

Attach your candidate label here

MATHEMATICS METHODS

MTM415117

Section **A**

Pages: 20

Questions: 18

Information Sheet: 1

Preparation time for this exam: 15 minutes

Suggested working time: 80 minutes

Instructions:

Calculators are not allowed to be used in this section.

Section A will be collected after 80 minutes.

- There are **five (5) parts** to this section.
- Answer **all** questions and **all** items within each question.
- Write your answers in the spaces provided in this exam paper.
 - Spare diagrams have been provided at the end of Parts 1 and 2. Indicate in the box provided if you have used the spare diagrams.
- The exam is **three (3) hours** in length. It is suggested that you spend **approximately 80 minutes** in total answering the questions in this exam booklet.
- During the first 80 minutes you may move onto Section B, but you **cannot** use your calculator until told by your supervisor(s).
- The Mathematics Methods Information Sheet can be used throughout the exam.
- All answers must be written in **English**.
- You **must** make sure your answers address the listed criteria.

Marker use	
C4	/ 16
C5	/ 16
C6	/ 16
C7	/ 16
C8	/ 16

Additional Exam Instructions

For questions worth **one (1)** mark, you are not required to show workings. Markers will look at the presentation of the answer(s) and at the argument(s) leading to the final answer(s).

For questions worth **two (2)** or more marks **you are required** to show relevant workings.

Marks will be allocated:

- according to the degree to which workings convey a logical line of reasoning, and
- for suitable justifications and explanations of methods and processes when requested.

Criteria

You **must** make sure your answers address:

- Criterion 4 understand polynomial, hyperbolic, exponential and logarithmic functions
- Criterion 5 understand circular functions
- Criterion 6 use differential calculus in the study of functions
- Criterion 7 use integral calculus in the study of functions
- Criterion 8 understand binomial and normal probability distributions and statistical inference.

Guide to Exam Structure

		Parts	Questions available	Questions to answer	Suggested working time	Marks available
Section A	Part 1		4	4	16 minutes	16 marks
	Part 2		4	4	16 minutes	16 marks
	Part 3		3	3	16 minutes	16 marks
	Part 4		4	4	16 minutes	16 marks
	Part 5		3	3	16 minutes	16 marks
Totals			18	18	80 minutes	80 marks
Section B	Part 1		4	4	20 minutes	20 marks
	Part 2		3	3	20 minutes	20 marks
	Part 3		4	4	20 minutes	20 marks
	Part 4		4	4	20 minutes	20 marks
	Part 5		4	4	20 minutes	20 marks
Totals			19	19	100 minutes	100 marks
Totals			37	37	180 minutes (3 hours)	180 marks

Part 1

- Answer **all** questions in this part.
- This part assesses **Criterion 4**.

Question 1

Given that $f(x) = 4e^{2x-3}$

a) State the domain and range of $f(x)$.

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b) Find the inverse $f^{-1}(x)$.

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c) State the domain of $f^{-1}(x)$.

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Question 2

Marker use

Determine the coefficient of x^3 in $(2x - 3)^5$.

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Question 3

Using Figure 1, sketch the graph of $f(x) = \log_2(x + 2) - 2$.

Include asymptotes and intercepts in your sketch.

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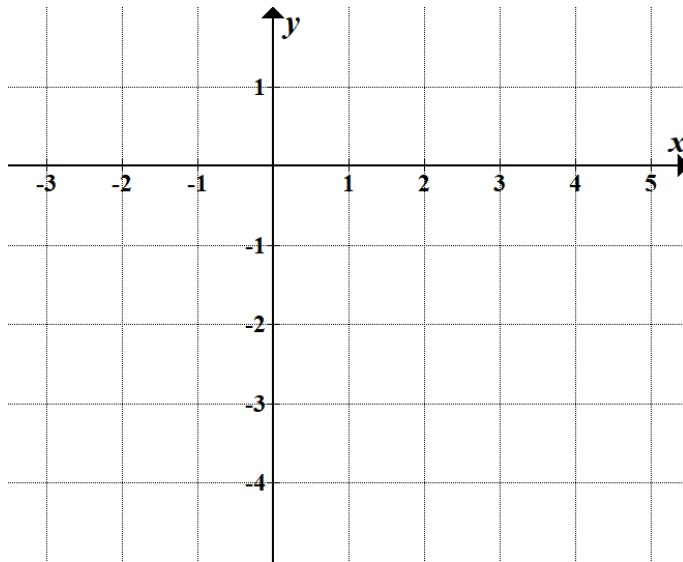


Figure 1

Spare diagram used (X)

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Question 4

Marker use

For the function $f(x) = \sqrt{9(4x - 8)} + 2$

a) Using algebra express $f(x)$ in the form $y = a\sqrt{x - h} + k$.

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b) State the transformations required (in order) from $y = \sqrt{x}$ to transform $f(x)$ into the form $y = a\sqrt{x - h} + k$.

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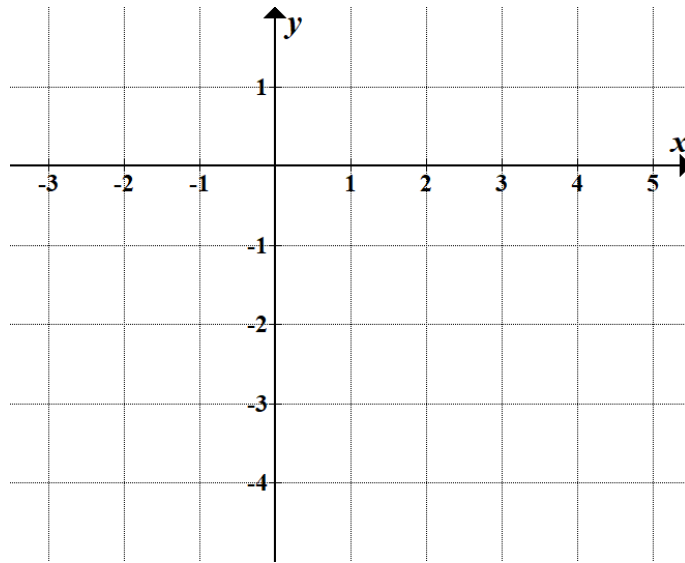
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/16

Spare Diagram

Marker use

Question 3



Part 2

Marker use

- Answer **all** questions in this part.
- This part assesses **Criterion 5**.

Question 5

a) Convert -135° into radians.

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b) Convert $\frac{11\pi}{6}$ into degrees.

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Question 6

Give exact values of the following:

a) $\cos(-150^\circ)$

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b) $\tan\left(\frac{3\pi}{4}\right)$

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Question 7

Given the graph of $y = \sin(x)$ below, for $x \in [-2\pi, 2\pi]$

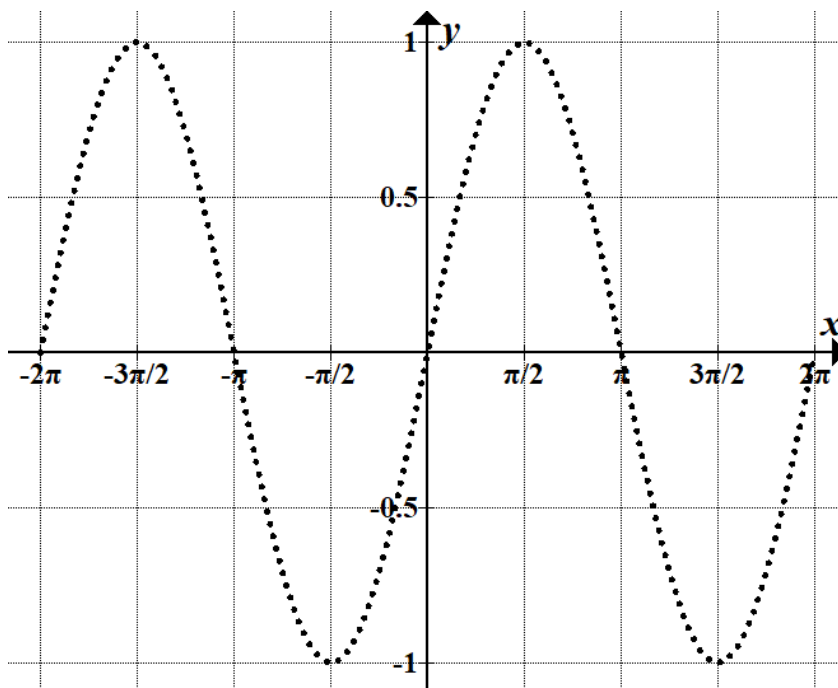


Figure 2

Spare diagram used (X)

a) Using Figure 2, sketch on the same axes the graph of $y = -\sin\left(x - \frac{\pi}{2}\right)$.

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b) Which trigonometric function has the same graph as the one you drew in a)?

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c) Calculate $-\sin\left(\frac{\pi}{3} - \frac{\pi}{2}\right)$.

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Question 8

Marker use

Given that $\sin x = -\frac{4}{7}$ and $-\pi \leq x \leq -\frac{\pi}{2}$, calculate:

a) $\cos x$

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b) $\tan x$

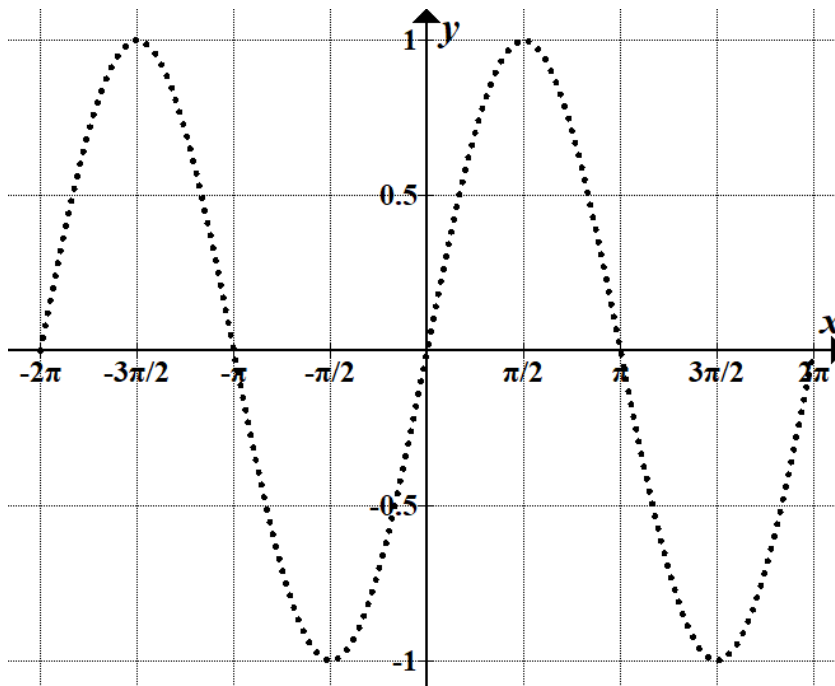
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Question 7 a)



Part 3

- Answer **all** questions in this part.
- This part assesses **Criterion 6**.

Question 9

Differentiate the following

a) $y = 3 \cos(\pi x) + 6 \tan(2x)$

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b) $y = 6x^3 - x + \pi - \frac{2}{x^3} + x^{\frac{3}{2}}$

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c) $y = e^{4x+1} \log_e(2x - 3)$

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Question 10

Show that the derivative of $f(x) = \frac{\ln(x)}{e^x}$ is equal to $f'(x) = \frac{1-x\ln(x)}{xe^x}$.

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Question 11

A normal to the curve $f(x) = \frac{1}{x^2}$ is drawn at $x = k$.

One example of a solution is shown in Figure 3.

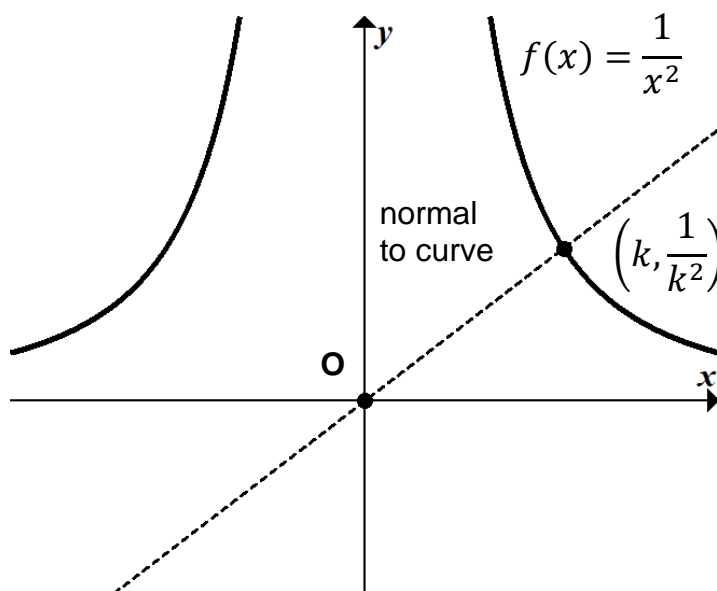


Figure 3

- a) Using Figure 3, show that the gradient of the normal at $x = k$ is equal to $\frac{k^3}{2}$.

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- b) Find the exact values of k such that the normal passes through the origin, O.

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Total
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Part 4

Marker use

- Answer **all** questions in this part.
- This part assesses **Criterion 7**.

Question 12

a) Evaluate

$$\int \sqrt{2x - 3} dx$$

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b) Evaluate

$$\int \frac{e^{5x+1}}{e^{2x-3}} dx$$

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Question 13

Marker use

a) Show that the derivative of $y = (\ln x)^2$ is $\frac{dy}{dx} = \frac{2 \ln x}{x}$.

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b) Hence evaluate the integral:

$$\int_1^e \frac{\ln x}{2x} dx$$

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Question 14

a) Determine

$$\int_{-1}^2 (6x - 6x^2) dx$$

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b) Explain why the answer to a) is negative.

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Question 15

Marker use

The graph of $f(x)$ has a stationary point at $(2, 5)$.

It also has a y -intercept at -5 .

Its derivative is in the form $f'(x) = -3x^2 + ax + b$, for $a, b \in \mathbb{R}$.

Determine the function $f(x)$

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Total
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Part 5

- Answer **all** questions in this part.
- This part assesses **Criterion 8**.

Question 16



A new casino game is being featured where the player rolls a four and a six-sided die simultaneously. The game costs \$1 to play.

If the 4-sided die rolls higher than the 6-sided die, the player wins \$3 (including the \$1 to play).

If the 4-sided die rolls the same as the 6-sided die, the player receives their \$1 back.

If the 4-sided die rolls lower than the 6-sided die, the player loses their \$1.

a) Complete the Table 1 for the earnings for the player:

Earnings for Player		Six-Sided Die Roll					
		1	2	3	4	5	6
Four-Sided Die Roll	1	0	-1	-1	-1	-1	-1
	2	2					
	3	2		0			
	4	2					

Table 1

b) A random variable x is defined as the earnings for the player.

Complete the probability distribution table (Table 2) for the random variable x .

x	-1	0	2
$P(X = x)$			

Table 2

c) Calculate $E(X)$. Is this a fair game?

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Question 16 continues

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Question 16 continued

Marker use

d) Calculate $var(X)$.

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Question 17

A random variable X is normally distributed with a mean of 10 and a standard deviation of 2.



Figure 4

Using the approximate values for areas of the normal distribution, find values for the following:

a) $P(6 \leq X \leq 14)$

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Question 17 continues

Question 17 continued

Marker use

b) $P(6 \leq X \leq 8)$

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c) Find the value of x so that $P(X \geq x) = 0.0015$.

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Question 18

The probability of a machine in a factory producing a defective item is $\frac{1}{10}$.

X is a random variable given by the number of defective items produced when 4 items are made.

Calculate exact values for the following probabilities:

a) $P(X = 3)$

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b) $P(X < 3)$

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End of Section A

**Total
P5
/16**



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External Assessment 2024

MATHEMATICS METHODS

MTM415117

Section **B**

Pages: 32

Questions: 19

Information Sheet: 1

Suggested working time: 100 minutes

Instructions:

Calculators are allowed to be used in this section.

- There are **five (5) parts** to this section.
- Answer **all** questions and **all** items within each question.
- Write your answers in the spaces provided in this exam paper.
 - Spare diagrams have been provided at the end of Parts 1, 2, 3 and 5. Indicate in the box provided if you have used the spare diagrams.
- The exam is **three (3) hours** in length. It is suggested that you spend **approximately 100 minutes** in total answering the questions in this exam booklet.
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Marker use	
C4	/ 20
C5	/ 20
C6	/ 20
C7	/ 20
C8	/ 20

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Totals			18	18	80 minutes	80 marks
Section B	Part 1		4	4	20 minutes	20 marks
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	Part 3		4	4	20 minutes	20 marks
	Part 4		4	4	20 minutes	20 marks
	Part 5		4	4	20 minutes	20 marks
Totals			19	19	100 minutes	100 marks
Totals			37	37	180 minutes (3 hours)	180 marks

Part 1

- Answer **all** questions in this part.
- This part assesses **Criterion 4**.

Question 19

Solve

$$2 \log_3 x - \log_3(x - 1) = \log_3(x + 3)$$

Show all algebraic working in your answer.

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Question 20

The number of Carbon-14 atoms in a sample is given by the equation

$$A = A_0 e^{-kt}$$

where A_0 is the initial number of atoms, k is a constant, and A is the number of atoms remaining t years later.

- a) If A is equal to half the value of A_0 after 5730 years, find the value of k .

Give your answers to 6 decimal places.

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Question 20 continues

Question 20 continued

Marker use

b) Hence find the age of a sample of Carbon-14 atoms if there is instead 70% of the initial number remaining.

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Question 21

The graph in Figure 5 is in the form:

$$f(x) = a \times b^x + k, \quad x \in \mathbb{R}$$

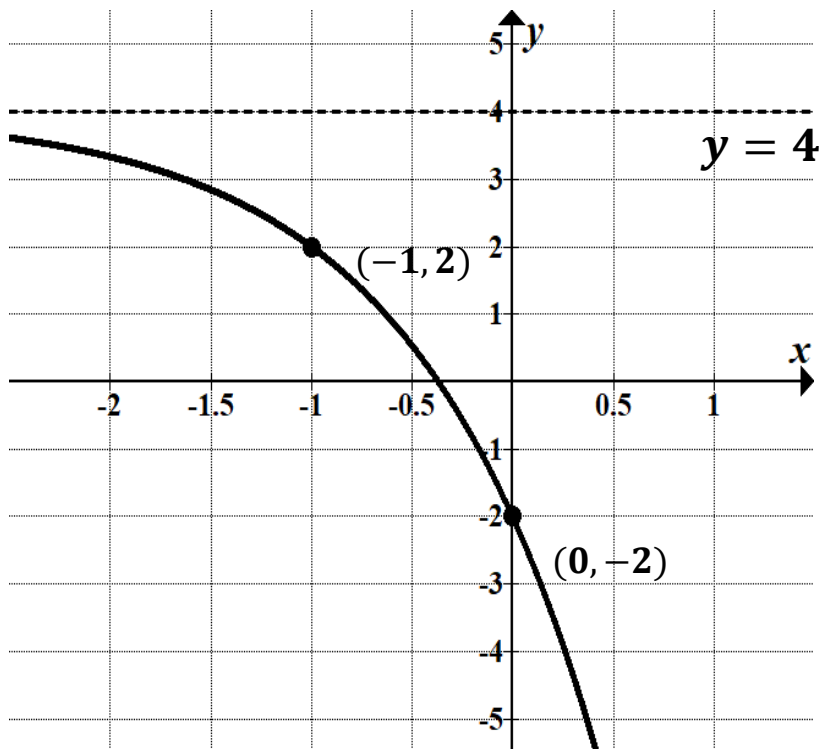


Figure 5

a) Using Figure 5, State the value of k .

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b) Determine the equation of $f(x)$.

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Exam continues over the page

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Question 22

a) Using Figure 6, sketch the graph of the hyperbola:

$$f(x) = \frac{a}{x - b}$$

for $a > 0$ and $b > 0$ where $x \in \mathbb{R} \setminus \{b\}$.

Include asymptotes and exact values for intercepts in your sketch.

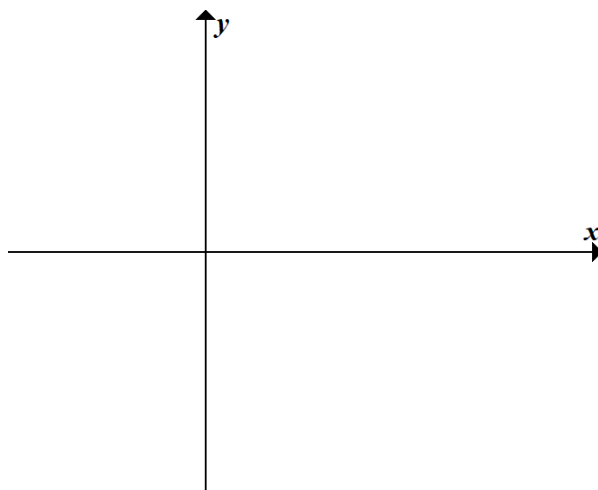


Figure 6

Spare diagram used (X)

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b) Consider $g(x) = x^{-2}$ and $f(x) = \frac{a}{x-b}$.

Determine the function $g(f(x))$

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Question 22 continues

Question 22 continued

Marker use

c) State the domain and range for $g(f(x))$.

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d) Find the function $f(x)$ such that:

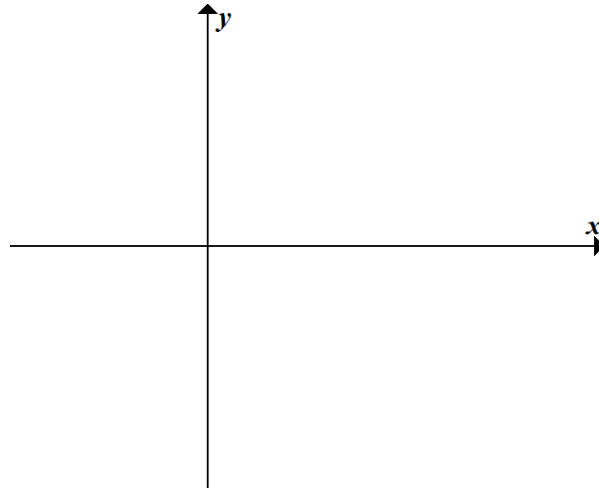
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$$g(f(x)) = 3(x - 4)^2$$

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Total
P1
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Question 22 a)



Exam continues over the page

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Part 2

- Answer **all** questions in this part.
- This part assesses **Criterion 5**.

Question 23

The graph below is in the form

$$y = a \tan n(x + b)$$

where x is given in radians.

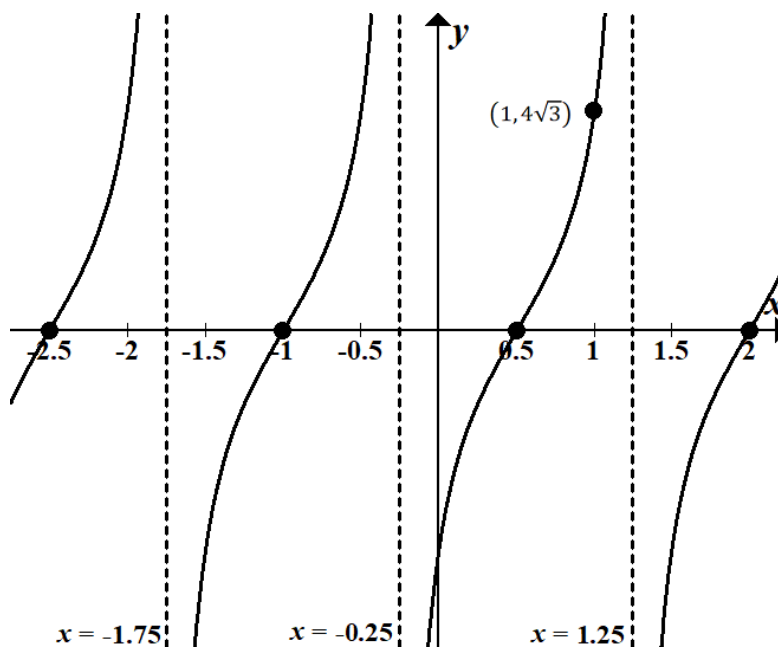


Figure 7

Spare diagram used (X)

a) What is the period for this graph?

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b) Find the equation for the graph in Figure 7.

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Question 24

Marker use

a) Solve algebraically the equation

$$\sin\left(3x - \frac{\pi}{2}\right) + 1 = \frac{1}{2}$$

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for $0 \leq x \leq \pi$.

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b) By considering the period of the equation $y = \sin\left(3x - \frac{\pi}{2}\right)$, find three more solutions in the interval $[\pi, 2\pi]$.

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Question 25

Marker use

The height of the water level at the entrance of a harbour follows a sinusoidal pattern.

Low tide has a height of 2 m and 6 hours later the following high tide is 8 m.

The height of the tide can be modelled by the equation:

$$H(t) = a \sin(n(t + b)) + c, \text{ where } t \text{ is hours after midnight.}$$

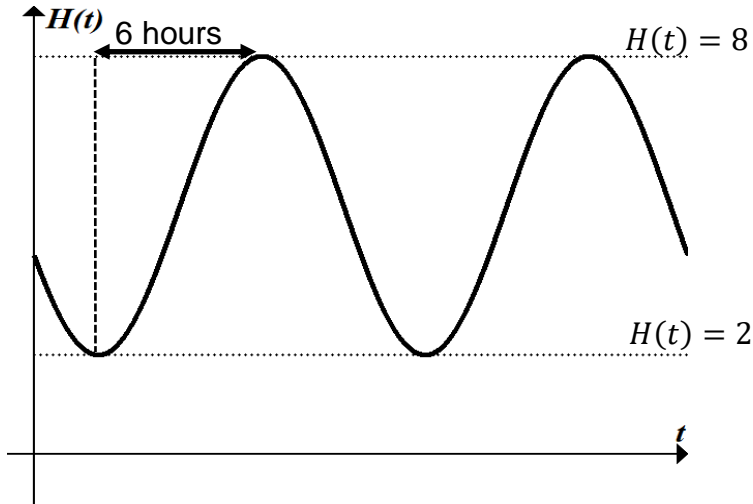


Figure 8

Spare diagram used (X)

a) Using Figure 8, calculate **first** the values of a , n and c .

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b) One night, the height of the water at midnight is 4 m and falling.

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Find an appropriate value for b to three decimal places.

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Question 25 continues

Question 25 continued

Marker use

c) A boat is only able to pass through the harbour when the depth of the water is 7 m or greater. During which times (to the nearest minute) of the day can the boat pass through?

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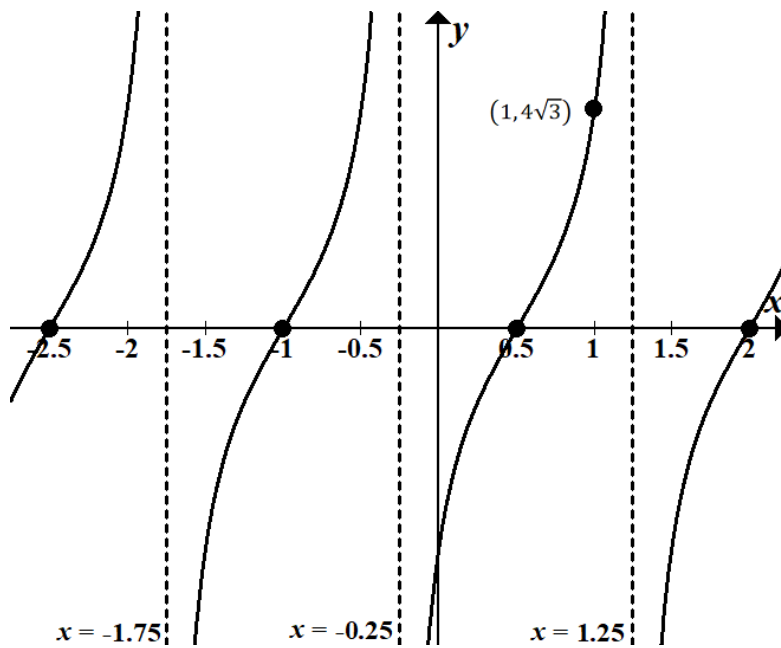
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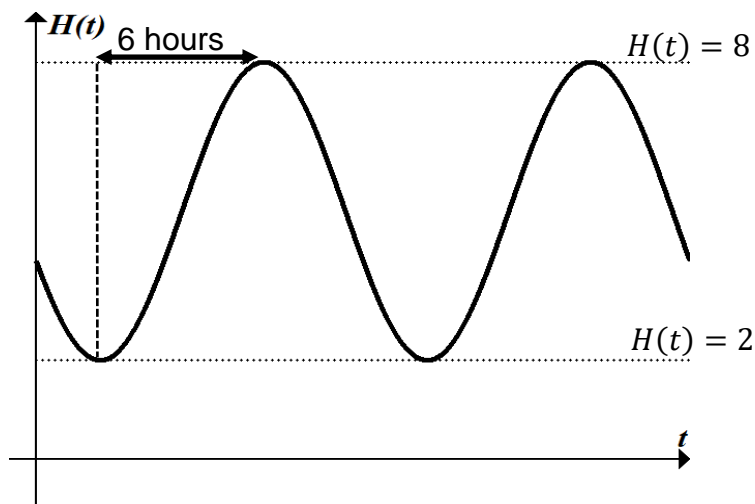
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Total
P2
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Question 23



Question 25



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Part 3

- Answer **all** questions in this part.
- This part assesses **Criterion 6**.

Question 26

Sketch the derivative of the function below on the same axes.

Note that no working is required to be shown for this question.

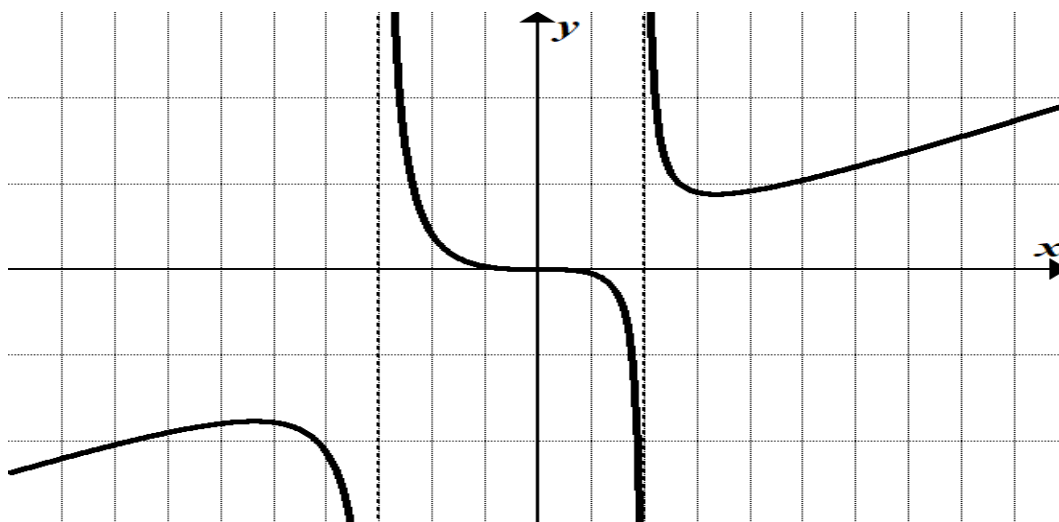


Figure 9

Spare diagram used (X)

Question 27

Find and classify the stationary point(s) of the function

$$f(x) = x e^{2x}$$

Show all algebraic working.

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Question 28

Marker use

A chain is hung across a canyon from the points $(0,100)$ and $(20,100)$.

The height of the chain above ground level can be modelled by the equation

$$H(x) = a(x - b)^2 + c \text{ for } a \neq 0$$

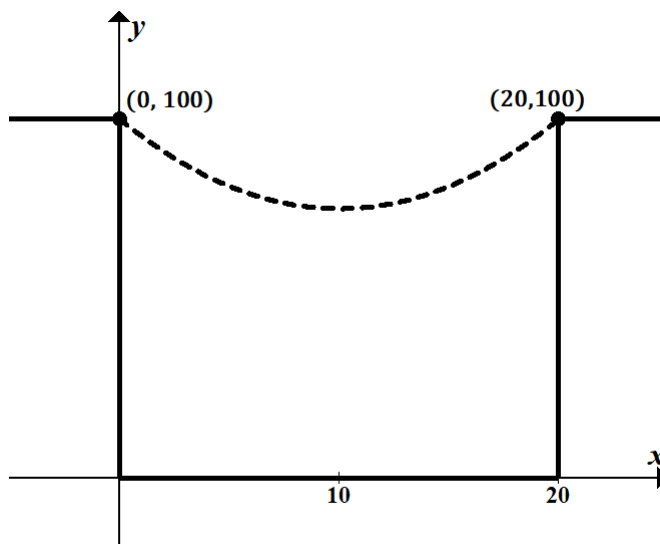


Figure 10

So that the chain is not too steep, the value of the rate of change at $x = 20$ is 5 m/m

(i.e. vertical metres per horizontal metres).

Find the minimum value of the height of the chain above ground level.

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Question 29

A camping equipment manufacturer wants to design a new tent.

A rear view is shown below.

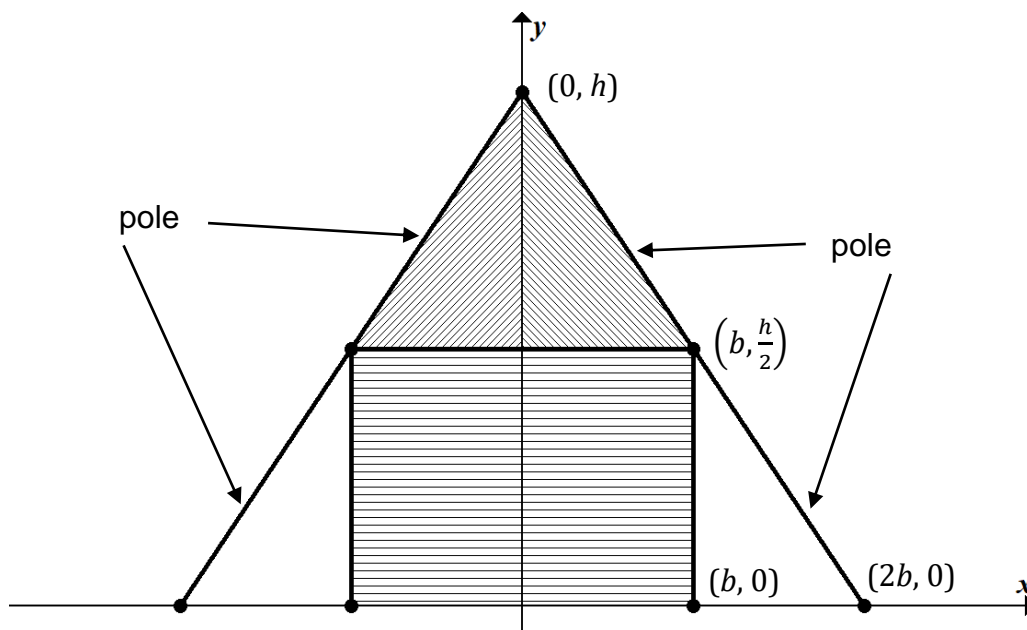


Figure 11

A pole that attaches from the ground $(2b, 0)$ to the top $(0, h)$ is 4 m long.

a) Show that $h = 2\sqrt{4 - b^2}$.

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b) Given that the shaded area of the tent is $\frac{3bh}{2}$,

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find an expression for the area in terms of b .

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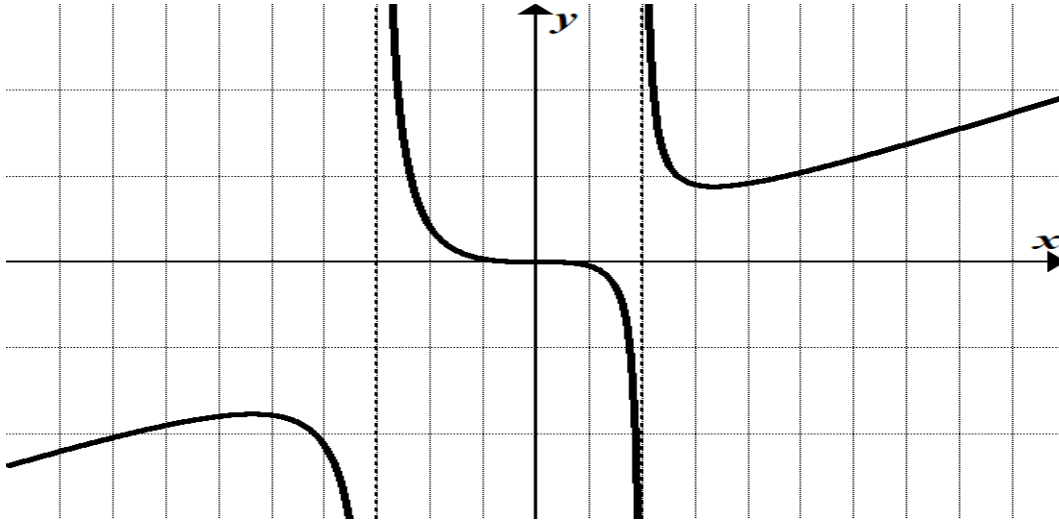
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Question 29 continues

Question 26



Part 4

- Answer **all** questions in this part.
- This part assesses **Criterion 7**.

Question 30

Write expressions for the shaded area enclosed by the functions in the following situations:

a)

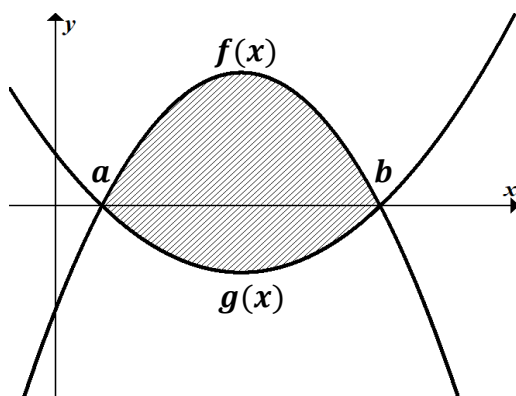


Figure 12

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b)

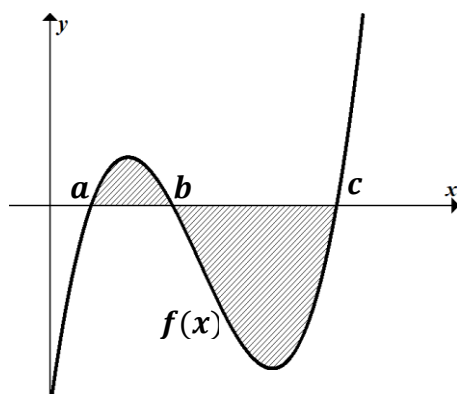


Figure 13

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Question 30 continues

Question 30 continued

Marker use

c)

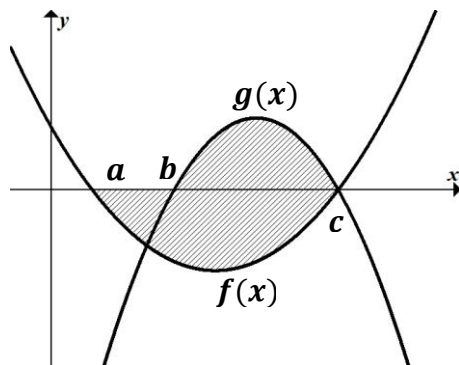


Figure 14

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Question 31

Find the value of k , where $k > 1$, such that the area enclosed between the line $x = k$ and the curves $y = \frac{1}{x^3}$ and $y = \frac{1}{x^2}$ is equal to $\frac{1}{8}$.

/4

Show all working in your answer.

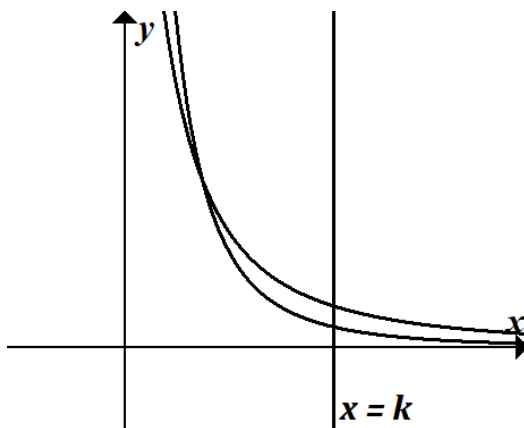


Figure 15

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Question 32

Marker use

A popcorn maker produces popcorn at a rate of

$$\frac{dP}{dt} = -15t^2 + 60t + 5\sin(2\pi t)$$

Where P is the number of pieces of popcorn produced in t minutes.

It begins producing popcorn at $t = 0$ and ends when $\frac{dP}{dt} = 0$ once more.

a) Find the value of $\frac{dP}{dt}$ at $t = 1$ minute.

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b) Find the time at which popcorn production ends.

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c) Find the equation of the function $P(t)$.

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d) Hence, find the total amount of popcorn pieces produced.

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Question 33

Marker use

The three functions are sketched in Figure 16 in the domain $[0, \pi]$:

$$f_1(x) = \sin(x)$$

$$f_2(x) = \sin\left(x - \frac{2\pi}{3}\right)$$

$$f_3(x) = \sin\left(x + \frac{2\pi}{3}\right)$$

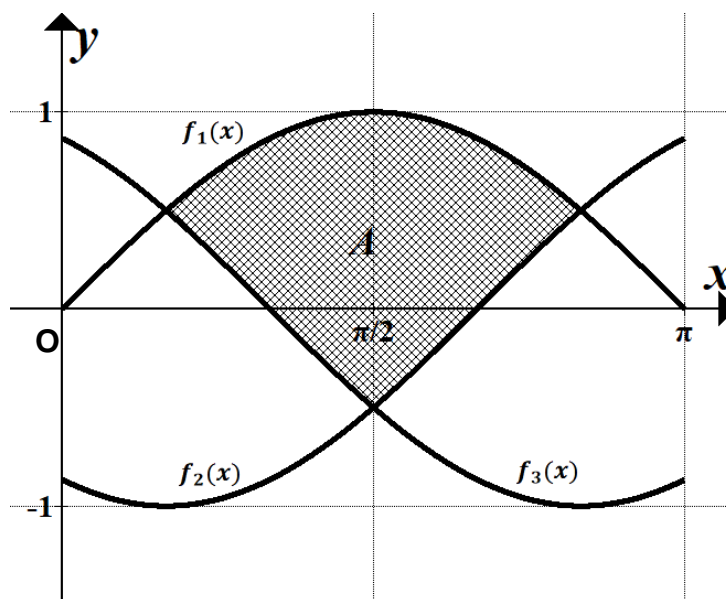


Figure 16

a) Identify all the boundaries of the area A above.

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b) Determine the exact value for the area of A.

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Total
P4
/20

Part 5

- Answer **all** questions in this part.
- This part assesses **Criterion 8**.

Question 34

A world-renowned chef randomly samples 10 vastly different ice-cream flavours.

Let X be the binomial random variable of how many of these she likes, each with a probability of $\frac{3}{10}$ of success.

From these 10 flavours:

- a) i. find the probability that she will like more than 5 flavours.

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/2

- ii. find the probability that she will like exactly 5 flavours.

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/1

- b) Calculate the mean and variance for this distribution.

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Question 35

200 random people are surveyed and 70 say that they prefer honey to jam on their toast.

A confidence interval for the true proportion of the population who prefer honey is (0.2945, 0.4055).

Find the level of confidence that generates this interval.

Show full working in your answer.

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Question 36

Marker use

A normal distribution has 10% above 30 and 25% below 10.

a) Using Figure 17, sketch a diagram representing the information above.

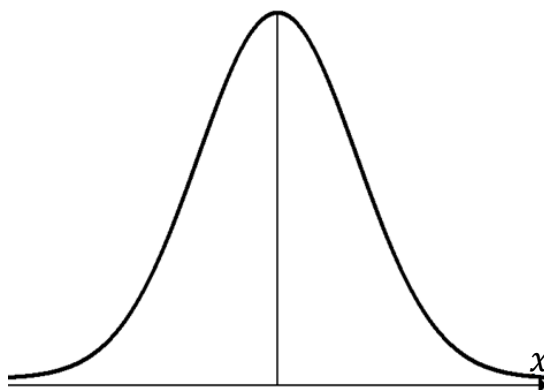


Figure 17

Spare diagram used (X)

/1

b) Find μ and σ to two decimal places.

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c) If instead, 25% of its area is above 30 and 10% of its area is below 10, state the new values of μ and σ .

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Question 37

Marker use

A recent survey of 1000 people undertaken by a particular puffer jacket company found that 50% of respondents own a puffer jacket made by that company.

- a) Show that the 95% confidence interval for this survey finding is (46.80%, 53.10%), i.e. a margin of error of 3.10%, to 2 decimal places.

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Company management want p to increase so that when another 1000 people are surveyed in a year's time, the margin of error is now 2%. The confidence interval for the new survey will still be 95%.

- b) What value of \hat{p} would be required for this to occur?

/3

Is this a reasonable increase in the proportion of puffer jacket ownership in 1 year?

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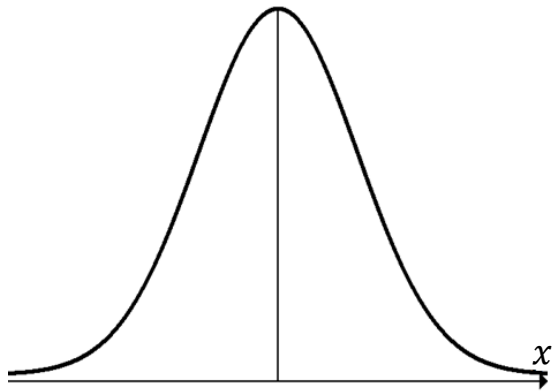
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Total
P5
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Spare Diagram

Question 36 a)



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