

Mathematics Methods (MTM415117)

External Assessment Information Sheet

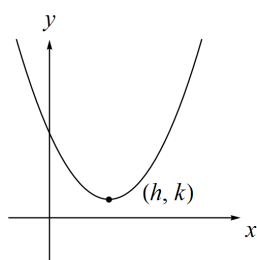
FUNCTION STUDY

Quadratic Formula: If $ax^2 + bx + c = 0$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Graph Shapes:

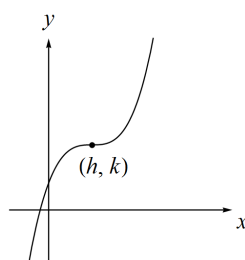
Quadratic

$$y = a(x-h)^2 + k$$



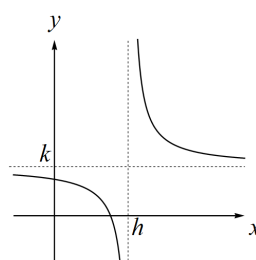
Cubic

$$y = a(x-h)^3 + k$$



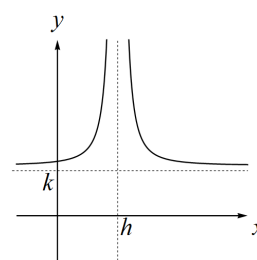
Hyperbola

$$y = \frac{a}{x-h} + k$$



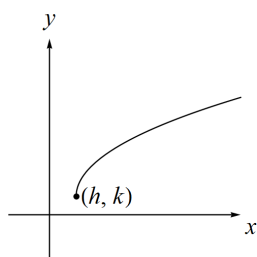
Truncus

$$y = \frac{a}{(x-h)^2} + k$$



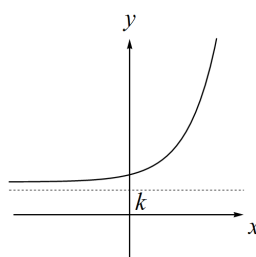
Square Root

$$y = a\sqrt{x-h} + k$$



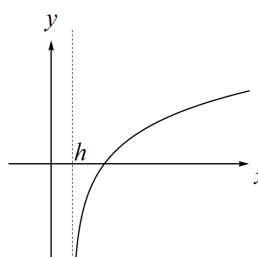
Exponential

$$y = a \times b^{x-h} + k$$



Logarithmic

$$y = a \log_n(x-h) + k$$



Graphical Transformations:

The graph of:

$y = -f(x)$ is a reflection of the graph of $y = f(x)$ in the x axis

$y = f(-x)$ is a reflection of the graph of $y = f(x)$ in the y axis

$y = af(x)$ is a dilation of the graph of $y = f(x)$ by factor a in the direction of the y axis

$y = f(ax)$ is a dilation of the graph of $y = f(x)$ by factor $\frac{1}{a}$ in the direction of the x axis

$y = f(x-h)$ is a translation of the graph of $y = f(x)$ by h units to the right

$y = f(x) + k$ is a translation of the graph of $y = f(x)$ by k units upwards

Index Laws

$$a^x \times a^y = a^{x+y}$$

$$a^x \div a^y = a^{x-y}$$

$$(a^x)^y = a^{x \times y}$$

$$(a^x)^{\frac{1}{y}} = \sqrt[y]{a^x}$$

$$(a^{\frac{x}{y}})^y = \sqrt[y]{a^x}$$

Log Laws

$$\log_a xy = \log_a x + \log_a y$$

$$\log_a \left(\frac{x}{y} \right) = \log_a x - \log_a y$$

$$\log_a x^n = n \log_a x$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Useful log results

Definition: If $y = a^x$ then

$$\log_a y = x$$

$$\log_a 1 = 0$$

$$\ln 1 = 0$$

$$\log_a a = 1$$

$$\ln e = 1$$

Inverse Functions

$$f\{f^{-1}(x)\} = f^{-1}\{f(x)\} = x$$

Binomial Expansion

$$(x+y)^n = {}^nC_0 x^n + {}^nC_1 x^{n-1}y + {}^nC_2 x^{n-2}y^2 + \dots + {}^nC_{n-1} x y^{n-1} + {}^nC_n y^n$$

CIRCULAR FUNCTIONS**Conversion:**

To convert from radians to degrees multiply by $\frac{180}{\pi}$

To convert from degrees to radians multiply by $\frac{\pi}{180}$

Basic Identities:

$$\sin^2 x + \cos^2 x = 1$$

$$\tan x = \frac{\sin x}{\cos x}$$

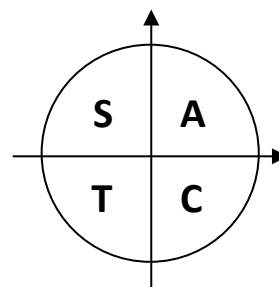
$$\cot x = \frac{1}{\tan x}$$

$$\sec x = \frac{1}{\cos x}$$

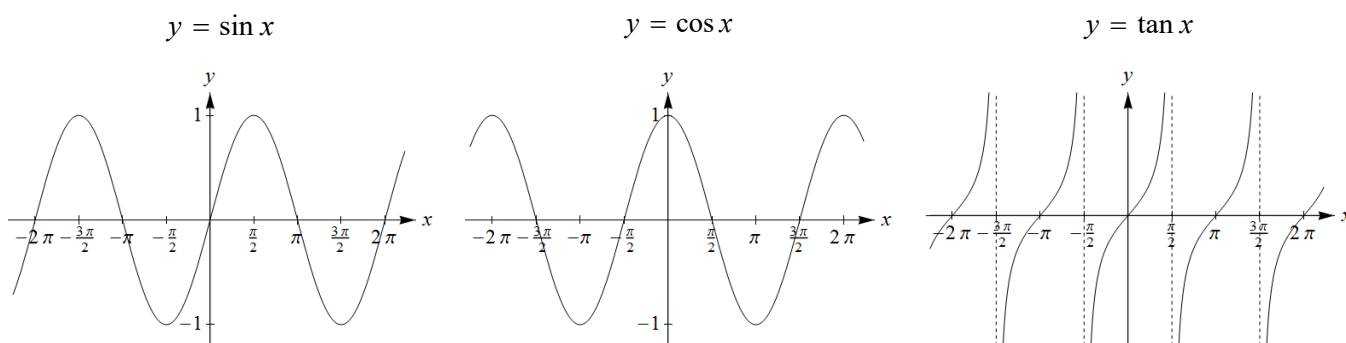
$$\operatorname{cosec} x = \frac{1}{\sin x}$$

Exact Values:

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\sin x$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
$\cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0	1
$\tan x$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	undefined	0	undefined	0

CAST Diagram:

Trigonometric Graphs:



Graphical Transformation:

The graph of

$y = a \sin n(x+b)+c$ or $y = a \cos n(x+b)+c$ has:

amplitude: $|a|$

period: $\frac{2\pi}{|n|}$

phase shift: b (shift of b units to the left)

vertical shift: c units upwards

The graph of

$y = a \tan n(x+b)+c$ has:

dilation: by factor a in the direction of the y axis

period: $\frac{\pi}{|n|}$

phase shift: b (shift of b units to the left)

vertical shift: c units upwards

Trigonometric Equations:

If $\sin x = a$ then $x = n\pi + (-1)^n \arcsin a$, $n \in \mathbb{Z}$

If $\cos x = a$ then $x = 2n\pi \pm \arccos a$, $n \in \mathbb{Z}$

If $\tan x = a$ then $x = n\pi + \arctan a$, $n \in \mathbb{Z}$

CALCULUS

Definition of Derivative: $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

Differentiation and Integration

Differentiation Formulae	
Function	Derivative
x^n	nx^{n-1}
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$ or $\frac{1}{\cos^2 x}$
e^x	e^x
$\log_e x$ or $\ln x$	$\frac{1}{x}$

Integration Formulae	
Function	Integral
a	$ax + c$
x^n	$\frac{x^{n+1}}{n+1} + c$
$(ax+b)^n$	$\frac{(ax+b)^{n+1}}{a(n+1)} + c$
e^x	$e^x + c$
$\frac{1}{x}$	$\ln x + c$
$\sin x$	$-\cos x + c$
$\cos x$	$\sin x + c$

Differentiation Rules

	Function	Rule	Function	Rule
Product Rule	$f(x).g(x)$	$f(x).g'(x) + f'(x).g(x)$	$u.v$	$v.\frac{du}{dx} + u.\frac{dv}{dx}$
Quotient Rule	$\frac{f(x)}{g(x)}$	$\frac{g(x).f'(x) - f(x).g'(x)}{\{g(x)\}^2}$	$\frac{u}{v}$	$v.\frac{du}{dx} - u.\frac{dv}{dx}$ v^2
Chain Rule	$g\{f(x)\}$	$g'\{f(x)\}.f'(x)$	$y = f(u)$ and $u = g(x)$	$\frac{dy}{du} \cdot \frac{du}{dx}$

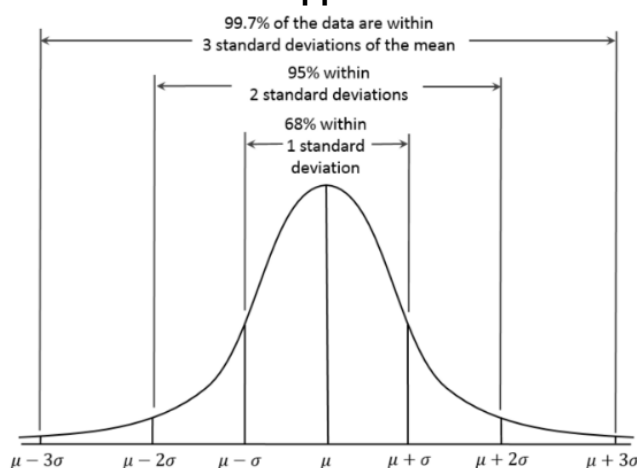
PROBABILITY DISTRIBUTIONS

Combinations: ${}^nC_r = \frac{n!}{r!(n-r)!}$ $n! = n(n-1)(n-2) \times \dots \times 3 \times 2 \times 1$

	Discrete Random Distribution	Binomial Distribution
$\Pr(X=x)$	as table	$\Pr(X=x) = {}^nC_x p^x (1-p)^{n-x}$
Expected Value	$E(X) = \sum (x \cdot \Pr(X=x))$	$\mu = np$
Variance	$\sigma^2 = E(X^2) - [E(X)]^2$	$\sigma^2 = np(1-p)$

Standard Normal: $z = \frac{x - \mu}{\sigma}$

The 68 – 95 – 99.7 approximations to the normal distribution



Sample Proportion

$$\hat{p} = \frac{X}{n}$$

mean: $E(\hat{p}) = p$

standard deviation: $\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$

Confidence intervals

$$\left(\hat{p} - z \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}, \hat{p} + z \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \right)$$

for a 95% confidence interval, $z = 1.96$

Where the Margin of error (M) is:

$$M = z \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$