2025 ASSESSMENT REPORT
MTS415118 MATHEMATICS SPECIALISED

General Comments

The 2025 Mathematics Specialised Exam contained a mixture of questions. Some questions were designed to be straightforward but there were also some difficult, discriminating questions.

Algebraic skills and basic numerical skills are an important part of the toolkit required in mathematics examinations. There is an expectation that students are proficient and accurate with the algebra skills learnt in previous courses. Whilst there were many students who demonstrated strong algebraic skills there were still some who made far too many basic algebra errors which meant that the problem became almost impossible to solve.

Section A - Criterion 4

Students generally performed well in this section, with around 60% scoring 21 or above. Most candidates understood the requirements for proving by induction and demonstrating convergence, though some struggled to clearly communicate their reasoning. The most challenging questions overall were 4c) and 6b).

Question 1

This question was well done with almost every student earning some marks and around 50% achieving full marks. There were a small number that used n instead of k in the assumption step. As we are proving the statement for all n, assuming it is true for n is not considered good mathematical communication. Some students struggled with algebraic manipulation and resorted to expanding both sides. While full marks were awarded if this was done correctly, it often resulted in unnecessary mistakes and time spent.

Question 2

a. This part was done well with approximately 60% of students achieving full marks. Most solved using the easiest method of $S_n - S_{n-1}$ while a few approached it by summing T_n to n terms.

b. Generally, this was well handled. There were a few that didn't include the expected final statement specifying the values for $a, b,$ and c, thereby losing a $\frac{1}{2}$ mark.

Question 3

a. Students generally understood what was expected for the 'formal' approach; however, quite a large proportion struggled with the rigging. Some incorrectly simplified the expression inside the absolute value, which made the rigging easier thereby requiring careful awarding of marks. A few also failed to include a final connection statement for the proof.

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b. For two marks, markers were looking for recognition that the sequence diverges and a reason, such as an exponential function dominating a polynomial.

Question 4

This question was not handled well overall. Many students spent too much time on the first two parts (worth only one mark each) and too little on the last one.

a. The solution was easily seen if they expanded the $(|x|-1)^2$.

b. When students didn't see the solution to this question immediately, they tended to spend excessive time on it.

c. Very few achieved full marks, as most addressed only one part – either finding the sum or solving for valid solutions – rather than both. Students who didn't consider what they had already found in part b) struggled to identify the valid solutions.

Question 5

a. This part was well done with at least 80% achieving full marks.

b. Students managed this one well too. There were a few students that didn't address the "hence" part of the question, which implied a connection to part a) was required for full marks.

c. More than half the students failed to recognise the initial step in this question and break the limit into

$$\sum_{k=1}^{N-1}\frac{1}{r(r+1)(r+2)}=\sum_{k=1}^{\infty}\frac{1}{r(r+1)(r+2)}-\sum_{k=1}^{N}\frac{1}{r(r+1)(r+2)}$$ to solve for N

Question 6

a. This part was very well done with at least 65% achieving full marks.

b. Many students did not attempt this part, possibly due to time constraints. Those who used simultaneous equations and recognised the relationship with the previous question were often successful in achieving full marks.

Section B – Criterion 5

In general, this section was reasonably well done by most students. Most coped well with the standard questions but often struggled when required to apply their understanding to less familiar contexts.

Question 7

a. Well done with most candidates using a LHS =….=RHS approach.

b. Also well done with the most common approach being to use two entries to set up two equations and then solve. The most common error being a sign error on λ or μ. Strictly speaking, if two matrix entries are used to solve for λ and μ, these values of λ and μ should be substituted into the other entries to verify they are indeed solutions.

The elegant

$$C^2 = C \times C^2 = C \times (4C - 17I) = 4C^2 - 17C = 4(4C - 17I) - 17C = 16C - 68I - 17C = -C - 68I$$

was not sighted.

Question 8

a. Whilst straightforward, this question relied on candidates knowing that a matrix being invertible is equivalent to the existence of an inverse matrix, which is equivalent to the matrix being non-singular, which is equivalent to the determinant, and is **not** 0. (Not all candidates did.)

   Errors seen were giving values for which the determinant **is** 0 and some puzzling arithmetic errors with $4 \times 9 = 35$ being seen more than once.

b. Well done by the majority who knew that $\det(A) = ad - bc$.

Question 9

Was well done by candidates who realized $S = Y \times X \Rightarrow Y^{-1} \times S = X$ and knew how to calculate the inverse. This gave a matrix that was readily identified as a rotation. If X was not correctly calculated, no clear geometric action is apparent (or present). Finding the angle (and direction) of rotation was required as well.

The most common errors were multiplying by Y^{-1} on the right not left, dropping the $-$ with the -46 determinant of Y (this results in a rotation that is `out' by 180°), not correctly swapping the main diagonal elements of Y or negating the non-main diagonal entries of Y when calculating Y^{-1}.

Some candidates set up four equations for $S = Y \times X$ by letting $X = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and equating the entries on the LH and RH sides. This was labour intensive and very error prone. It cannot be stated strongly enough that the reason matrices proliferate in mathematics is that they are an efficient way to process (and solve) linear equations in parallel. Nonetheless some candidates (considerably less than half) who adopted this approach were successful.

Question 10

a. Most candidates were successful. Interestingly, about half found a symmetric form and half a parametric form.

b. A little less well done than the above. Some didn't know how to approach this, others stumbled arithmetically and some did not use their parameter to then give the point of intersection.

c. Had a greater level of success than part b) with most candidates knowing that parallel planes will have the same (or parallel) direction vectors and thus solved $x + y - z = d$ for d using the midpoint. Sadly, the most common error for this question was the inability to recall the midpoint `formula' seen in Grade 9/10.

d. Was a trickier question and few had success (the fact that there are many correct answers stymied some). The two successful approaches employed were to find any second point on the plane Π_2 and use this and the midpoint to generate a line (in the same manner as part a)), or find a vector whose coefficients are multiples of t that always satisfies $x + y - z = 0$

and add this to the midpoint. (*This is equivalent to finding a vector parallel to Π_2 as it has dot product 0 with direction vector of Π_2 and so is perpendicular to the normal and adding an arbitrary multiple (t) of this to the point to generate the line*).

Question 11

a. Successfully done by most candidates. Errors included putting the matrices associated with the reflection and dilation in the wrong order, incorrectly attributing $\theta = \frac{\pi}{3}$ with the line $y = \frac{x}{\sqrt{3}}$ rather that the correct $\theta = \frac{\pi}{6}$ or the ubiquitous arithmetic error.

b. This question was challenging with the most common response being $d = -2$ from the slightly incorrect $-d = \frac{d^2}{2}$ $(\det(M) = -d)$ rather than the correct $|-d| = \frac{d^2}{2} \Rightarrow d = 2$.

c. The arithmetic involved in calculating $\det(M)$ was a source of error for some.

Question 12

a. Done moderately successfully. Most candidates demonstrated proficiency at using row operations to clear out columns and annotate these row operations.

   The most successful approach was to solve to upper triangular form, solve for z, back substitute and solve for y, and for x.

b. A disappointingly incomplete response from most candidates. (Most mentioned one or other of what is below, but rarely both. In addition, a crisp demarcation in terms of α and β between the two cases was often absent.)

   $\alpha = -1 \text{ and } \beta \neq -1$ giving (an inconsistent set of equations) no solution

   and

   $\alpha = -1 \text{ and } \beta = -1$ giving infinitely many solutions lying along a `line'.

   With the line being $(x, y, z) = \left(-\frac{2}{7} + \frac{8}{7}t, -\frac{1}{7} - \frac{3}{7}t, t\right)\ t \in R$.

   Of note is the observation that if the correct $z = \frac{\beta+1}{\alpha+1}$ from part a) is substituted into this the `ugly' expressions for x and y are found without undue pain.

Section C – Criterion 6

Overall, this section was done well but basic algebra skills and manipulation of expressions let candidates down. Often quantum leaps were made in working out without justifying steps. Proofs and "show that" style questions must clearly indicate all steps.

Question 13

Most candidates managed to use the chain rule to find the derivative. Only a few then managed to successfully manipulate the expression to create the "y" that was required.

Question 14

Done very well and most candidates managed full marks. Some failed to show their steps and showed minimal working out.

Question 15

Generally done very well. Marks were lost for not justifying the inflection points or the turning points. Some forgot to draw the graph.

Question 16

a. Done very well. Most used implicit differentiation correctly and rearranged the expression to find the derivative.

b. Not done well. Some candidates recognised that the derivative must be undefined and then made the denominator equal to zero. Very few then substituted this back into C to solve for the coordinates.

Question 17

a. Not done well. Most students managed to gain a mark for attempting to find the derivative. Very few then managed to manipulate the expression correctly by getting a common denominator and simplifying.

b. Generally done well. Most candidates got the correct expression for the area bounded by the curve. They then recognised that this was the result from part a). There were a few errors with substituting in numbers and simplifying the result.

Question 18

a. Generally done well. Most candidates got the correct expression for the volume around the x-axis. Basic algebra skills let some down in their final answer.

b. Done very poorly. Very few recognised the region needed to be split up into parts to correctly calculate the volume. The volume is found by rotating the line $x = 1$ around the y-axis to gain a cylinder. Then subtract from this the volume generated by the parabola around the y-axis. Candidates weren't sure which y-values to use.

Section D – Criterion 7

In this section, it is particularly easy to obtain/check answers using calculators. Candidates needed to clearly indicate that they knew how to solve the problem algebraically. Marks were deducted if excessive calculator use was evident, without appropriate working.

Question 19

a. This question was done reasonably well. Many were able to manipulate the algebra to simplify the fraction $\left(\text{i.e. } \frac{3+2s}{1+2s} = 1 + \frac{2}{1+2s}\right)$.

b. If students realised that they needed to use the double angle formula, then this question was done well. Otherwise, there were some “interesting” algebraic manipulations, or no attempt was made at all.

Question 20

If students made the substitution as suggested, then they were usually able to reach the correct answer.

Common errors:

- some students forgot to change the boundaries when making the substitution and therefore found the wrong value for the definite integral
- some “interesting” algebraic manipulations.

Question 21

This question was very poorly done.

a. Very few students attempted this part. Of those who did, only a very small number were successful. Part marks were awarded for recognising that Integration by Parts was the required technique.

b. If students did not attempt part a) then this was often also not done. A few students were able to use part a) successfully.

Question 22

This question was done extremely well. It was expected that the final answer was given in explicit form as it was very simple to manipulate algebraically.

Question 23

Most students recognised that this was a homogeneous differential equation. Those that substituted $y = Vx$ sooner were far more successful than those that tried to manipulate the algebra before substituting. Many algebraic errors were made by many students leading to disaster!

Question 24

This question was very poorly done and often omitted.

a. Surprisingly, most students were unable to calculate the gradients of the two straight lines and then multiply them together to get the required result.

b. Very few students were successful on this question. Part marks were awarded for recognising the Separable DE and then for using partial fractions. Only two students obtained full marks.

Section E – Criterion 8

Question 25

Generally well attempted by most candidates. Substituting $w = x + yi$ and $\overline{w} = x - yi$, expanding, and then equating real and imaginary terms was implemented by most candidates. Algebraic errors then tended to follow, but 2 out of 3 marks were awarded in these cases.

Question 26

a. Most candidates performed well with this proof. LHS and RHS methods were used in most cases, which was pleasing. Some candidates kept both sides of the proof through their working, and some did not clearly show the final step, with marks being deducted in these instances.

b. Most candidates did not attempt this question and were unfamiliar with the identity involving the diagonals of a parallelogram. Candidates who drew a parallelogram with adjacent sides labelled OP and OQ were awarded 1 mark. If both diagonals of the parallelogram were drawn and labelled $z + w$ and $z - w$ respectively, full marks were awarded, which only occurred for one candidate.

Question 27

a. Most candidates could correctly use the quadratic formula to solve for z^4 in rectangular form. Full marks were only awarded if candidates then clearly indicated that these values were equivalent to $e^{\pm\frac{2i\pi}{3}}$.

   This question could also be done using a geometric series, but most candidates who attempted this method tended to find all eight values of z, rather than the required two values of z^4. Part marks were awarded in these cases.

b. Responses were varied here. Some candidates ignored part a) and solved part b) using geometric series; marks were generously awarded in these cases if executed correctly. Most candidates who completed part a) were cognisant of what was required in part b), but algebraic errors tended to produce incorrect solutions.

Question 28

a. Generally well done. Most candidates used $P(2 + 3i) = 0$ to solve for α and β. Expansion of the polynomial using graphics calculators was accepted, but candidates who then showed no methods of solving in obtaining values of α and β were deducted one mark.

   Algebraic errors occasionally led to incorrect values of α and β, which made part b) less straightforward.

b. If part a) was correct, candidates seemed to solve part b) with relative ease. Identifying the conjugate-root theorem and then expanding the conjugate factors was awarded 1.5 marks. The other quadratic factor could then be obtained either by long division or by equating coefficients. If no working was shown in this step, one mark was deducted.

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Too many candidates showed the fully factorised polynomial but did not state the roots as required. A surprisingly large number of candidates correctly obtained the second quadratic factor as $(z-1)^2$, but then incorrectly found corresponding roots as $z = \pm 1$.

Some candidates combined parts a) and b) to factorise the $P(z)$, and full marks were awarded if executed correctly.

Question 29

a. Responses varied. A small but significant number of candidates drew circles centred at -4 and $3i$, or lines parallel to the axes. Zero marks were awarded in these cases.

   Some candidates indicated in their sketch that the required set corresponds to the perpendicular bisector of -4 and $3i$, without finding the equation of the line. The majority of candidates obtained the line algebraically. In either case, correct axes intercepts were required on the sketch for full marks.

b. Majority of the candidates who attempted this question successfully drew two circles with respective radii of 1 and 2. A surprisingly large number of candidates drew the circles in incorrect locations, which caused issues with the remainder of the sketch.

   The absolute value seemed to confuse candidates, as most obtained arguments of $\frac{\pi}{4}$ *or* $\frac{3\pi}{4}$, but relatively few drew both. Candidates who correctly identified the required region of $Arg(z) \geq \frac{3\pi}{4}$ and $Arg(z) \leq \frac{\pi}{4}$ were in the minority. Again, intercepts were required for full marks, which were awarded to approximately 10% of candidates.

Question 30

a. Students who attempted this question seemed to know exactly how to execute this proof. If attempted, the success rate was high.

b. Most candidates who attempted this question identified that a reciprocal was required in some way, but relatively few could finish the proof with a justifiable step.

c. The vast majority of candidates did not gain any traction with this question, if attempted at all. Approximately 10 candidates solved the equation to obtain $\theta = \frac{\pi}{18}$. A few left their answers at this point, rather than stating that this gives one solution of $x = \operatorname{cosec}\frac{\pi}{18}$.