
- The **Mathematics Methods Information Sheet** can be used throughout the exam.
- All answers must be written in **English**.
- You **must** make sure your answers address:
  - Criterion 4 understand polynomial, hyperbolic, exponential and logarithmic functions
  - Criterion 5 understand circular functions
  - Criterion 6 use differential calculus in the study of functions
  - Criterion 7 use integral calculus in the study of functions
  - Criterion 8 understand binomial and normal probability distributions and statistical inference.

Marker Use	
C4	/ 16
C5	/ 16
C6	/ 16
C7	/ 16
C8	/ 16

Additional Exam Instructions

For questions worth **one (1)** mark, you are not required to show workings. Markers will look at the presentation of the answer(s) and at the argument(s) leading to the final answer(s).

For questions worth **two (2)** or more marks **you are required** to show relevant workings.

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- for suitable justifications and explanations of methods and processes when requested.

Guide to Exam Structure

	Parts	Questions available	Questions to answer	Suggested working time	Marks available
Section A	Part 1	3	3	16 minutes	16
	Part 2	3	3	16 minutes	16
	Part 3	4	4	16 minutes	16
	Part 4	3	3	16 minutes	16
	Part 5	3	3	16 minutes	16
Totals		**16**	**16**	**80 minutes**	**80**
Section B	Part 1	5	5	20 minutes	20
	Part 2	4	4	20 minutes	20
	Part 3	4	4	20 minutes	20
	Part 4	5	5	20 minutes	20
	Part 5	3	3	20 minutes	20
Totals		21	21	100 minutes	100
Totals		**37**	**37**	**180 minutes (3 hours)**	**180**

Part 1

- Attempt **all** questions in this part.
- This part assesses **Criterion 4**.

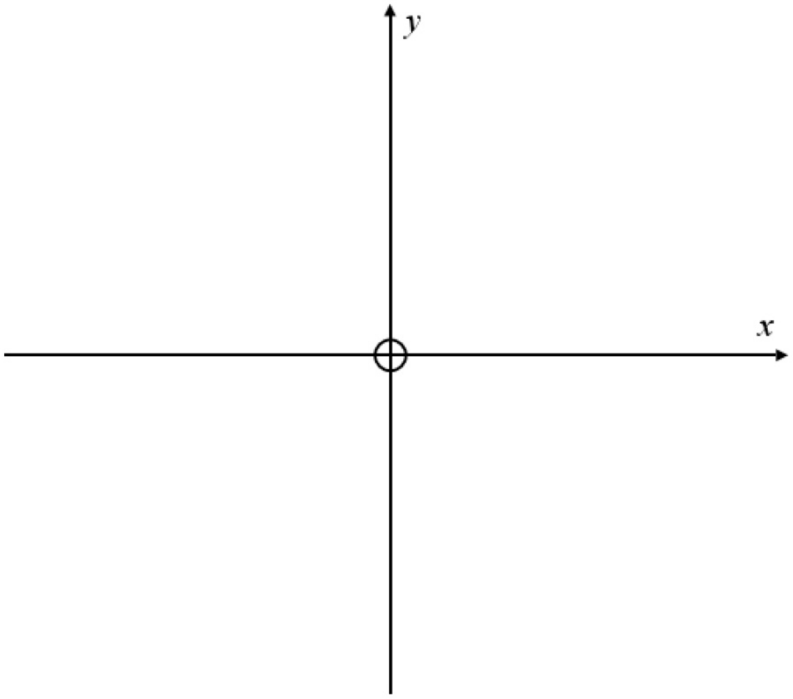
Question 1

Marker use

a) Sketch a graph of $f(x) = \dfrac{-5}{x-1} - 2$ on the axes below.

3

Label any asymptotes and axes intercepts.



Spare diagram used (✓)

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b) Hence, determine the domain where $f(x)$ is positive.

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Part 1 continues

Marker use

Question 2

A quadratic function, $f(x)$, has a turning point at $(4,1)$ and intersects the y axis at 2.

a) Determine the equation for this parabolic function in the form $f(x)=a(x-h)^2+k$.

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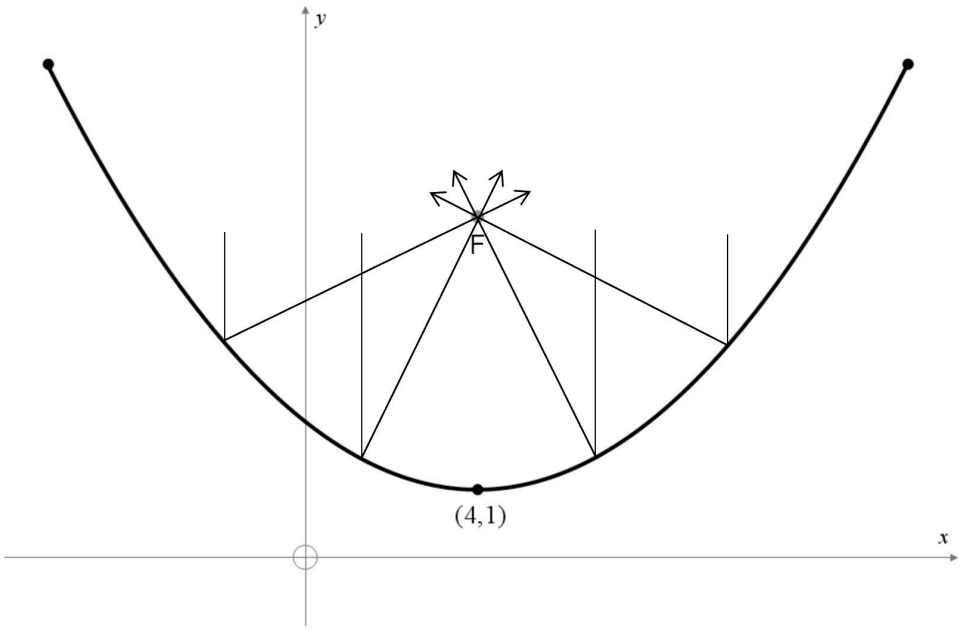
Parabolic solar devices reflect direct sunlight to a focal point.

The diagram below illustrates this focal point F, for $f(x)$ where $x\in[-6,14]$.



b) Determine the range of $f(x)$ for the sketch above.

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c) Determine the coordinate for the focal point, F which is given by $\left(h,k+\frac{1}{4a}\right)$.

1

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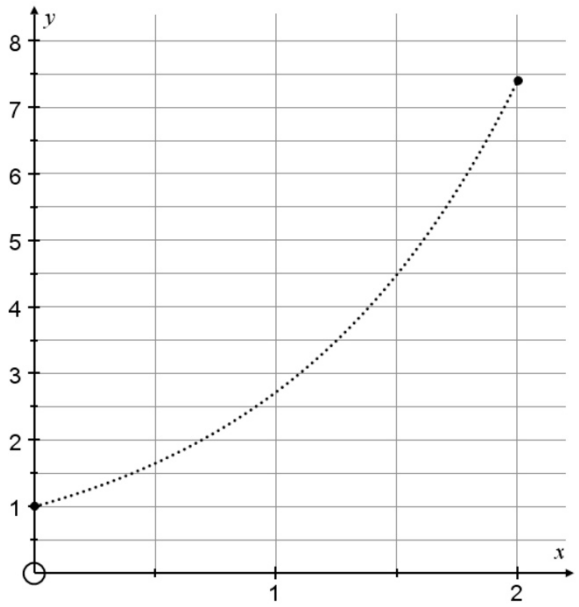
Part 1 continues

Marker use

Question 3

A graph of $y = e^x$ for $x \in [0,2]$ is shown below.



Spare diagram used (✓)

☐

a) On the same axes, **plot** points to accurately graph $y = 2^{(2x-1)}$ over the same domain.
No working required.

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b) Hence, determine an **approximate** graphical solution for $e^x = 2^{(2x-1)}$.

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c) Determine an **exact** solution for $e^x = 2^{(2x-1)}$ in terms of natural logarithms.

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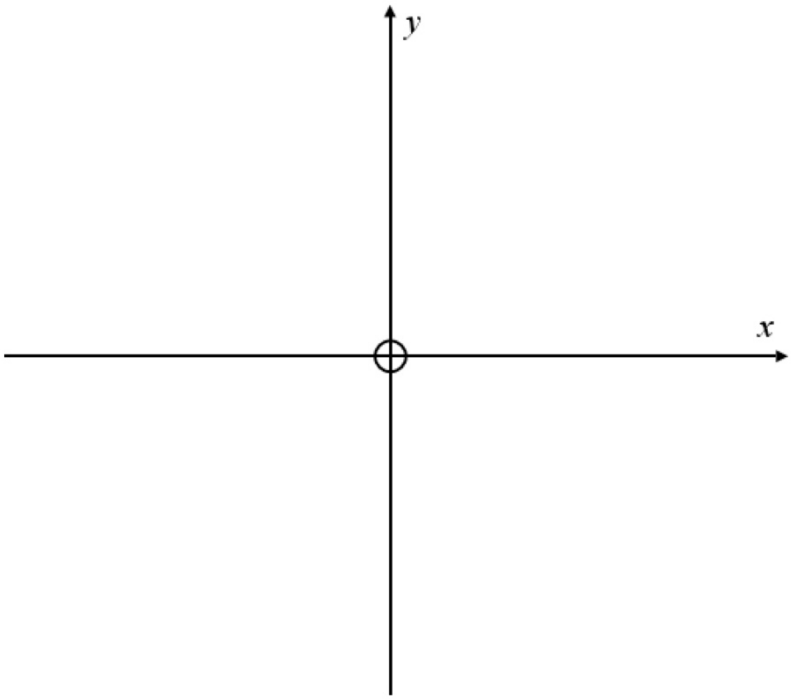
Total C4

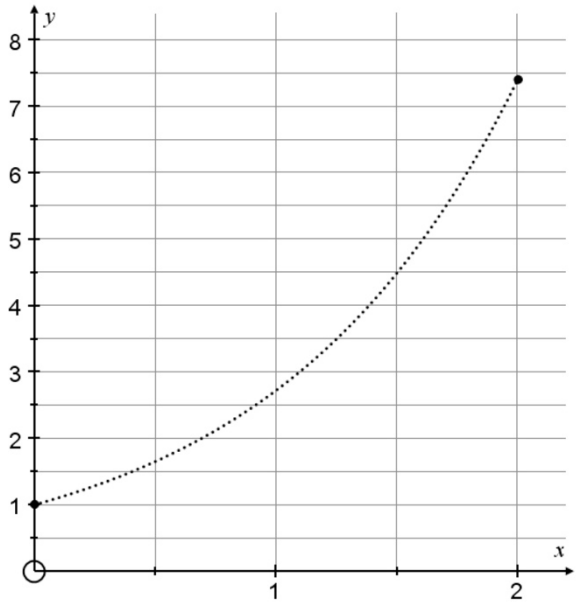
16

Spare Diagrams

Question 1



Question 3



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Exam continues over the page

Part 2

- Attempt **all** questions in this part.
- This part assesses **Criterion 5**.

Marker use

Question 4

a) Convert $\frac{8\pi}{5}$ radians into degrees. 1

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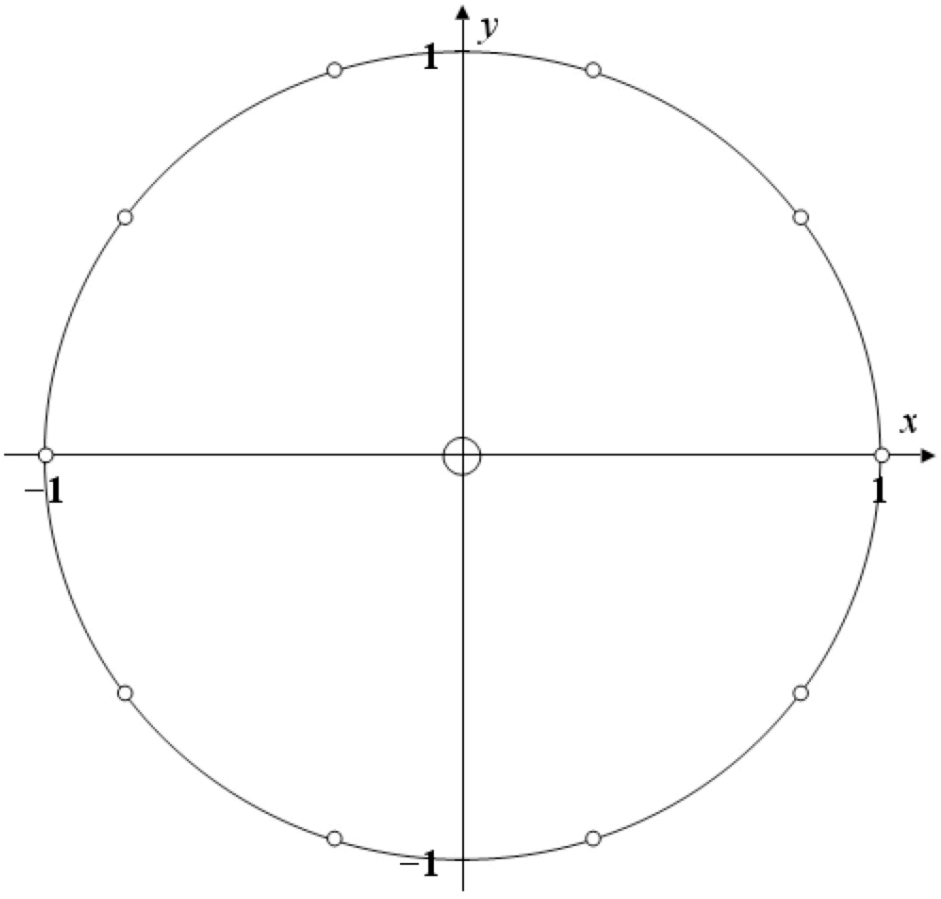
b) Accurately label $\sin\left(\frac{2\pi}{5}\right)$, $\cos\left(\frac{6\pi}{5}\right)$ and $\cos\left(\frac{8\pi}{5}\right)$ on the unit circle diagram below. 3

Dots are evenly spaced around the unit circle.



Spare diagram used (✓)

☐

Part 2 continues

Marker use

Question 5

a) Label the axes below with appropriate scales and accurately sketch the graph of $y = 2\sin(4x)$ over the domain $x \in \left[\frac{-\pi}{8}, \frac{\pi}{2}\right]$.

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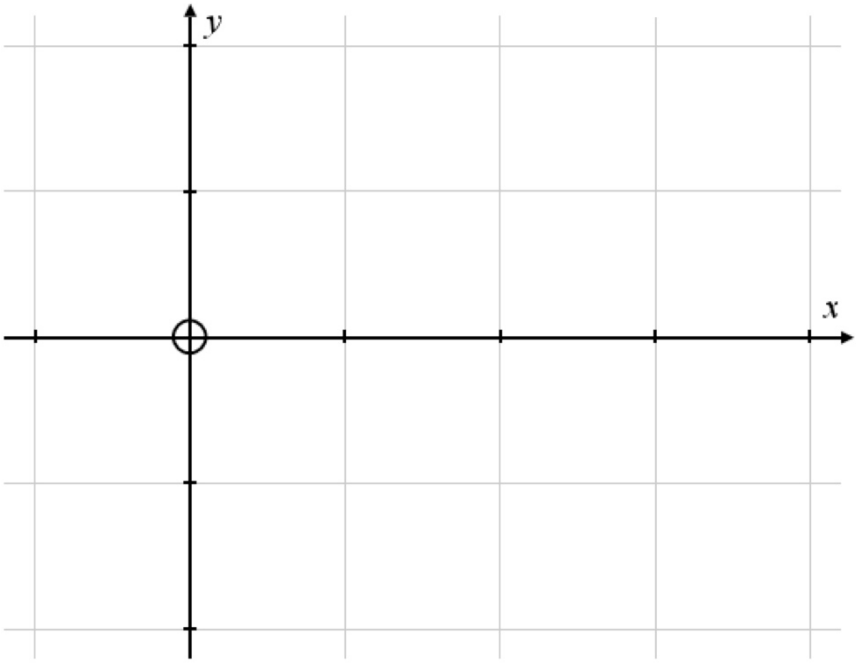

Spare diagram used (✓)

☐

b) Hence, use your graph to determine the **number** of solutions to the equation $2\sin(4x) = -\sqrt{3}$ for the domain $x \in \left[\frac{-\pi}{8}, \frac{\pi}{2}\right]$.

2

Label these solutions on the sketch.

There is no need to determine values for these solutions.

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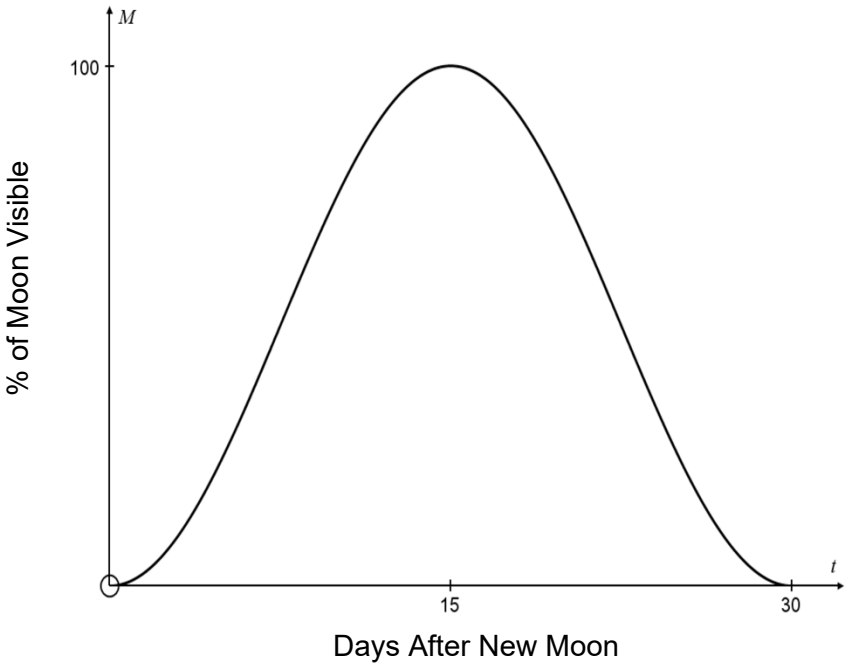
Part 2 continues

Question 6

The visibility of the moon from earth varies from 0% to 100% and back to 0% over 30 days.

The images and graph below model this variation.



a) Determine a function of the form $M = \mathrm{a}\cos\left(\mathrm{b}t\right) + \mathrm{c}$ where M represents the percentage of the moon visible, t days after a new moon. **3**

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Question 6 continues

Marker use

3

b) Hence, determine when more than 75% of the moon is visible.

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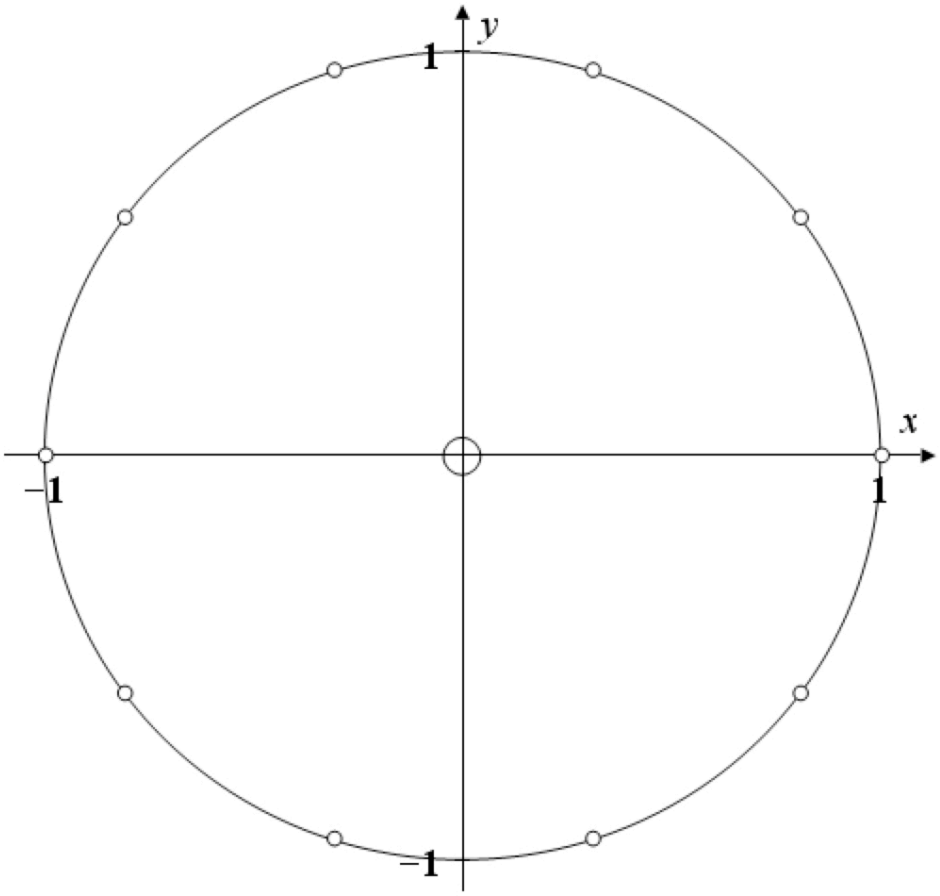
Total C5

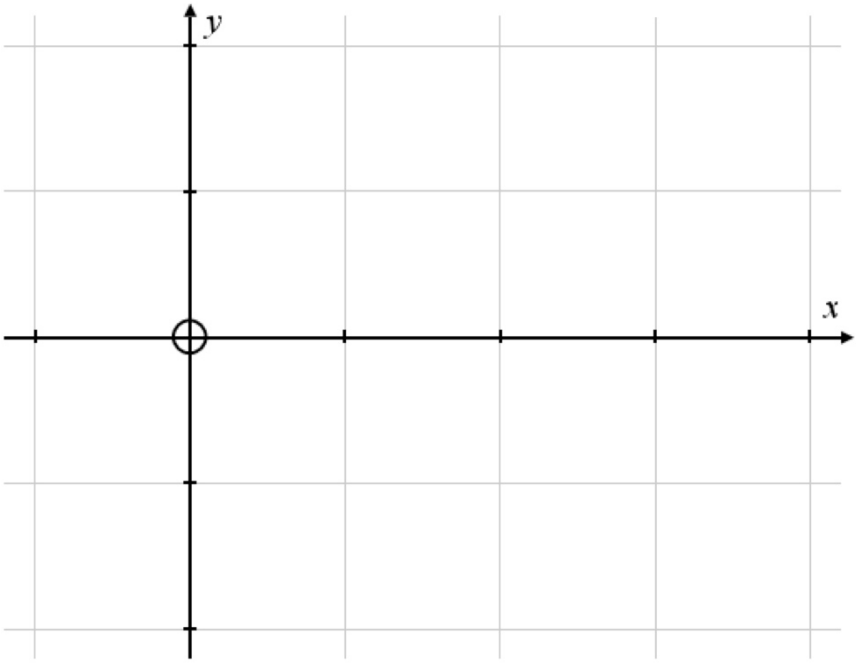
16

Spare Diagrams

Question 4



Question 5



Part 3

- Attempt **all** questions in this part.
- This part assesses **Criterion 6**.

Marker use

Question 7

a) Given $f(x)=\dfrac{\sin(x)}{x}$, use the quotient rule to show that $f'(\pi)=-\dfrac{1}{\pi}$.

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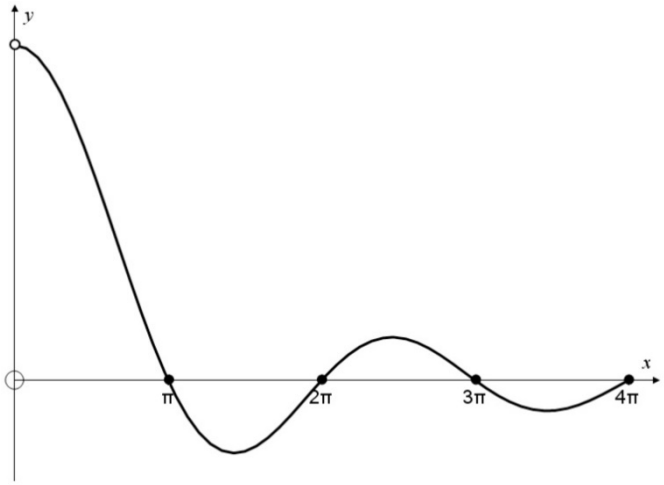
Consider the following graph and derivative values for $f(x)=\dfrac{\sin(x)}{x}$.



x	π	2π	3π	4π
$f'(x)$	$-\dfrac{1}{\pi}$	$\dfrac{1}{2\pi}$	$-\dfrac{1}{3\pi}$	$\dfrac{1}{4\pi}$

b) State a **variation**, across the tabled $f'(x)$ values, relevant to the graph having:

i. Alternating minima and maxima.

1

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ii. A decreasing amplitude.

1

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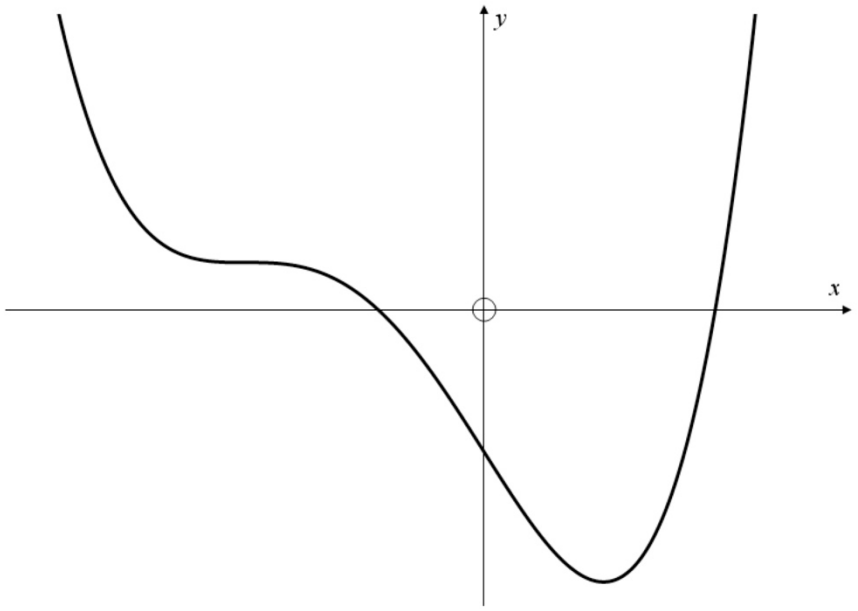
Part 3 continues

Marker use

Question 8

Sketch possible **derivative graphs** on the respective axes for the functions below.

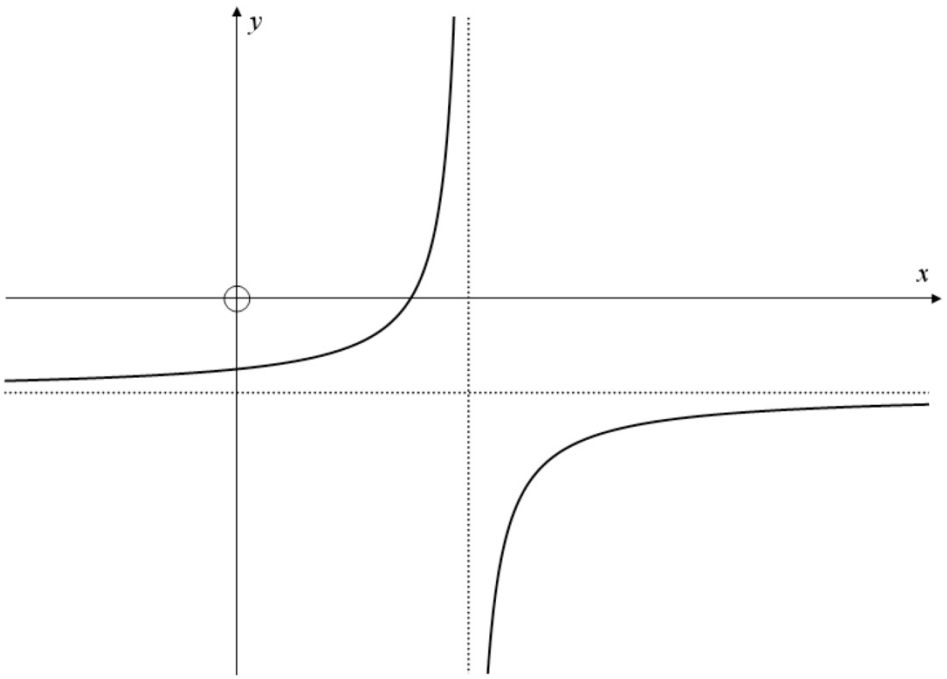
a)



2

Spare diagram used (✓) ☐

b)



2

Spare diagram used (✓) ☐

Part 3 continues

Marker use

Question 9

Determine the point where the line $ex - y + e = 0$ is a tangent to $f(x) = e^x + e$.

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Question 10

Show that $f(x) = \ln(x) - \frac{1}{x} + \frac{1}{x^2}$ has only **one** stationary point at $(1, 0)$. Justify its nature.

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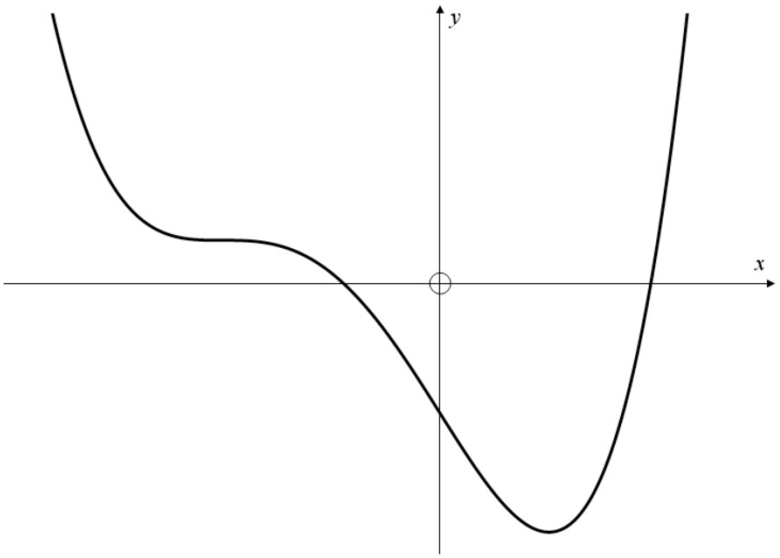
Total C6

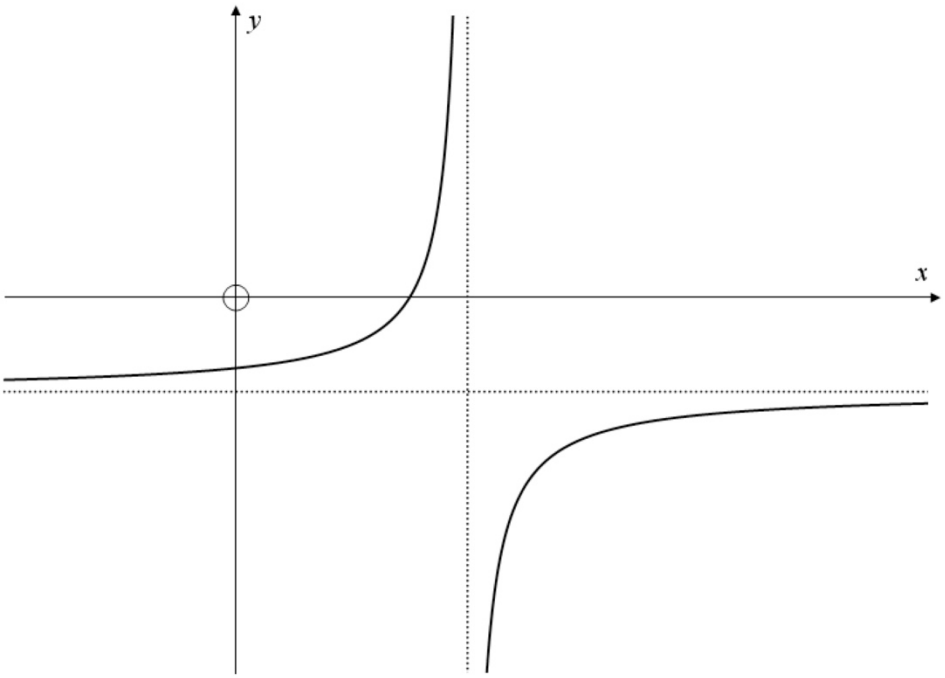
16

Spare Diagrams

Question 8 a)



Question 8 b)



Part 4

- Attempt **all** questions in this part.
- This part assesses **Criterion 7**.

Question 11

Marker use

Apply the appropriate index law(s) or basic identity before determining the following integrals.

a) $\int_1^9 \frac{1}{\sqrt{x}}\,\mathrm{d}x$.

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b) $\int \frac{e^{5x}}{\left(e^x\right)^2}\,\mathrm{d}x$.

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c) $\int_2^5 \left(3\cos^2 x + 3\sin^2 x\right)\,\mathrm{d}x$.

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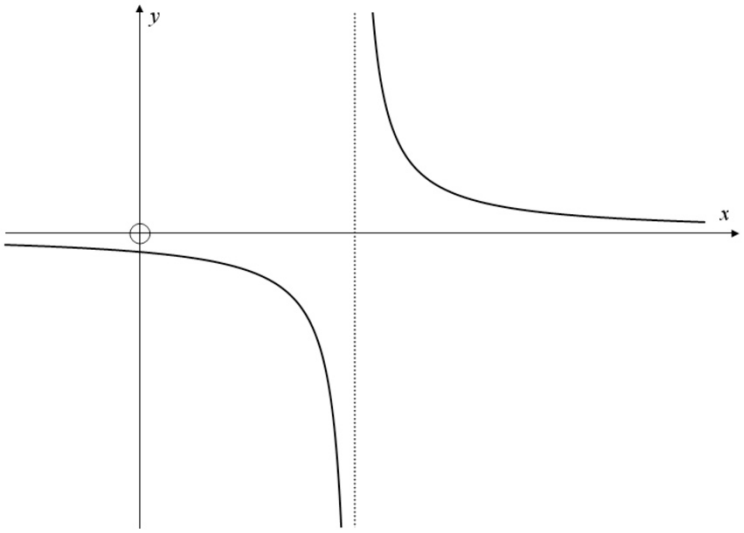
Part 4 continues

Marker use

Question 12

The function $f(x)=\dfrac{1}{x-2}$

is sketched to the right.



a) State $\int \dfrac{1}{x-2}\,dx$. **2**

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b) Circle the appropriate word to the right of the following definite intergrals to identify whether they are: *POSITIVE*, *NEGATIVE*, *ZERO* or *UNDEFINED*. **4**

No working required.

i. $\int_3^5 f(x)\,dx$ *POSITIVE* *NEGATIVE* *ZERO* *UNDEFINED*

ii. $\int_{-2}^0 f(x)\,dx$ *POSITIVE* *NEGATIVE* *ZERO* *UNDEFINED*

iii. $\int_1^3 f(x)\,dx$ *POSITIVE* *NEGATIVE* *ZERO* *UNDEFINED*

iv. $\int_0^1 f(x)\,dx+\int_3^4 f(x)\,dx$ *POSITIVE* *NEGATIVE* *ZERO* *UNDEFINED*

Part 4 continues

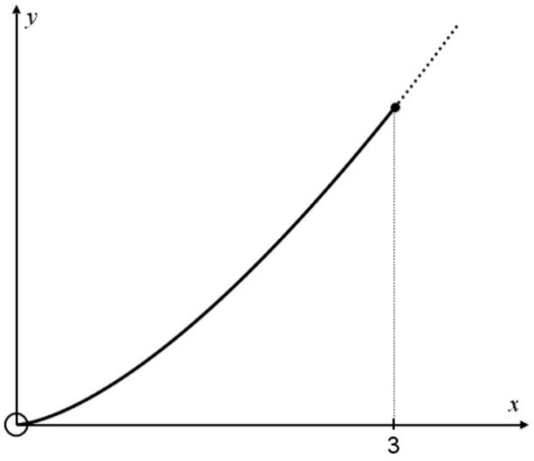
Marker use

Question 13

For a continuous function $f(x)$, the **length** of the curved section over $x \in [a,b]$, is given by:

$$\int_a^b \sqrt{1+[f'(x)]^2}\,dx.$$

The function $f(x)=\frac{2\sqrt{x^3}}{3}$ is graphed below with the curved section over $x \in [0,3]$ highlighted.



The derivative of $f(x)=\frac{2\sqrt{x^3}}{3}$ equals $\sqrt{x}$.

Determine the **length** of the curved section highlighted above.

4

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Total C7

16

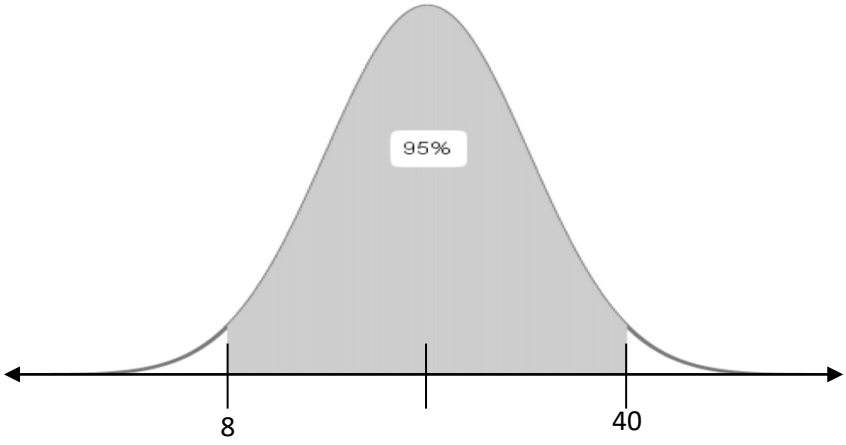
Part 5

- Attempt **all questions** in this part.
- This part assesses **Criterion 8**.

Question 14

Marker use

The shaded area is **approximately** 95% of the normal distribution below and symmetrical about the mean.



a) Determine approximations for the mean and standard deviation for this distribution.

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b) For this distribution, determine an approximation for $\Pr(X > 32)$.

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Part 5 continues

Marker use

Question 15

A partially completed table for a binomial distribution $X \sim Bi(3, p)$ is given below:

x	0	1	2	3
$\Pr(X = x)$				$\frac{1}{27}$

a) Determine p for this distribution and hence, complete all entries in the table. /4

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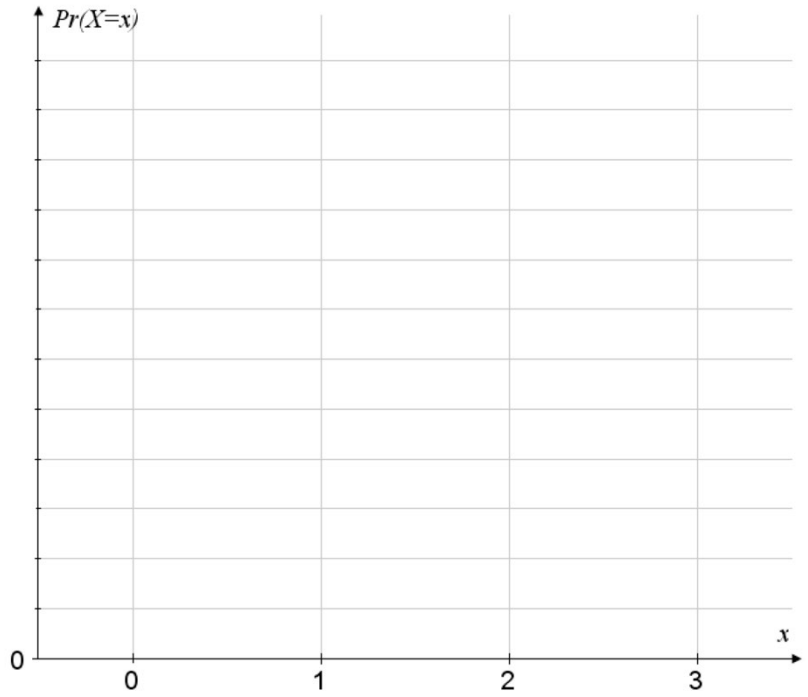
b) Choose an appropriate **fractional** scale for the y axis. Represent this distribution on the diagram below. /2



Spare diagram used (✓) ☐

c) Determine $\Pr(0 < X < 3)$ for this distribution. /1

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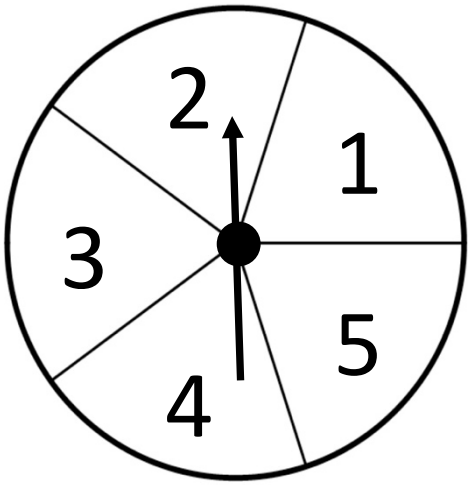
Part 5 continues

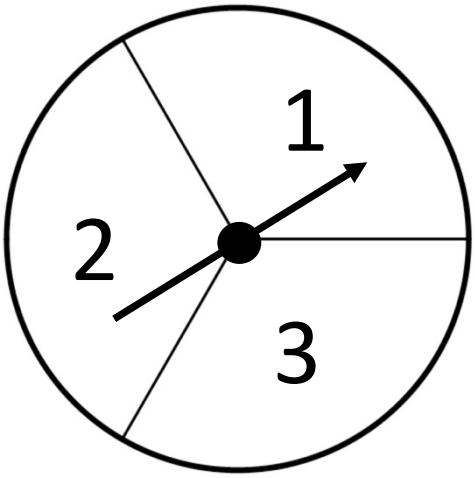
Marker use

Question 16

The diagrams below represent two different "**spinners**" to be used in a game of chance.

All sectors within each spinner are the same size.





In a game, a player spins the arrows which point to a numbered sector when stopped.

If the numbers are **different**, the player **loses** 1 token.

If the numbers are the **same**, the player **gains** 3 tokens.

a) Calculate the expected value to show this game results in a net loss of tokens.

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b) A player starts with 15 tokens. Calculate the likely number of games before all tokens are lost.

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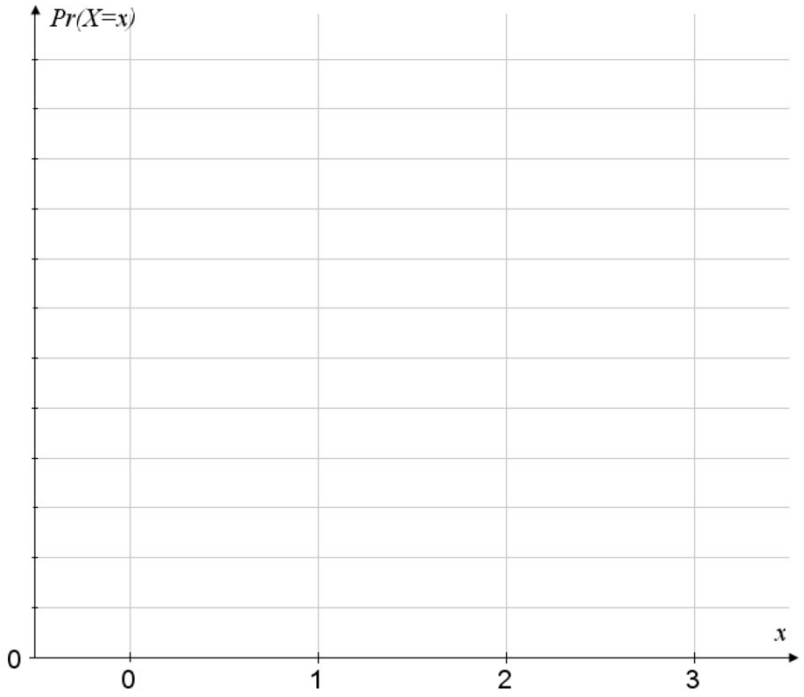
Total C8

/16

Spare Diagram

Question 15 b)



End of Section A

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(including answer booklets, cover sheets, rough note paper, or information sheets)
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OFFICE OF TASMANIAN ASSESSMENT, STANDARDS & CERTIFICATION

External Assessment 2022

Attach your candidate label here

MATHEMATICS METHODS

MTM415117

Section B

Pages	28
Questions	21
Information Sheet	1

Suggested working time: 100 minutes

Instructions:

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Marker Use	
C4	/ 20
C5	/ 20
C6	/ 20
C7	/ 20
C8	/ 20

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Totals		16	16	80 minutes	80
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	Part 3	4	4	20 minutes	20
	Part 4	5	5	20 minutes	20
	Part 5	3	3	20 minutes	20
Totals		**21**	**21**	**100 minutes**	**100**
Totals		**37**	**37**	**180 minutes (3 hours)**	**180**

Part 1

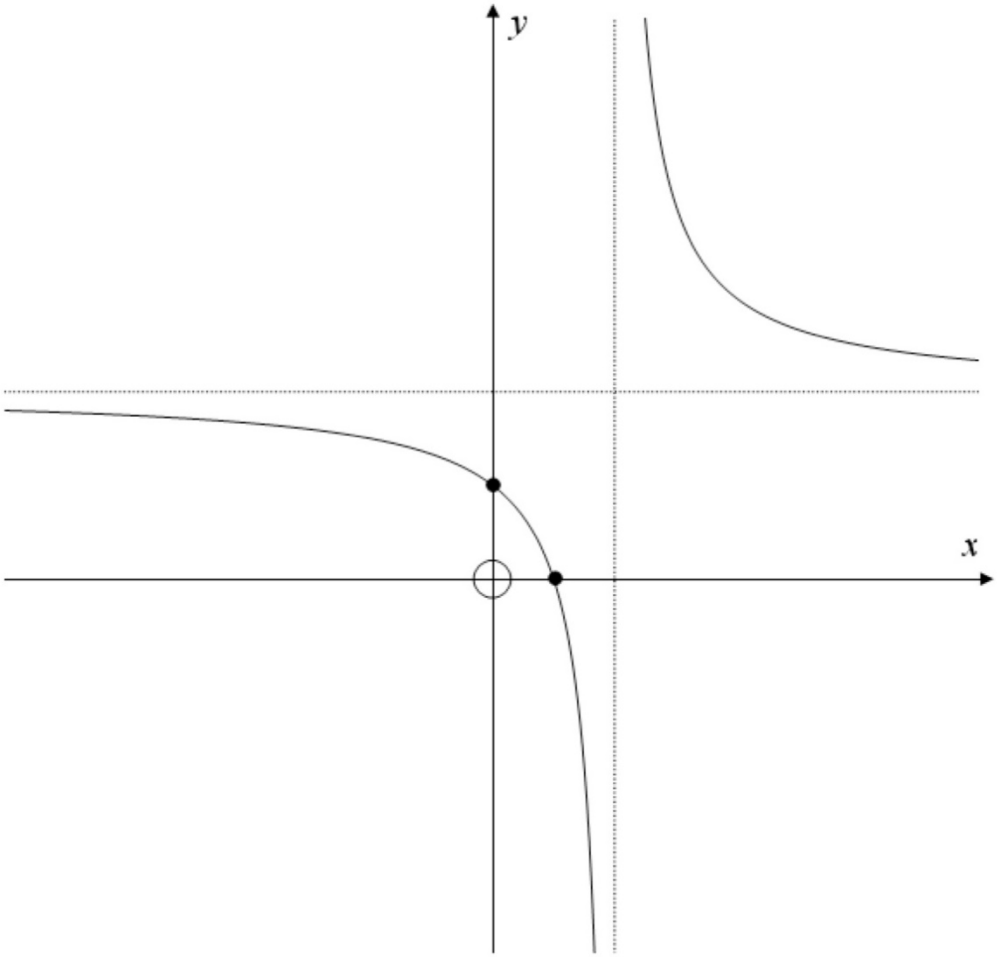
- Attempt **all** questions in this part.
- This part assesses **Criterion 4**.

Question 17

Marker use

A hyperbola of the form $f(x) = \dfrac{a}{x-h} + k$ is sketched below.

The function intersects the x and y axes at p and q respectively.



Spare diagram used (✓)

☐

On the axes above, sketch a graph of $y = f(-x)$.

Label the asymptotes and axes intercepts with the appropriate pronumerals after this graphical transformation.

3

Part 1 continues

Marker use

Question 18

3

Use log laws to show that $x = 3$ is the only valid solution to the equation $\log_3(x-2) + \log_3(x+5) = \log_3 8$.

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Question 19

4

A square root function and its inverse are defined by:

$f(x) = 4 - 3\sqrt{x}$ and $f^{-1}(x) = \dfrac{(4-x)^2}{9}$ where $x \geq 4$.

Show that $f\{f^{-1}(x)\} = f^{-1}\{f(x)\} = x$.

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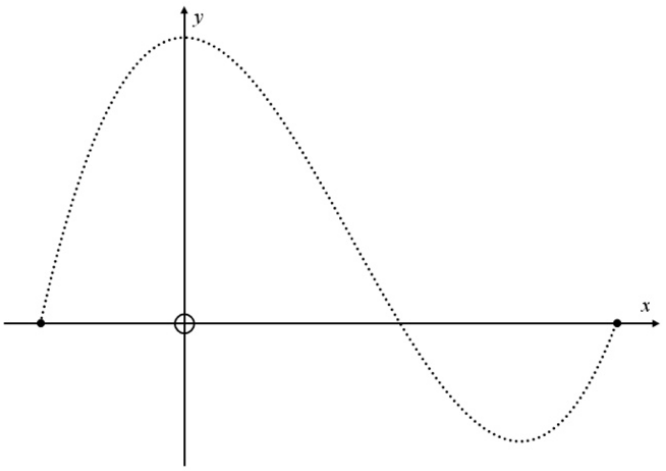
Part 1 continues

Marker use

Question 20

The function $f(x)=(x+1)(2x-3)(x-3)$ is defined over a restricted domain and sketched below.



a) Determine the **largest** domain so the inverse of $f(x)$ is a function.

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b) Hence, determine the domain for this inverse function, $f^{-1}(x)$.

2

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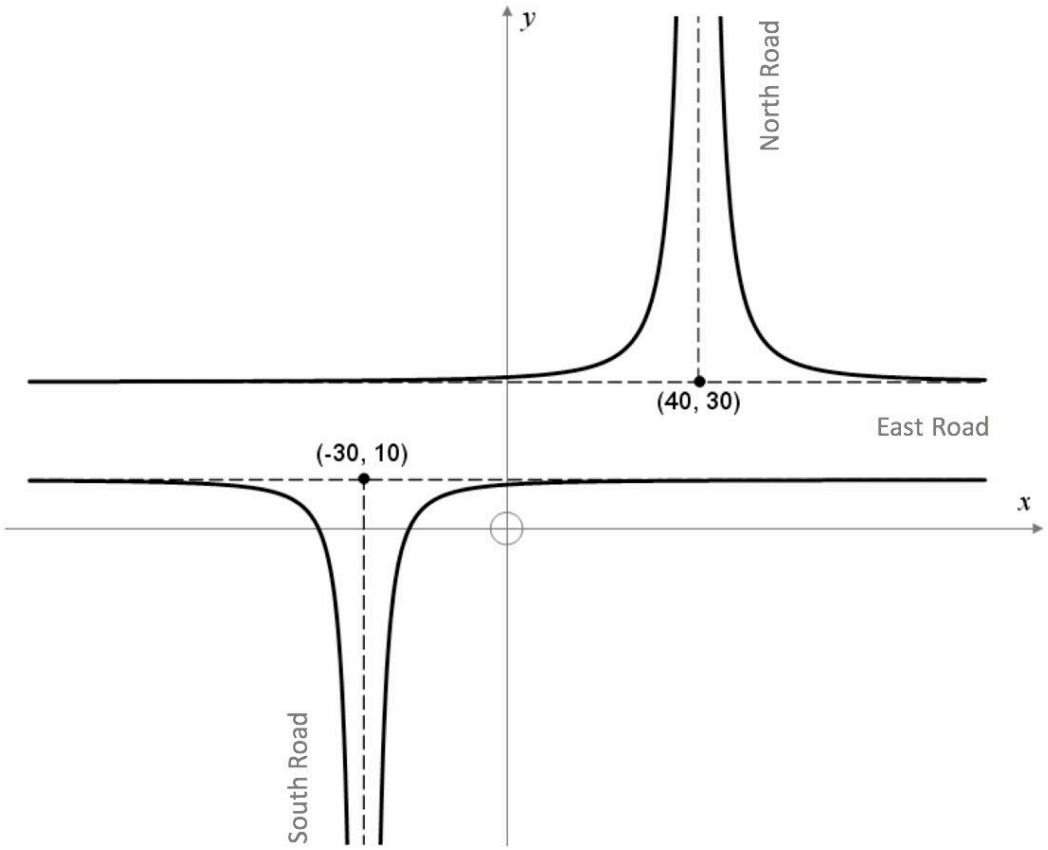
Part 1 continues

Marker use

Question 21

$f(x)=\dfrac{1600}{(x-40)^2}+30$ and $g(x)=\dfrac{a}{(x-h)^2}+k$ model roads as shown below.



- Vertical asymptotes model the mid-lines of North Road and South Road.
- Horizontal asymptotes model the width approached by East Road for large distances from the origin.
- All coordinates are measured in metres.

Question 21 continues

Marker use

a) State the width approached by East Road for large distances from the origin.

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In Question 21 b) and c), you may use your calculator to solve any relevant equations.

b) Determine the equation for $g(x)$ given $(-40,1)$ is a point on the function.

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c) North Road ends 64m above the point $(40,30)$.

Determine the **final width** of North Road.

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Total C4

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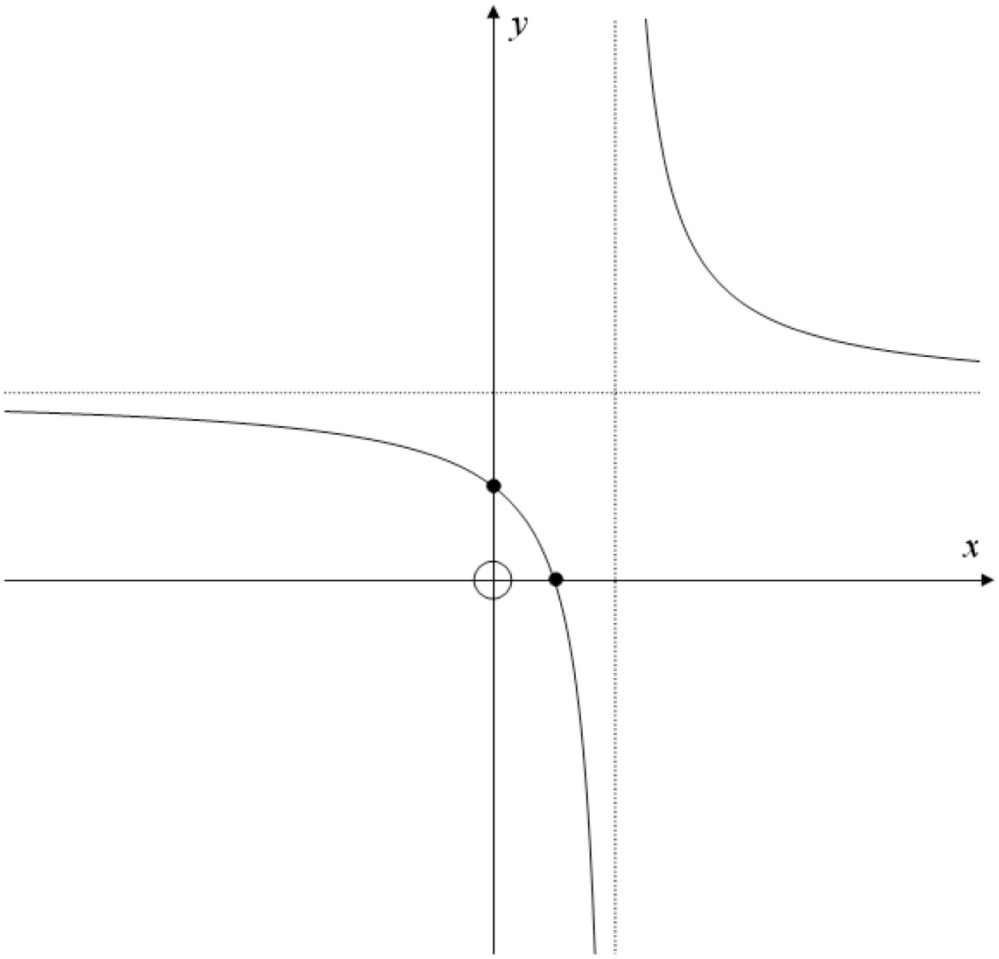
Spare Diagram

Question 17



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Exam continues over the page

Part 2

- Attempt **all** questions in this part.
- This part assesses **Criterion 5**.

Marker use

Question 22

Determine an exact value for $\cos\theta$ given $\sin\theta = \frac{3}{7}$ and $\frac{\pi}{2} < \theta < \pi$. /3

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Question 23

Consider the trigonometric equation:

$$\cos\left(\frac{\pi}{2}+\theta\right) + \sin\left(-\pi+\theta\right) = -\frac{1}{2}, \quad \text{where} \quad 0 < \theta < \frac{\pi}{2}.$$

a) Provide appropriate reasoning to show the equation can be simplified to $\sin(\theta) = \frac{1}{4}$. /3

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b) Hence, evaluate a solution for θ in **radians**. /1

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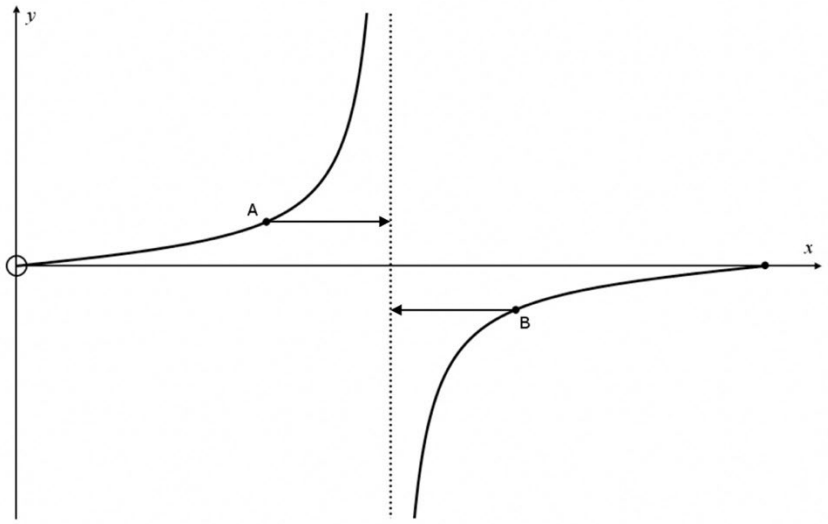
Part 2 continues

Marker use

Question 24

The sketch below is for one period of $f(x) = \tan\left(\frac{4\pi x}{3}\right)$.



The two **arrowed lines** each represent a horizontal distance of $\frac{1}{8}$ from points A and B to either side of the asymptote.

Determine the x intercepts, asymptote and hence the coordinates of points A and B.

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Part 2 continues

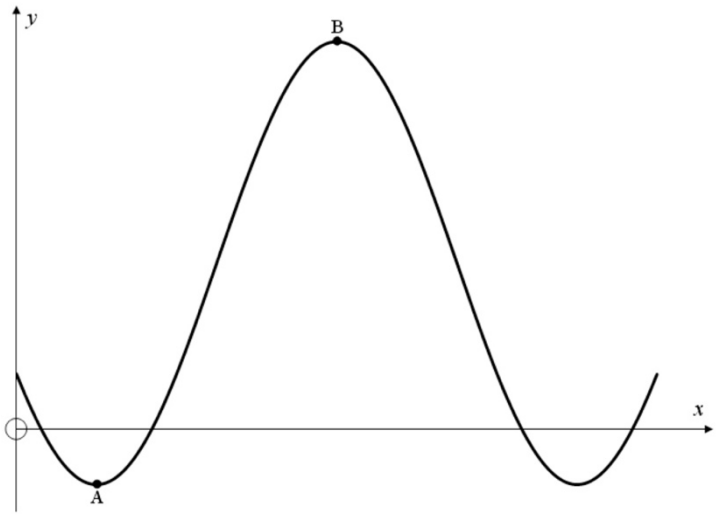
Marker use

Question 25

a) Determine an equation of the graph below in the form $y = a\sin\left[n\left(x+b\right)\right]+c$.

6

The function has a minimum at point A of $\left(\frac{\pi}{6},-1\right)$ and maximum at point B of $\left(\frac{2\pi}{3},7\right)$.



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Question 25 continues

Marker use

2

b) There are many possible correct solutions for the equation found in Question 25 a).

Determine the number of possible correct responses for a, n, b and c by circling the appropriate word(s) in the table below.

No working required.

Variable	Number of Possible Correct Responses		
a	One	Two	More than two
n	One	Two	More than two
b	One	Two	More than two
c	One	Two	More than two

Total C5

20

Part 3

- Attempt **all** questions in this part.
- This part assesses **Criterion 6**.

Question 26

Marker use

A company is producing and selling mathematical tee shirts.

The **profit**, is given by $P(x) = 24x - \frac{x^2}{3} - \left(45 + \frac{x^2}{6} - 25\ln(x)\right)$.

The number of tee shirts made and sold in any given week is denoted by x, where x is greater than zero.

a) Show that $P'(x) = 24 - x + \frac{25}{x}$

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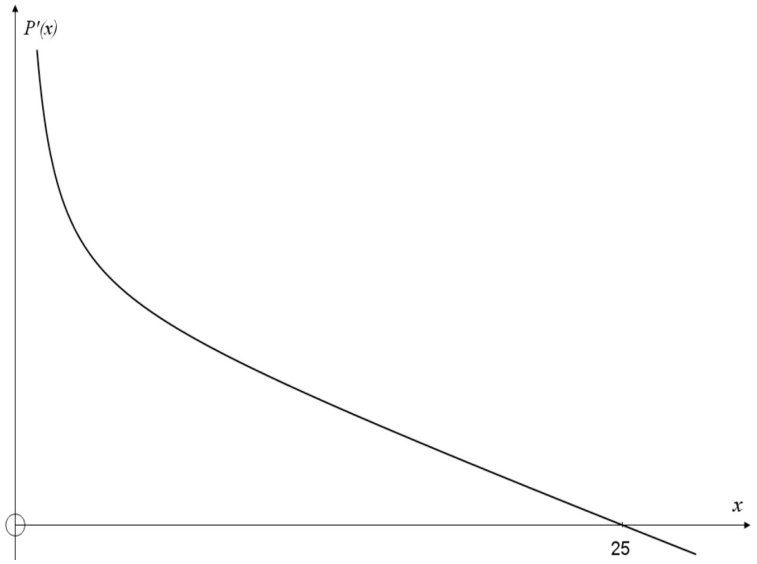
b) Use the sketch of $P'(x)$ to justify the number of tee shirts required to **maximise** profits.



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Part 3 continues

Marker use

Question 27

Show that the gradient of the function $y = \cos(4x).\tan(2x)$ is decreasing at $x = \frac{\pi}{6}$.

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Use the **product** and **chain** rules to provide reasoning.

You may use your calculator to evaluate any substitutions.

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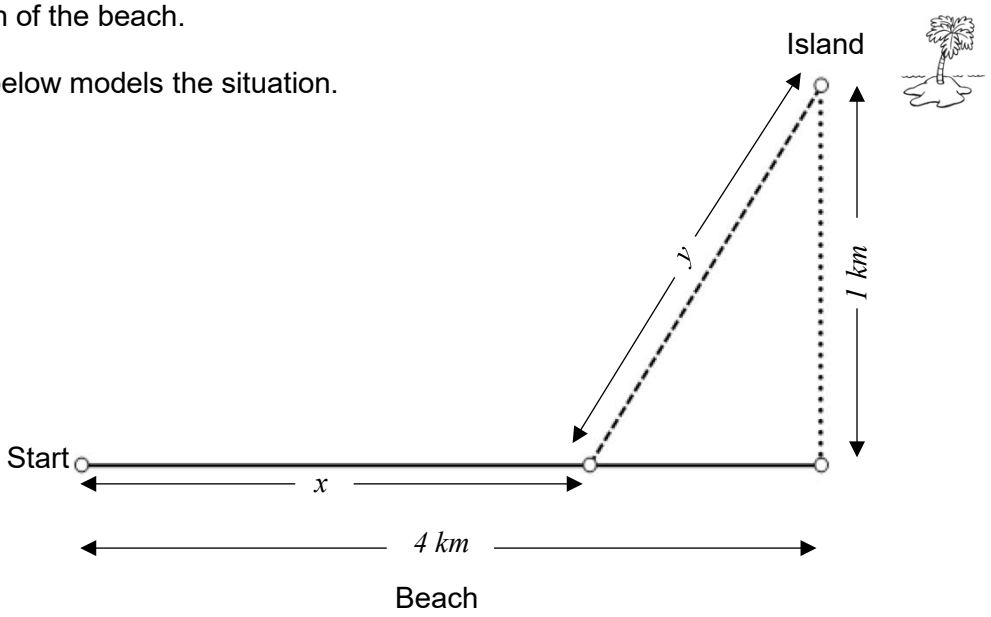
Part 3 continues

Marker use

Question 28

A person starts to walk from one end of a 4 km beach and then swims to an island which is 1 km due north of the beach.

The diagram below models the situation.



They walk at a rate of 5 km/hr and swim at a rate of 3 km/hr.

They want to **minimise** the time taken by walking x km then swimming y km.

The time they take is given by $T = \frac{x}{5} + \frac{\sqrt{x^2 - 8x + 17}}{3}$ hours.

a) Determine the value for x that minimises the time taken.

There is no need to justify your solution as a minimum.

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b) Hence, determine the minimum time.

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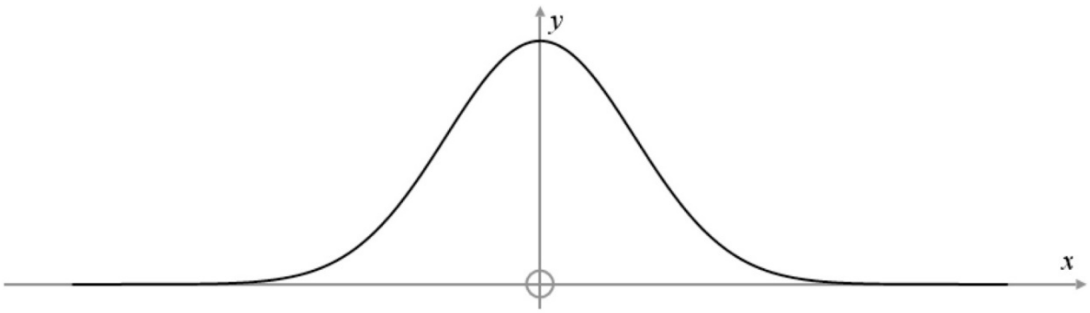
Part 3 continues

Marker use

Question 29

The function $f(x) = e^{-x^2}$ is graphed below.



Spare diagram used (✓) ☐

a) Sketch the **normals** to $f(x)$ that pass through the **origin** on the graph above.

/ **1**

b) Show the **gradient** of the **normal** that passes through the point $\left(\frac{\sqrt{\ln 2}}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ equals $\frac{\mathbf{1}}{\sqrt{\ln 2}}$.

/ **4**

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c) Hence, show that this **normal** passes through the **origin**.

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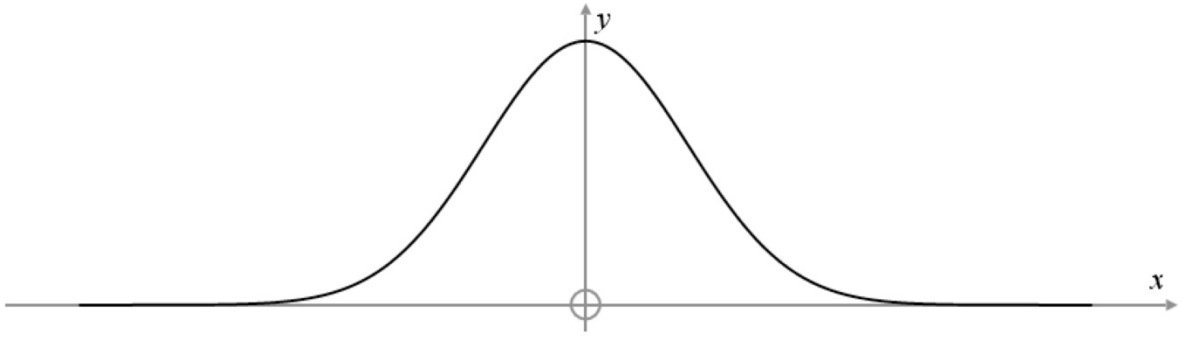
Total C6

/ **20**

Spare Diagram

Question 29 a)



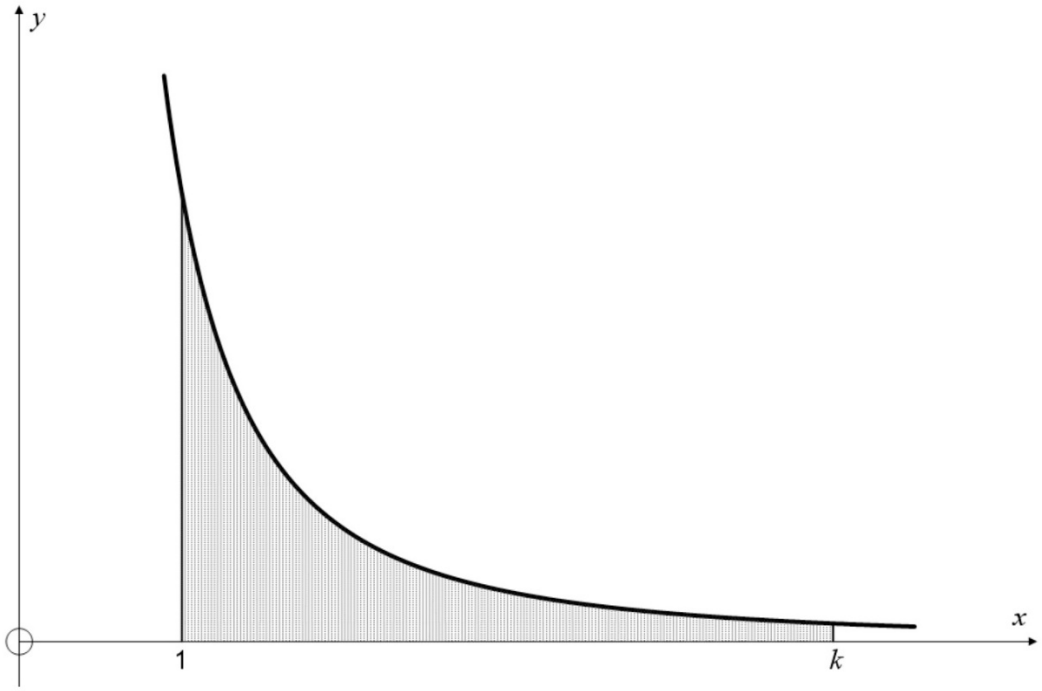
Part 4

- Attempt **all** questions in this part.
- This part assesses **Criterion 7**.

Question 30

Marker use

A section of the function $f(x)=\frac{10}{x^2}$ is graphed below.



Determine the value for k where the shaded area equals 8 units^2.

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Part 4 continues

Marker use

Question 31

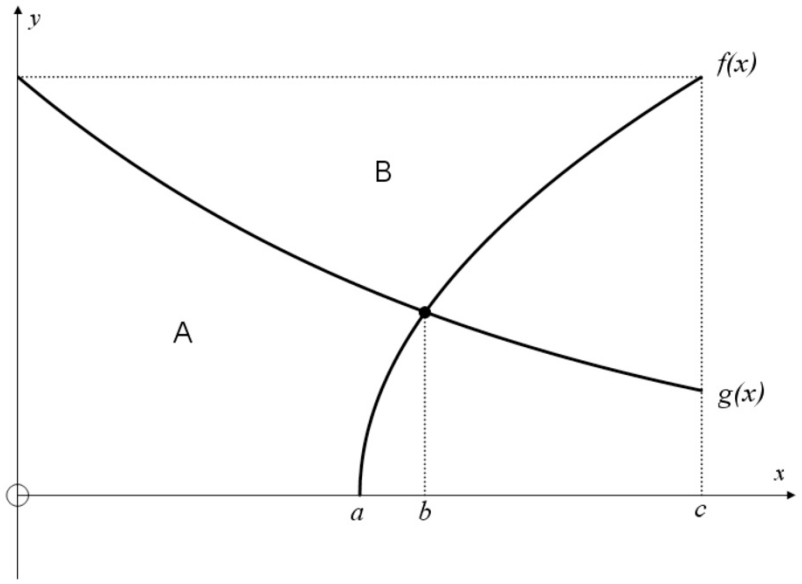
The graph to the right sketches

$f(x)$ for $x \in [a,c]$

and

$g(x)$ for $x \in [0,c]$.

The functions **intersect** where $x = b$.



Determine expressions to evaluate the enclosed areas A and B.

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Question 32

The functions $f(x)$ and $g(x)$ are continuous, where:

$f(x) = \frac{1}{4}g(x)$, $\int_0^4 f(x)\,dx = -3$ and $\int_4^{10} g(x)\,dx = 8$.

Evaluate $\int_0^{10} f(x)\,dx$.

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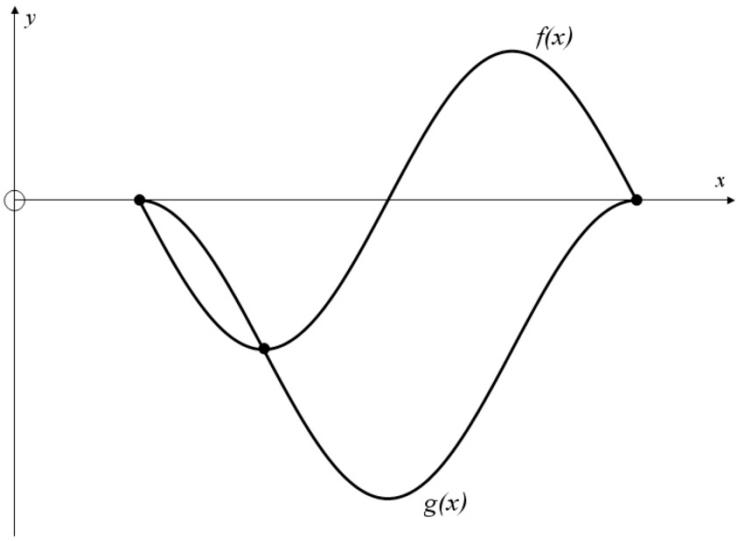
Part 4 continues

Marker use

Question 33

The sketch below has two enclosed areas between the functions:

$f(x) = \cos(3x)$ and $g(x) = \sin(3x) - 1$ over the domain $\left[\frac{\pi}{6}, \frac{5\pi}{6}\right]$.



Show that the total of the two enclosed areas equals $\frac{4+\pi}{3}$.

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Part 4 continues

Question 34

Marker use

A coal-fired power station emits sulfur dioxide.

Concentrations of the gas C, in parts per million (ppm), are monitored at various ground level positions x km away from the site.

The rate of change is modelled by $\frac{dC}{dx} = -\frac{1}{2x^3}$.

a) Determine an equation for the concentration given $C = 0$ when $x = 50$.

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b) Long term human exposure to this gas should not exceed concentrations of 0.03 ppm. Evaluate the minimum distance deemed safe for residents living nearby.

2

Express your answer accurate to the nearest kilometre.

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Total C7

20

Part 5

- Attempt **all** questions in this part.
- This part assesses **Criterion 8**.

Question 35

Marker use

A discrete random variable X is defined in the following table:

x	-2	0	3
$\Pr(X=x)$	$2a^2$	$\frac{5a}{2}$	a

a) Show the only valid solution for the constant a is $\frac{1}{4}$.

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b) Hence, determine the variance for the distribution X.

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Part 5 continues

Marker use

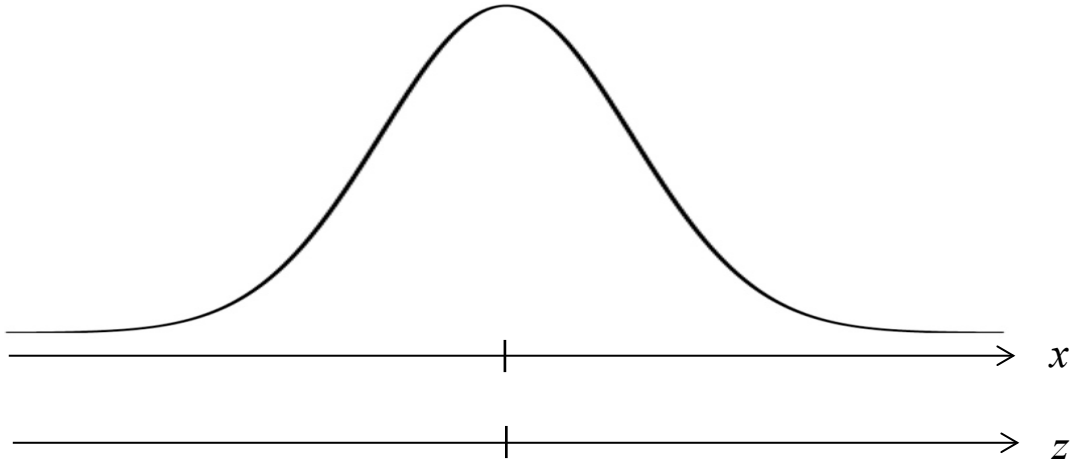
Question 36

The weight of blocks of a particular brand of chocolate is normally distributed with a mean of 100 grams.

2% of blocks produced weighed under 95 grams and were deemed unacceptable.

a) Show this information on the normal distribution graph below.

2



b) Show that the standard deviation for the weight of blocks is approximately 2.435 grams.

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c) Hence, determine the probability of blocks with a weight between 96 and 104 grams.

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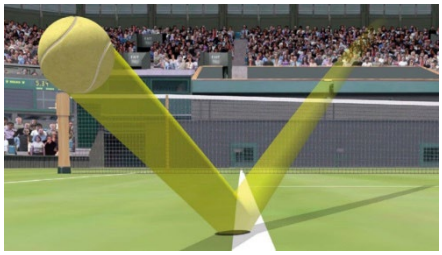
Part 5 continues

Marker use

Question 37



An electronic system keeps track of the ball in tennis matches and allows a player to challenge a decision.

A successful challenge leads to a decision being overturned.

A sample of 600 challenges had 168 overturned.

a) Determine the proportion of overturned decisions for this sample. 1

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The table below compares statistics relevant to this sample.

Confidence Interval C%	Z_{values}	Margin of Error M
90%	1.645	
	2.576	0.04722

b) Complete the table. Show working below. 3

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c) Find and interpret the 90% confidence interval in the context of this tennis scenario. 3

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Total C8

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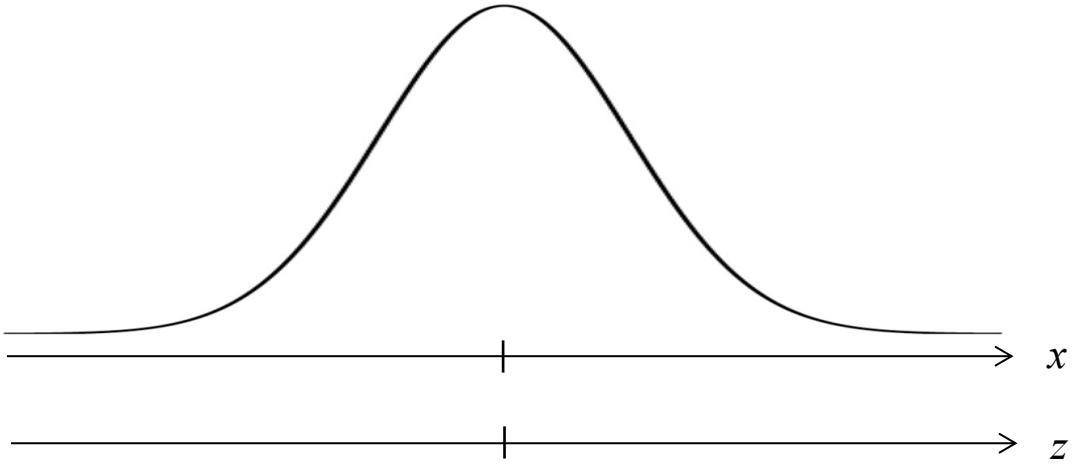
Spare Diagram

Question 36



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End of Section B

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